Exotic Hadrons and the Doubly Heavy Tetraquarks from Lattice QCD

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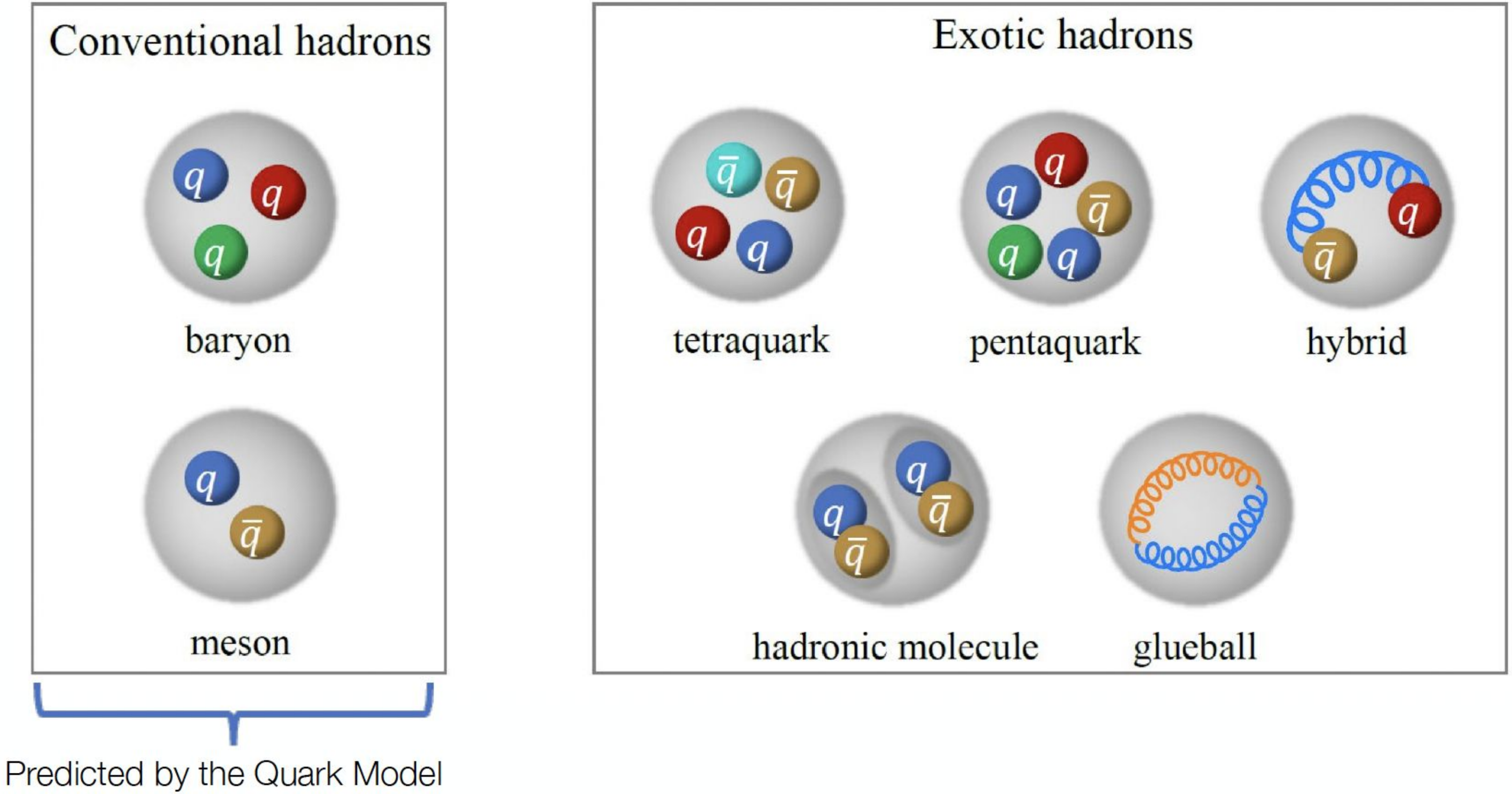
Reviewing 2504.03473 and 2411.06266

ELTE Eötvös Loránd University, Budapest

December 9, 2025

Hadrons


Conventional hadrons
q
q
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baryon
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q̄
meson
Exotic hadrons
q̄
q̄
q
q
tetraquark
q
q
q̄
q
q
pentaquark
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hybrid
q
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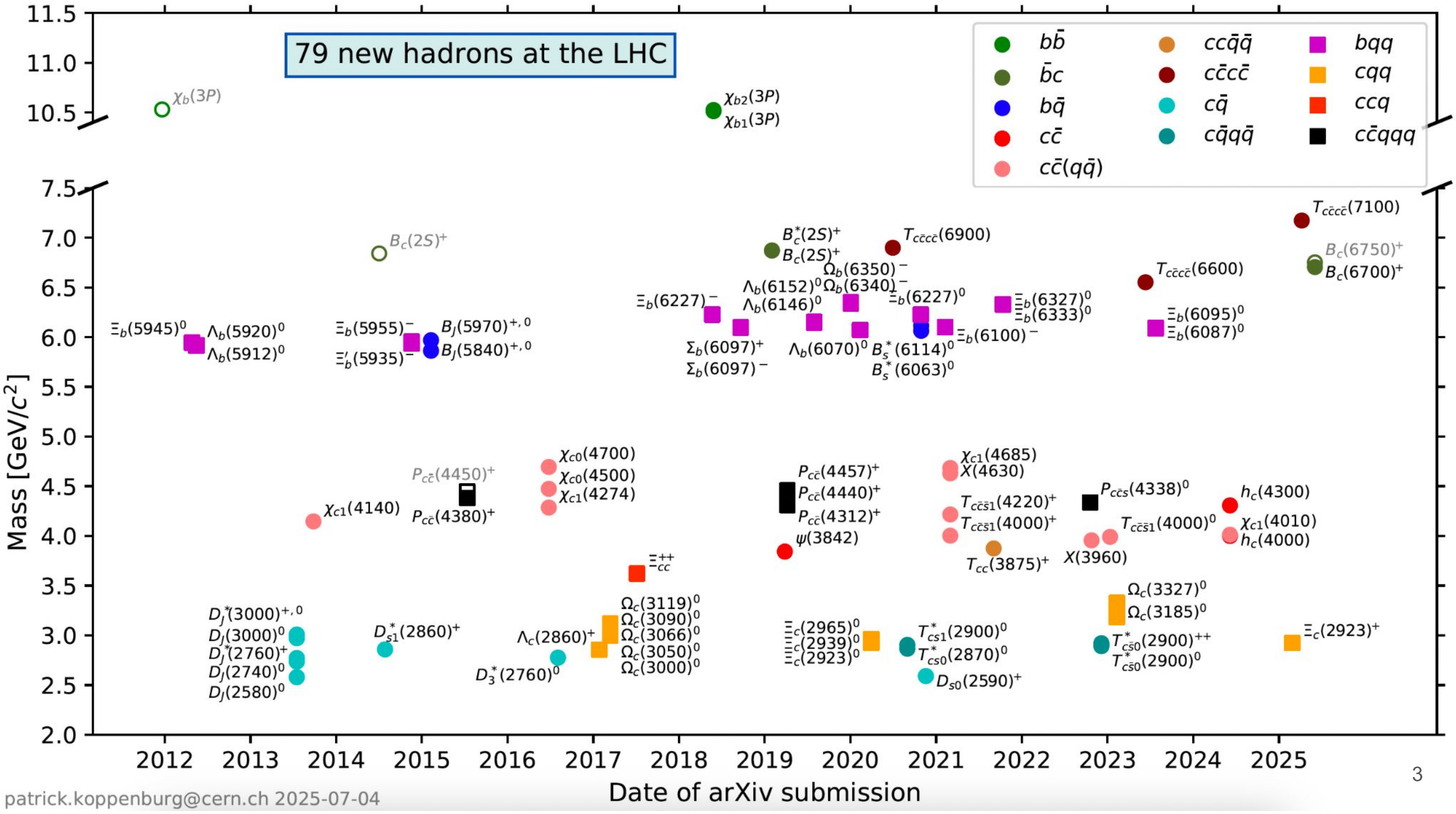
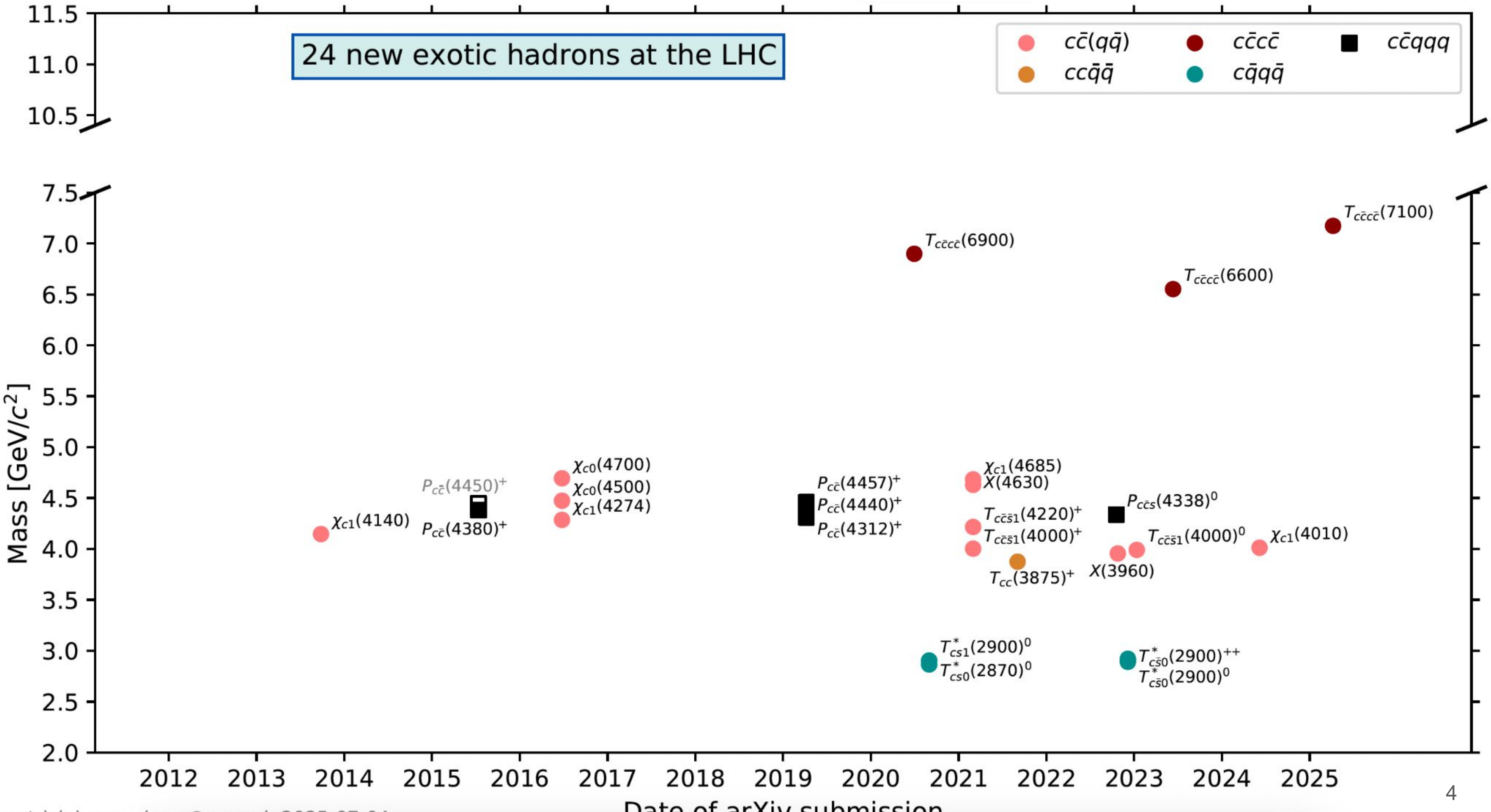
q
q̄
hadronic molecule
glueball
Predicted by the Quark Model


79 new hadrons at the LHC
Mass [GeV/c^2]
Date of arXiv submission
$b\bar{b}$
$\bar{b}c$
$b\bar{q}$
$c\bar{c}$
$c\bar{c}(q\bar{q})$
$cc\bar{q}\bar{q}$
$c\bar{c}c\bar{c}$
$c\bar{q}$
$c\bar{q}q\bar{q}$
bqq
cqq
ccq
$c\bar{c}qqq$
$\chi_b(3P)$
$\chi_{b2}(3P)$
$\chi_{b1}(3P)$
$B_c(2S)^+$
$B_c^*(2S)^+$
$B_c(2S)^+$
$T_{c\bar{c}c\bar{c}}(6900)$
$T_{c\bar{c}c\bar{c}}(6600)$
$T_{c\bar{c}c\bar{c}}(7100)$
$B_c(6750)^+$
$B_c(6700)^+$
$\Xi_b(5945)^0$
$\Lambda_b(5920)^0$
$\Lambda_b(5912)^0$
$\Xi_b(5955)^-$
$\Xi_b'(5935)^-$
$B_J(5970)^{+,0}$
$B_J(5840)^{+,0}$
$\Xi_b(6227)^-$
$\Sigma_b(6097)^+$
$\Sigma_b(6097)^-$
$\Lambda_b(6152)^0$
$\Lambda_b(6146)^0$
$\Lambda_b(6070)^0$
$\Omega_b(6350)^-$
$\Omega_b(6340)^-$
$\Xi_b(6227)^0$
$B_s^*(6114)^0$
$B_s^*(6063)^0$
$\Xi_b(6100)^-$
$\Xi_b(6327)^0$
$\Xi_b(6333)^0$
$\Xi_b(6095)^0$
$\Xi_b(6087)^0$
$\chi_{c1}(4140)$
$P_{c\bar{c}}(4450)^+$
$P_{c\bar{c}}(4380)^+$
$\chi_{c0}(4700)$
$\chi_{c0}(4500)$
$\chi_{c1}(4274)$
$P_{c\bar{c}}(4457)^+$
$P_{c\bar{c}}(4440)^+$
$P_{c\bar{c}}(4312)^+$
$\psi(3842)$
$\chi_{c1}(4685)$
$X(4630)$
$T_{c\bar{c}\bar{s}1}(4220)^+$
$T_{c\bar{c}\bar{s}1}(4000)^+$
$T_{cc}(3875)^+$
$P_{c\bar{c}s}(4338)^0$
$T_{c\bar{c}\bar{s}1}(4000)^0$
$X(3960)$
$h_c(4300)$
$\chi_{c1}(4010)$
$h_c(4000)$
Ξ_{cc}^{++}
$D_J^*(3000)^{+,0}$
$D_J(3000)^0$
$D_J^*(2760)^+$
$D_J(2740)^0$
$D_J(2580)^0$
$D_{s1}^*(2860)^+$
$\Lambda_c(2860)^+$
$D_3^*(2760)^0$
$\Omega_c(3119)^0$
$\Omega_c(3090)^0$
$\Omega_c(3066)^0$
$\Omega_c(3050)^0$
$\Omega_c(3000)^0$
$\Xi_c(2965)^0$
$\Xi_c(2939)^0$
$\Xi_c(2923)^0$
$T_{cs1}^*(2900)^0$
$T_{cs0}^*(2870)^0$
$D_{s0}(2590)^+$
$\Omega_c(3327)^0$
$\Omega_c(3185)^0$
$T_{c\bar{s}0}^*(2900)^{++}$
$T_{c\bar{s}0}^*(2900)^0$
$\Xi_c(2923)^+$
patrick.koppenburg@cern.ch 2025-07-04


24 new exotic hadrons at the LHC
$c\bar{c}(q\bar{q})$
$cc\bar{q}\bar{q}$
$c\bar{c}c\bar{c}$
$c\bar{q}q\bar{q}$
$c\bar{c}qqq$
$T_{c\bar{c}c\bar{c}}(7100)$
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$T^*_{cs1}(2900)^0$
$T^*_{cs0}(2870)^0$
$T^*_{c\bar{s}0}(2900)^{++}$
$T^*_{c\bar{s}0}(2900)^0$
Mass [GeV/c^2]
Date of arXiv submission
11.5
11.0
10.5
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
2012
2013
2014
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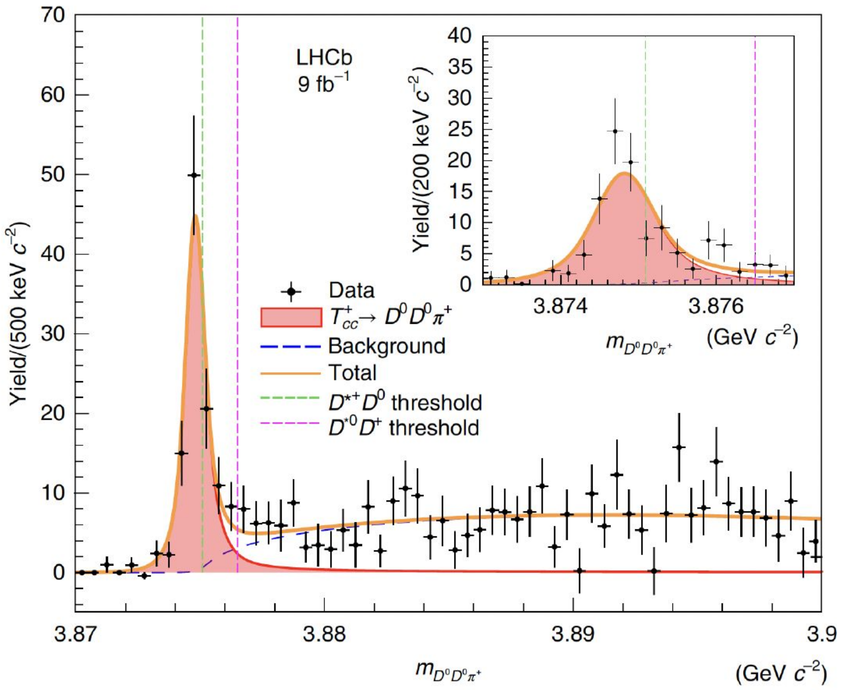
2021
2022
2023
2024
2025

LHCb : Double-charm tetraquark $T_{cc}^{+}\ [\,cc\bar{u}\bar{d}\,]$

2021: Signal in $D^0D^0\pi^+$ just 0.4 MeV below D^0D^{*+} threshold. [1] [2]



$$\delta m_{\text{pole}} = m_{T_{cc}^{+}} - \left(m_{D^{*+}} + m_{D^0}\right),$$

$$\begin{aligned}\delta m_{\text{pole}} &= -360 \pm 40\,^{+4}_{-0}\ \text{keV}/c^2\,,\\ \Gamma_{\text{pole}} &= 48 \pm 2\,^{+0}_{-14}\,\text{keV}\end{aligned}$$

From different models expected:

$$I(J^P) = 0(1^+)$$

Does this state exist in QCD?

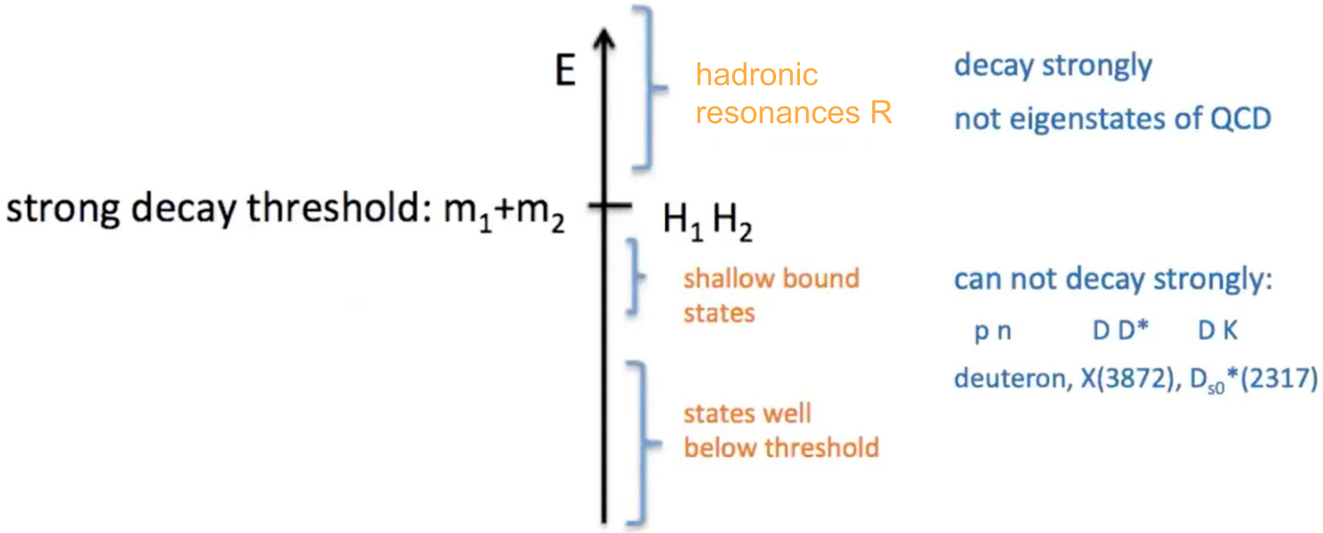
- What is its mass?

- Quantum numbers?

[1] R. Aaij *et al.* (LHCb Collaboration), Nature Physics **18**, 751 (2022). arXiv:2109.01038v4.

[2] R. Aaij *et al.* (LHCb Collaboration), Nature Communications **13**, 3351 (2022). arXiv:2109.01056v4.

Classification of hadron states



Most of the hadrons are strongly decay resonances.

*They need to be inferred from scattering in the experiment and/or the lattice.

Hadrons from lattice QCD

- QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$

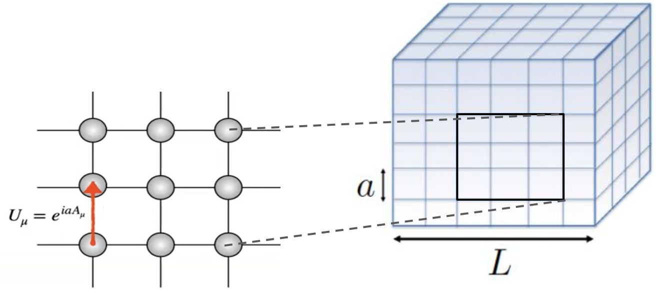
Confinement

Hadron spectrum

$\vec{x}, t$ (Minkovsky) → $\vec{x}, -it_E$ (Euclidean)

- Lattice QCD is a first principle numerical approach to the strong interactions

$$\langle O(x)\, O(0) \rangle = \frac{1}{Z} \int \mathcal{D}A\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\; O(t)\, O(0)\, e^{-S_E(\bar{\psi},\psi,A_\mu)}$$



Correlator functions: main quantity extracted for study the spectroscopy

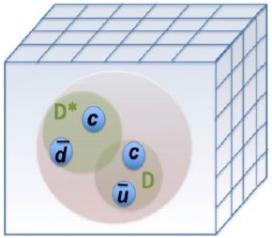
$\sum_n |n\rangle\langle n|$

$$C^{2pt}_{ij}(t) = \langle 0|O_i(t) O^\dagger_j(0)|0\rangle = \sum_n \langle 0|O_i|n\rangle e^{-E_n t_E} \langle n|O^\dagger_j|0\rangle = \sum_n Z^n_i Z^{n*}_j\, e^{-E_n t} \xrightarrow{t \to \infty} A_0 e^{-E_0 t}$$ **(ground state)**

Interpolators

Our basis of operators is chosen good enough i.e. span over all possible configurations and such that our approach reproduces the quantum numbers and describe the energies of hadrons

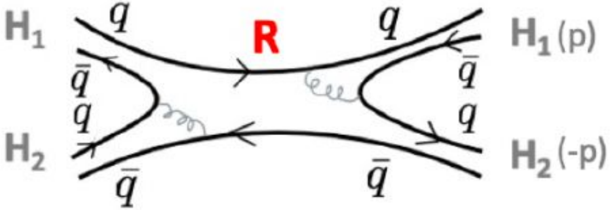
T^+_{cc}

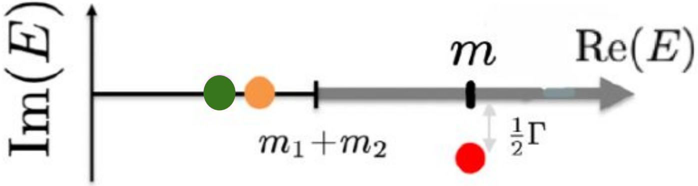


- Variational techniques
  Generalized Eigenvalue Problem (GEVP)

The spectrum

Hadron states near the threshold





Virtual bound st.
p = -i|p|
Riemann sheet II

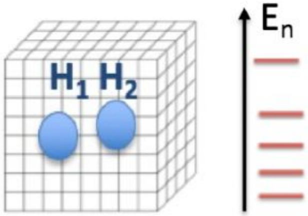
Bound st.
p = i|p|
Riemann sheet I

Resonance
Riemann sheet II

$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

➢ Scattering amplitude T(E) from the lattice QCD

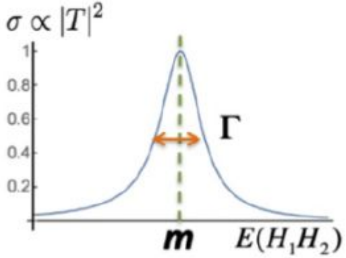


real E   for real E   for complex E

$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

analytic contin.
to complex E



Lattice Setup

Simulation details:

- $N_f = 2 + 1$ CLS ensambles.
- $m_\pi \simeq 280\ MeV$
- Spatial lattice extent $N_L = 24, 32$
- a $\simeq 0.086$ fm

We employ two heavy quark masses m_Q for the system $QQ\bar{u}\bar{d}$ with $J^P = 1^+$, $I = 0$:

m$_Q \simeq m_c : m_D \simeq 1.931\, GeV \quad m_{D*} \simeq 2.051\, GeV$

m$_Q \simeq m_{“b”} : m_{“B”} \simeq 4.042\, GeV \quad m_{“B*”} \simeq 4.075\, GeV$

The heavier the quark mass is close the b quark mass.

T_{cc}^{+} isospin-0 : $M(\vec{p}_1)M(\vec{p}_2)$ and $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ interpolators

- Total momenta : $P = 0$ (irrep T_1^+), $P = 1$ (irrep A_2)
- Color singlet Meson-Meson interpolators $[\bar{u}c]_{1_c}[\bar{d}c]_{1_c}$

$$O^{D^{(*)}D^*}(\vec{p_1},\vec{p_2}) = D^{(*)}(\vec{p_1})D^*(\vec{p_2}) = \sum_{\vec{x}_1}\bar{u}^a_A(\Gamma_1)_{AB}e^{i\vec{p}_1\vec{x}_1}c^a_B \quad \sum_{\vec{x}_2}\bar{d}^b_C(\Gamma_2)_{CD}e^{i\vec{p}_2\vec{x}_2}c^b_D - \{u \leftrightarrow d\}, \qquad \mathbf{N_v^{MM} = 60}$$

Several operators

- **Diquark-antidiquark interpolators** $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$

$$O^{4q}(\vec{P}) = \sum_{\vec{x}}\epsilon_{abc}c^b_A(\vec{x})(C\gamma_i)_{AB}c^c_B(\vec{x}) \ \ \epsilon_{ade}\bar{u}^d_C(\vec{x})(C\gamma_5)_{CD}\bar{d}^e_D(\vec{x})e^{i\vec{P}\vec{x}}, \qquad \mathbf{N_v^{4q} = 45}$$

- Distillation method - all quarks fields are smeared $\longrightarrow$ spectral decomposition

$$q^b_A(\vec{x}) = \sum_{i=1}^{N_v} v_b^{(i)}(\vec{x})v_{\bar{b}}^{(i)\dagger}(\vec{y})q^{\bar{b}}_A(\vec{y}),$$

N_v is the **number of eigenvectors**

T_{cc}^{+} isospin-0 : $M(\vec{p}_1)M(\vec{p}_2)$ and $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ interpolators

Tensors (in distillation space) needed to compute correlators:

- MM : single meson kernel

$$\phi^{ij}(\vec{p}) = \sum_{\vec{x}} \sum_{c} v_c^{(i)}(\vec{x}) v_c^{(j)}(\vec{x}) e^{i\vec{p}\vec{x}}.$$

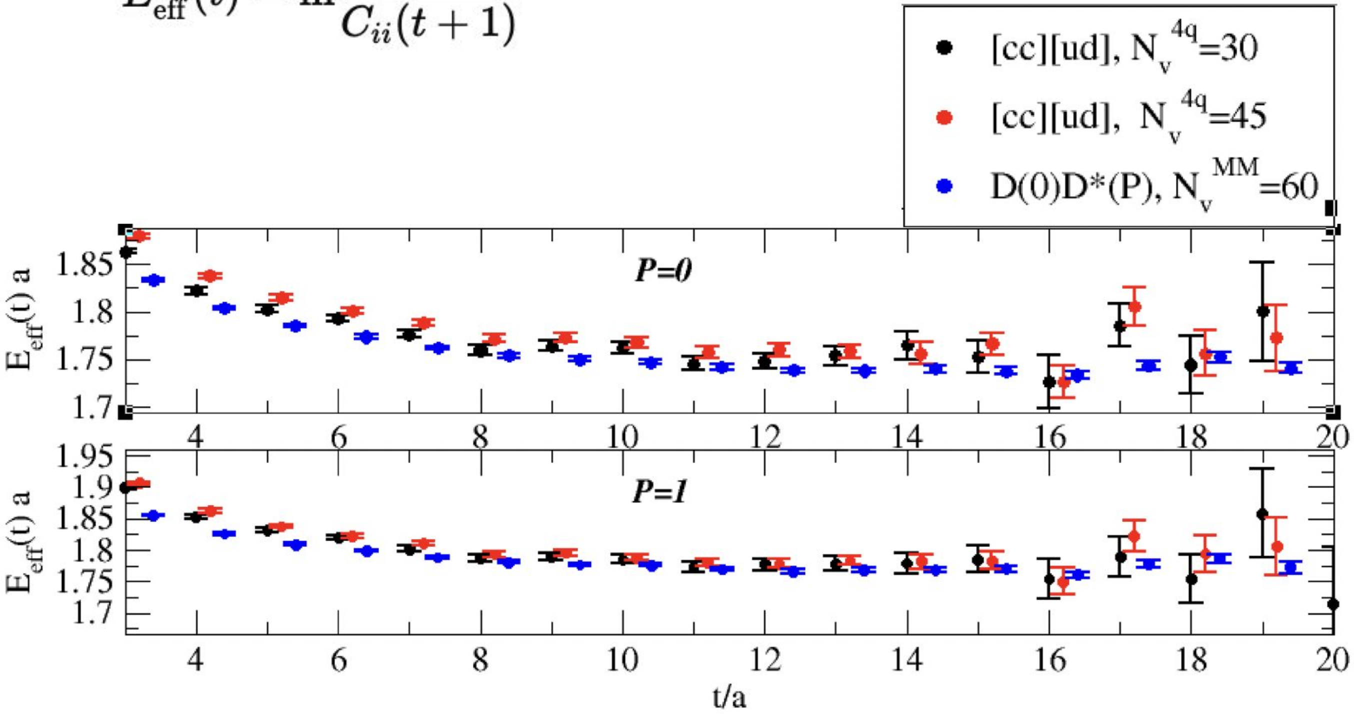
- $4q$: kernel for compact tetraquark $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$

$$\phi^{jklm}(\vec{p}) = \sum_{\vec{x}} \sum_{abcde} \epsilon_{abc}\epsilon_{ade} v_c^{(j)}(\vec{x}) v_c^{(k)}(\vec{x}) v_c^{(l)}(\vec{x})^{\dagger} v_c^{(m)}(\vec{x})^{\dagger} e^{i\vec{p}\vec{x}}.$$

- Costly summation over distillation indices for $4q \implies$ we employ $N_v^{4q} < N_v^{MM}$.

Effective masses of the diagonal correlators, dependence on N_v

$$E^{(i)}_{\text{eff}}(t) = \ln \frac{C_{ii}(t)}{C_{ii}(t+1)}$$



Effective energies of the diagonal correlators and the GEVP ground state

$$O^{MM}: \begin{array}{l} D(0)D^*(0) \\ D(1)D^*(-1)|_{l=0} \\ D(1)D^*(-1)|_{l=2} \\ D^*(0)D^*(0) \end{array}$$

$$O^{4q}: [cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$$

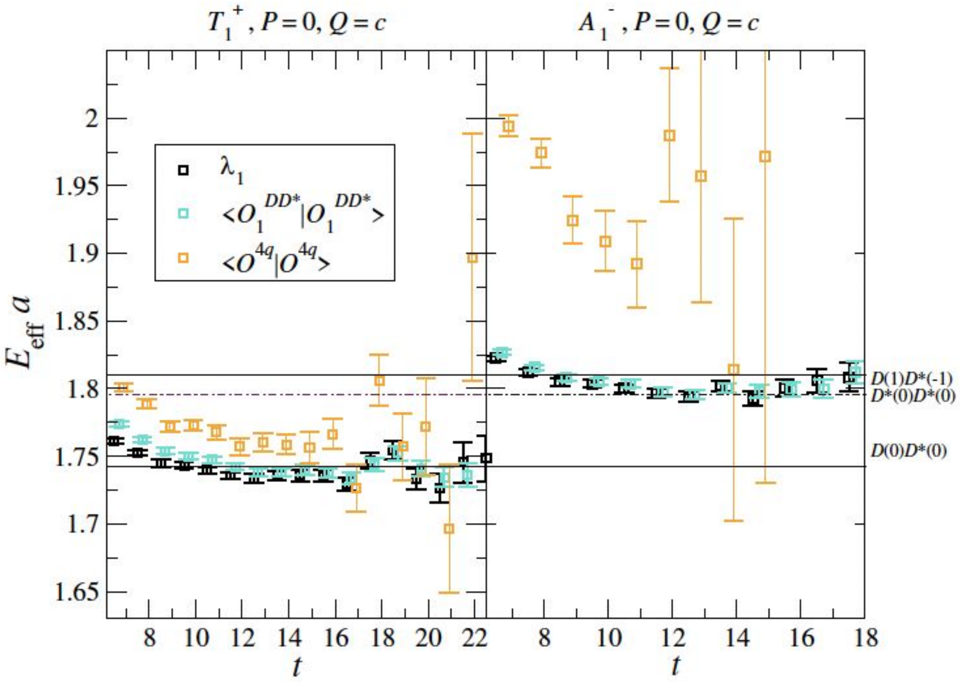
Solve the GEVP equation for λ and u

$$C(t)u^{(n)}(t) = \lambda^{(n)}(t, t_0)C(t_0)u^{(n)}(t)$$

Where t0 is the reference time after only N lowest eigenstates signi

Extractaction of En from

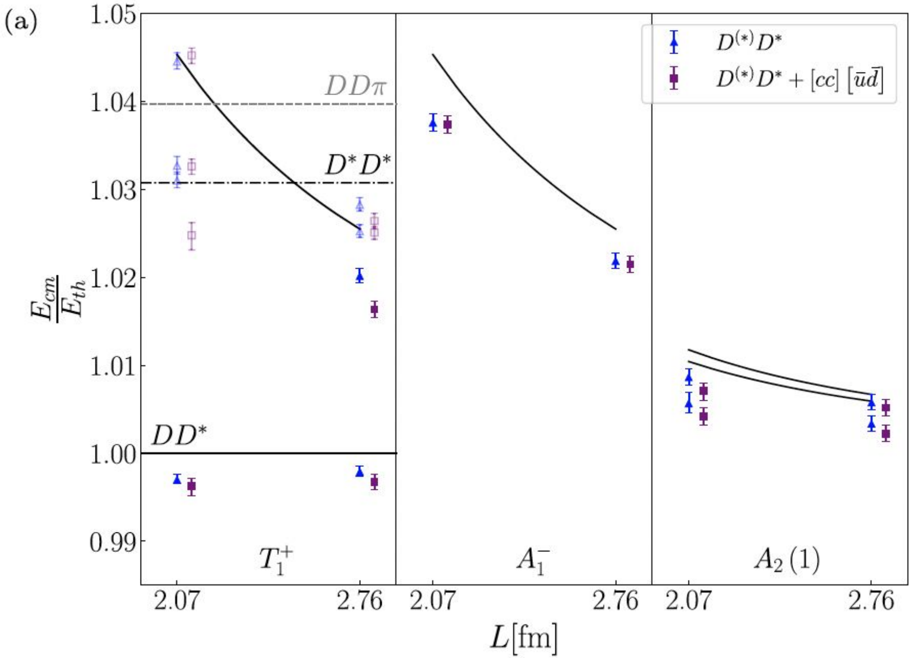
$$\lambda^{(n)}(t, t_0) \simeq Ae^{-E_n t}$$



Finite-volume energy spectrum of Tcc

O^{MM} : $D(0)D^*(0)$
$D(1)D^*(-1)|_{l=0}$
$D(1)D^*(-1)|_{l=2}$
$D^*(0)D^*(0)$

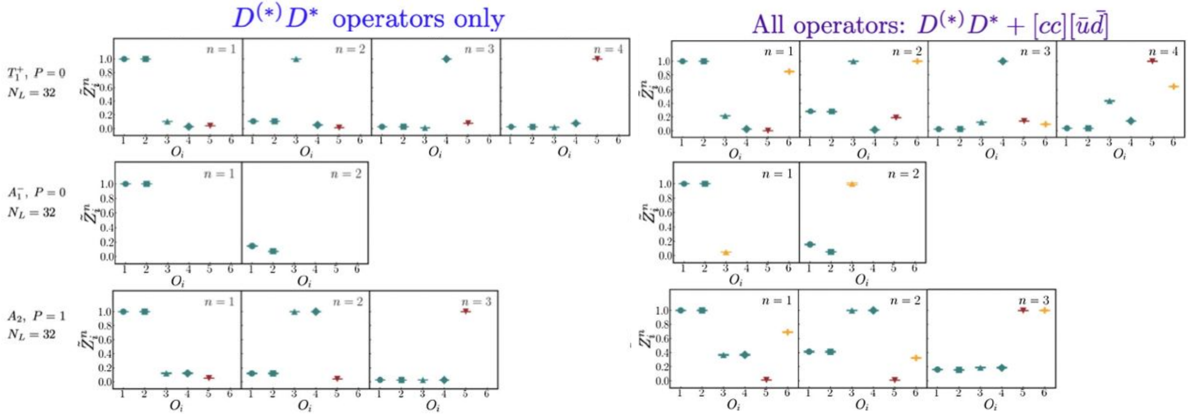
O^{4q} : $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$



$E_{th} = m_D + m_{D^*}$

- Overlap factors $Z_i^n = \langle n|O_i\rangle$

$\tilde{Z}_i^n \equiv Z_i^n / \max_{n'} Z_i^{n'}$



DD* Scattering amplitude with only meson-meson operators

Lüsher method:

$$t_l^{(J)} = \frac{E_{cm}}{2}\frac{1}{p\cot\delta_l^{(J)} - ip},$$

$$p\cot\delta_{l=0}^{(J=1)} = \frac{1}{a_0^{(1)}} + \frac{1}{2}r_0^{(1)}p^2$$

$$m_c^{(h)}:\ a_0^{(1)} = 1.04(29)\ \text{fm},\ r_0^{(1)} = 0.96(^{+0.18}_{-0.20})\ \text{fm}.$$

From the lattice

Relation between E and $\delta(E)$

$$p\cot\delta(p) = \frac{2}{\sqrt{\pi}L}Z_{00}(1,(\tfrac{pL}{2\pi})^2$$

$$E = \sqrt{m_D^2 + p^2} + \sqrt{m_{D^*}^2 + p^2}$$

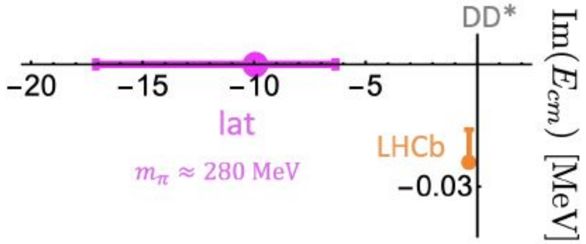
- t has a pole when $-i|p_B|*i = \frac{1}{a_0} - \frac{1}{2}r_0|p_B|^2$

  $p = -i|p_B| \qquad ip_B = p_B\cot\delta(p_B)\ ,$

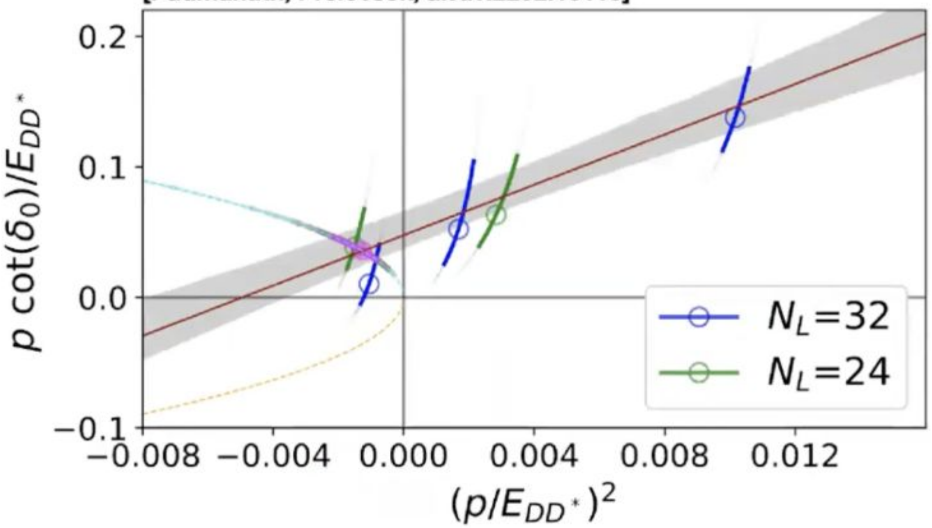
  at energy $E_{cm}^p = (m_D^2 - |p_B|^2)^{1/2} + (m_{D^*}^2 - |p_B|^2)^{1/2}$

which corresponds to a Virtual bound state at bind. energy $\delta m_{T_{cc}} = E_{cm}^p - m_D - m_{D^*} = -9.9^{+3.6}_{-7.1}$ MeV

$$\delta m_{T_{cc}} = \text{Re}(E_{cm}) - m_{D^0} - m_{D^{*+}}\ [\text{MeV}]$$



[Padmanath, Prelovsek, arXiv:2202.10110]

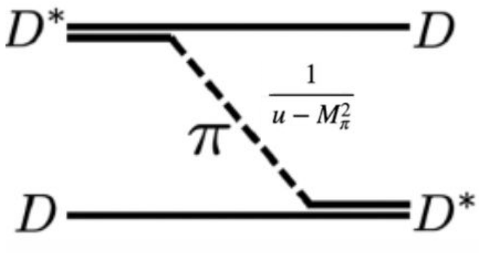


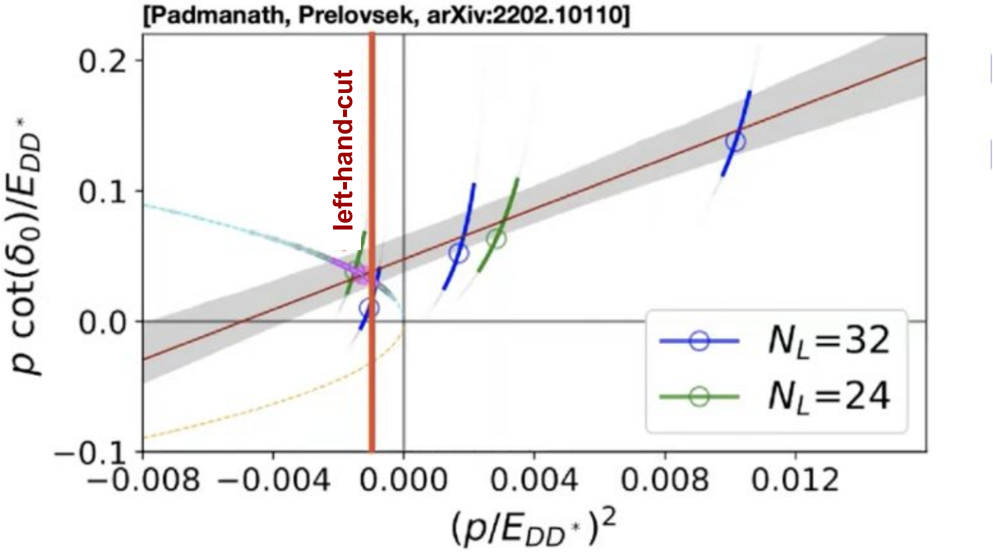
DD* Scattering amplitude with only meson-meson operators

Lhc Just 8 MeV below the threshold: $p^2_{lhc} \approx -\mu^2_\pi/4 \simeq -10^{-3}E^2_{th}$

Due to one-pion-exchange (OPE)

in the u-channel.

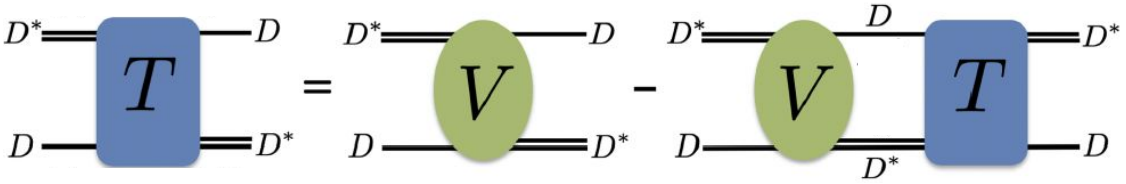




Additionally, is the Tcc basis with two-mesons sufficient?

DD* Scattering with EFT

➢ The Lippmann-Schwinger equation $T = V - V\mathcal{G}T$

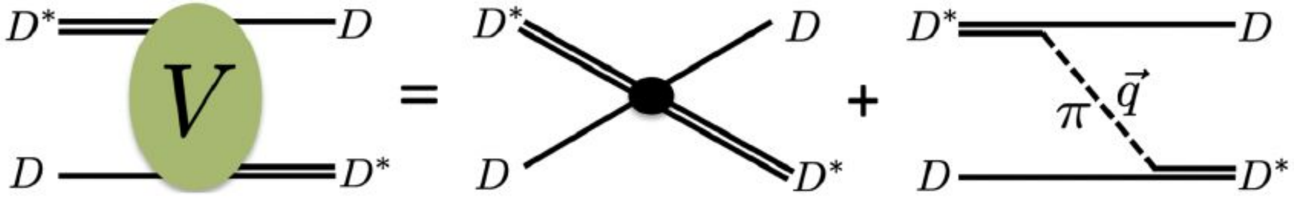


➢ Effective potential V derived derived from EFT to parametrize DD* interaction

$$V\left(\vec{p},\vec{\epsilon};\vec{p}',\vec{\epsilon}\,'\right) = \left[\left(2c_0^s + 2c_2^s(\vec{p}^2 + \vec{p}\,'^{\,2})\right)\left(\vec{\epsilon}\cdot\vec{\epsilon}\,'^*\right) + 2c_2^p\left(\vec{p}\cdot\vec{\epsilon}\right)\left(\vec{p}\,'\cdot\vec{\epsilon}\,'^*\right) + 3\left(\frac{g}{2f_\pi}\right)^2\frac{\left(\vec{\epsilon}\cdot\vec{q}\right)\left(\vec{\epsilon}\,'^*\cdot\vec{q}\right)}{q^2 - m_\pi^2}\right]\exp\left(-\frac{|\vec{p}|^n + |\vec{p}\,'|^n}{\Lambda^n}\right)$$

c_0^s, c_2^s, c_2^p low energy constants are parameters to be fitted to the FV lattice energies

➢ One-pion-exchange from the potential V



in nonrelativistic regime: **projected to various lattice irreps Λ:**

$$\textbf{poles}: \det\left(\mathcal{G}^{-1} + V\right) = 0 \longrightarrow \det\left(H - p^0 I\right) = 0 \longrightarrow \det\left(H^{\Gamma} - p^{0,\Gamma} I\right) = 0$$

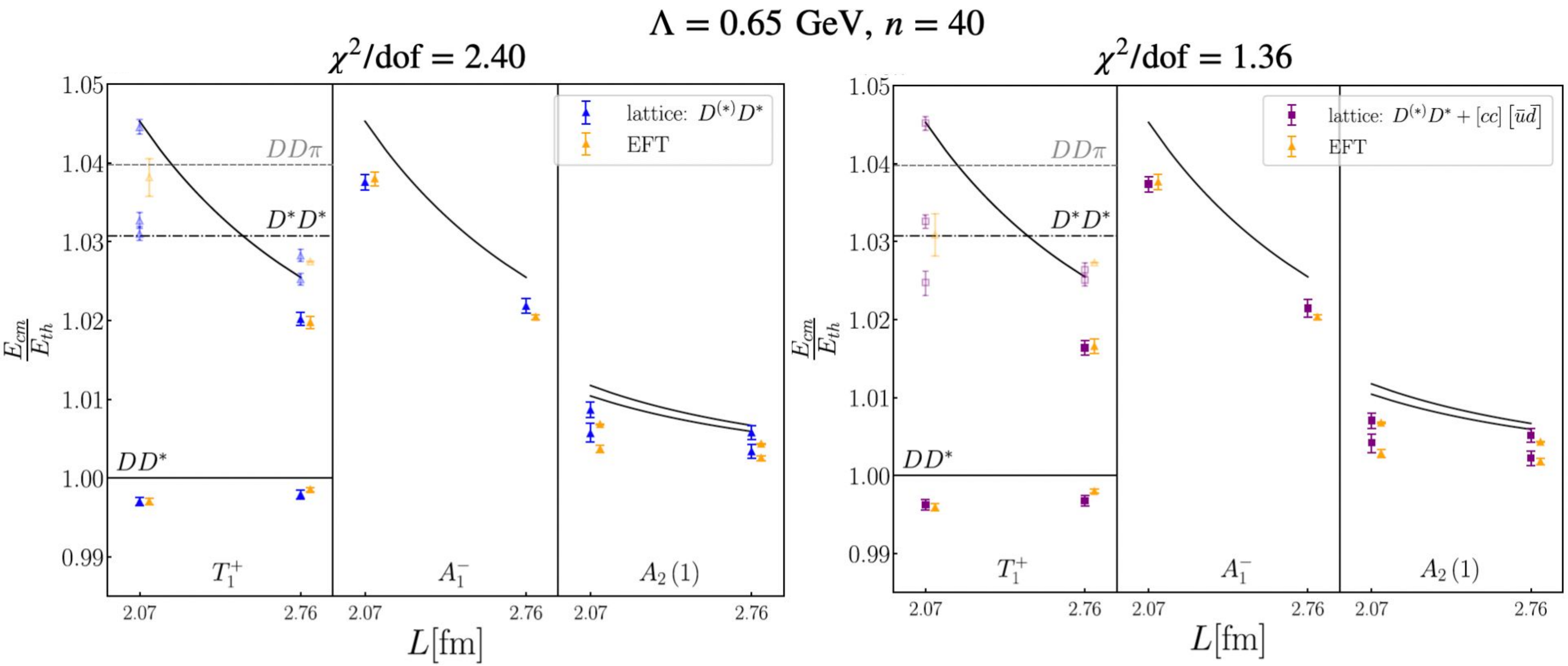
➢ Plane wave basis $|D(\vec{p}_D);\ D^*(\vec{p}_{D^*}, \vec{\epsilon}^{\,r})\rangle_{lat}, \quad \vec{P} = \vec{p}_D + \vec{p}_{D^*}$

$$\vec{p}_{D^{(*)}} = \tfrac{2\pi}{L}\vec{n}_{D^{(*)}},\ \vec{n}_{D^{(*)}} \in Z^3;\ r = x, y, z,$$

$$H = \frac{p^2}{2m_r} I + \frac{1}{L^3} V.$$

$$|D(\vec{k});\ D^*(-\vec{k}, \vec{\epsilon}^{\,r})\rangle_{cm}.$$

Fit of the low energy constants



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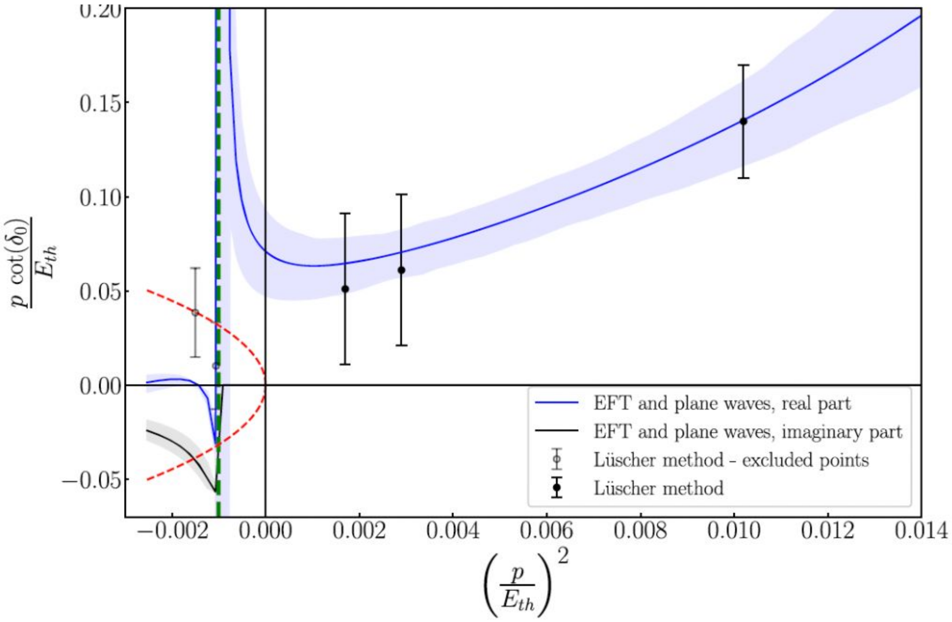
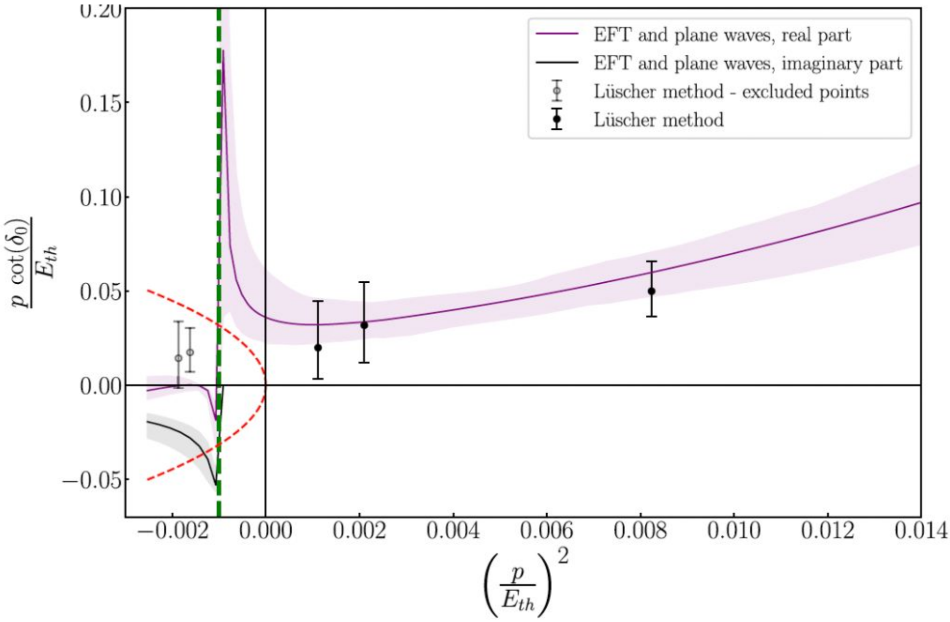
Comparison of the DD* scattering phase shifts

with $M(\vec{p}_1)M(\vec{p}_2)$



with $M(\vec{p}_1)M(\vec{p}_2)$ and $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$



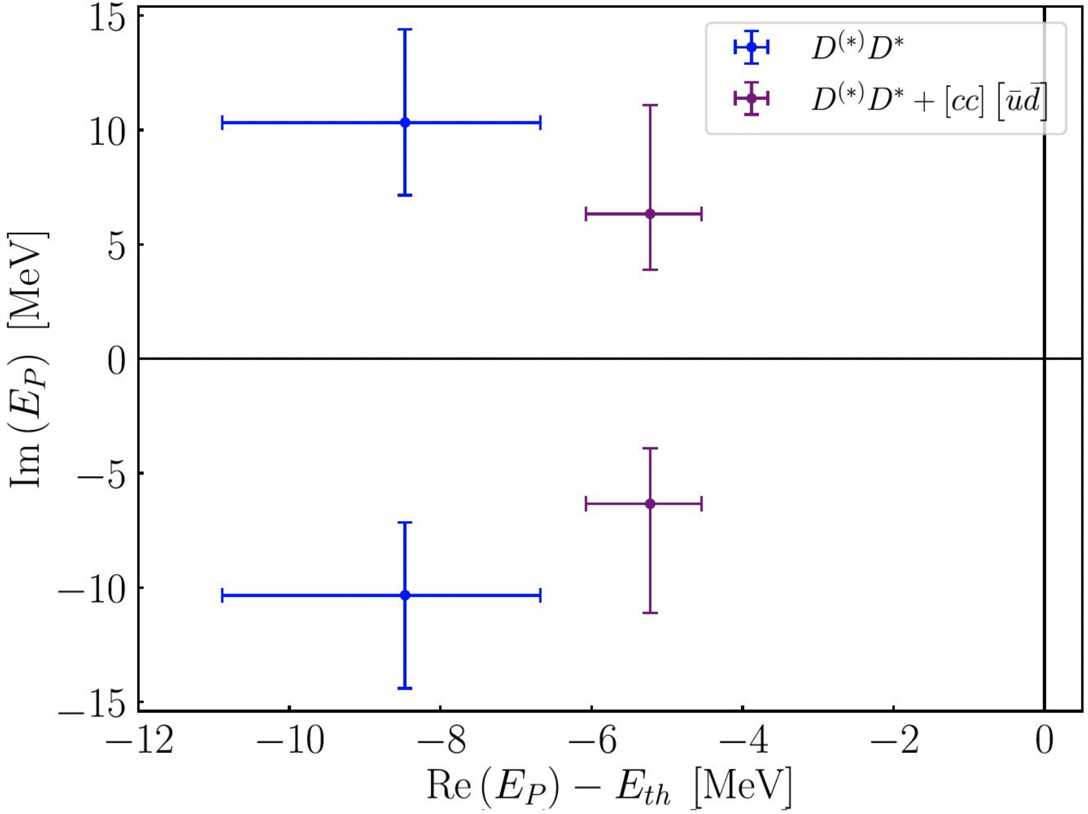
2504.03473

Tcc poles in the scattering amplitude

$$\mathrm{Re}\,(E_P) - E_{th} = -8.5^{+1.8}_{-2.4}\ \mathrm{MeV}$$
$$\mathrm{Im}\,(E_P) = -10.3^{+3.2}_{-4.1}\ \mathrm{MeV}$$

$$\mathrm{Re}\,(E_P) - E_{th} = -5.2^{+0.7}_{-0.8}\ \mathrm{MeV}$$
$$\mathrm{Im}\,(E_P) = -6.3^{+2.4}_{-4.8}\ \mathrm{MeV}$$

+ complex conjugate poles

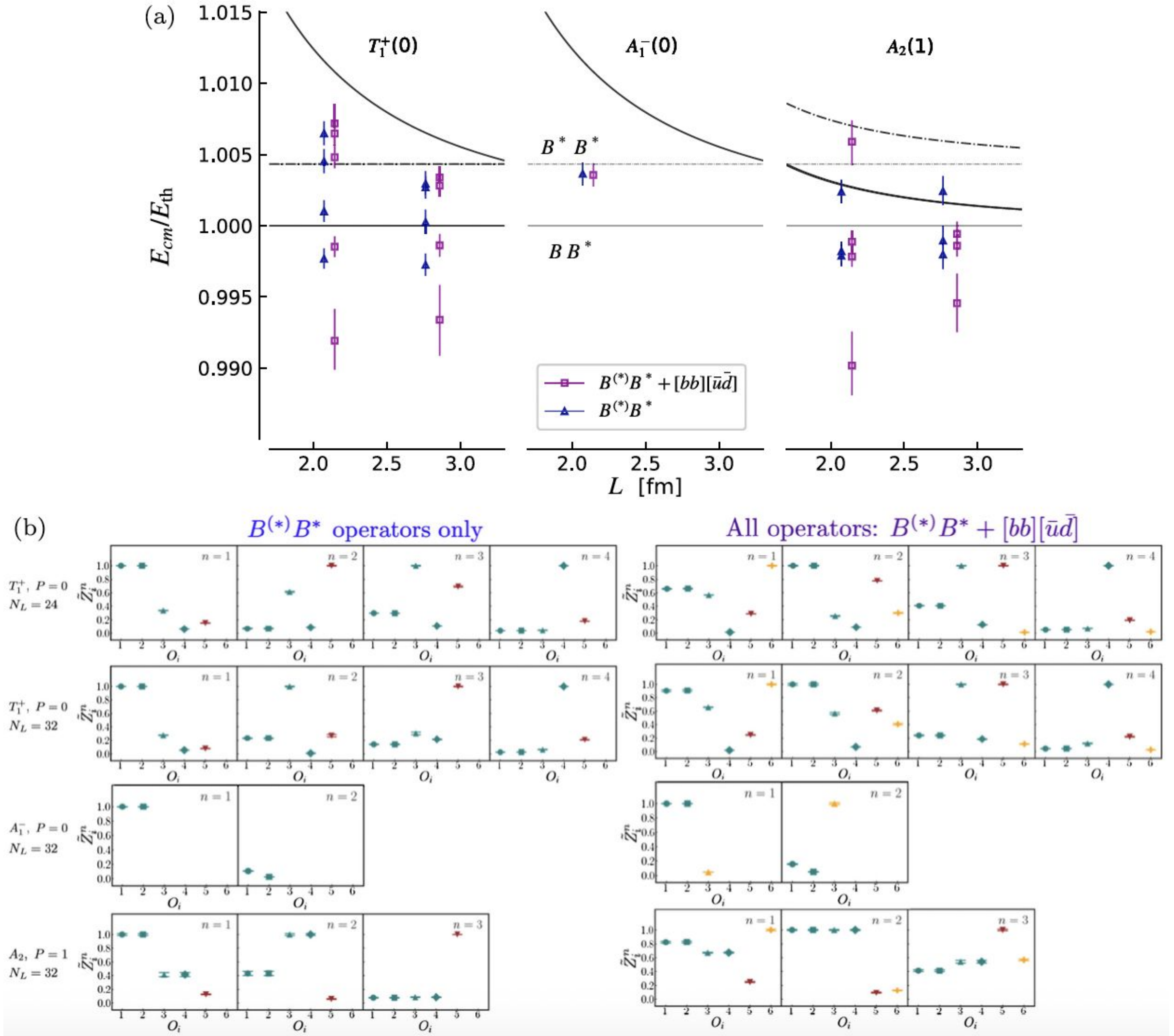


- ➢ Tcc is a subthreshold resonance

2504.03473

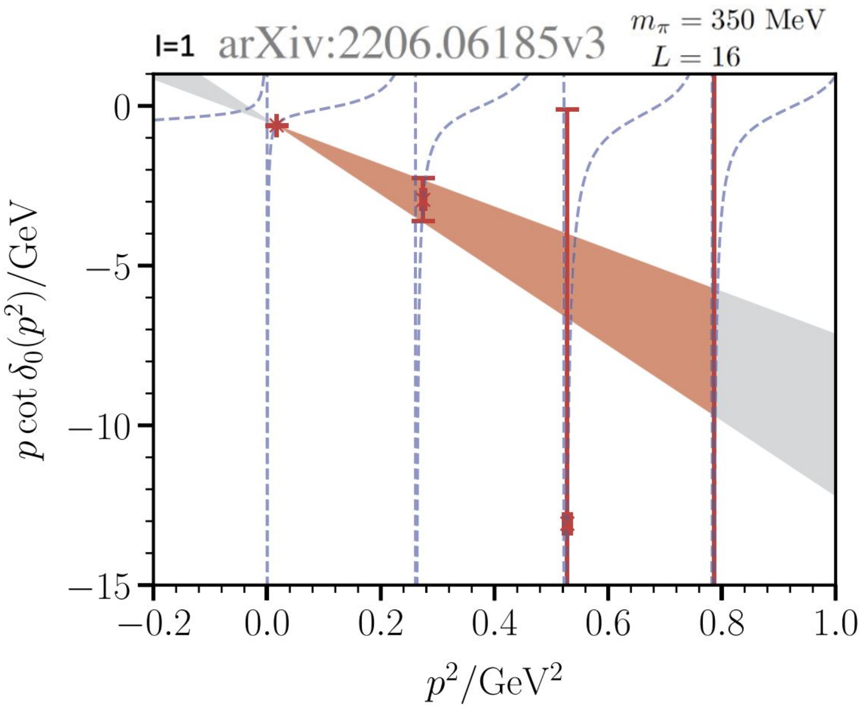
Finite-volume energy spectrum of the Doubly Bottom Tetraquark **Tbb**

- Tbb ground state shows a significant energy shift of ~100MeV when additionally diquark-antidiquark operators are considered

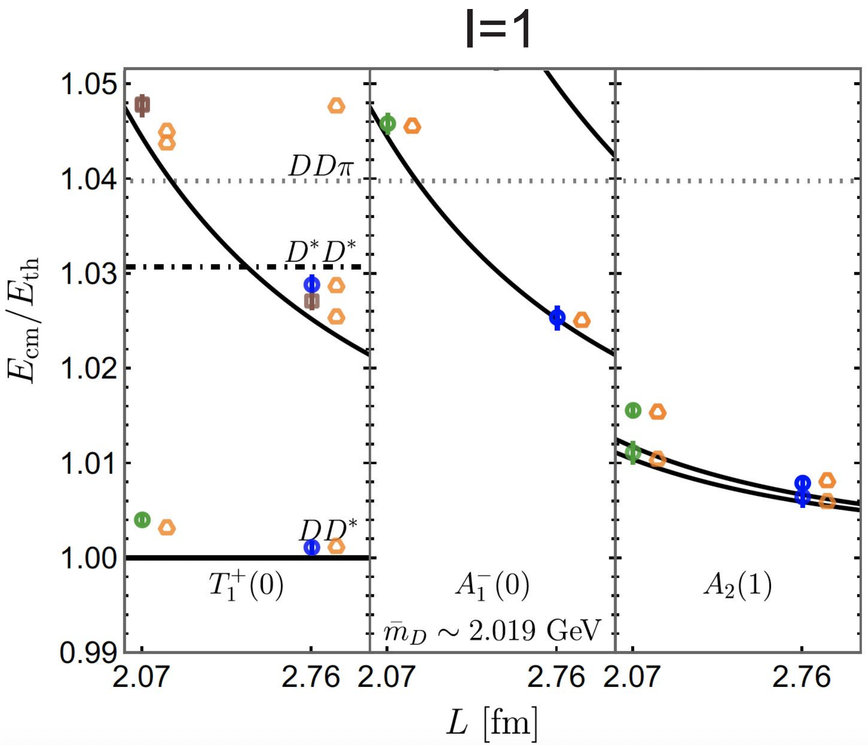


Isospin-1channel

- In a previous lattice QCD study the phase shifts were computed in red below which suggest a pole in the scattering amplitude near the DD* threshold. Such pole would correspond to a bound state.
- However, LHCb not observe any resonance in this channel.
- We decided to extract the energy spectrum in this isospin-1 channel with our lattice data.



Our work: finite-volume energy spectrum of I=1 channel

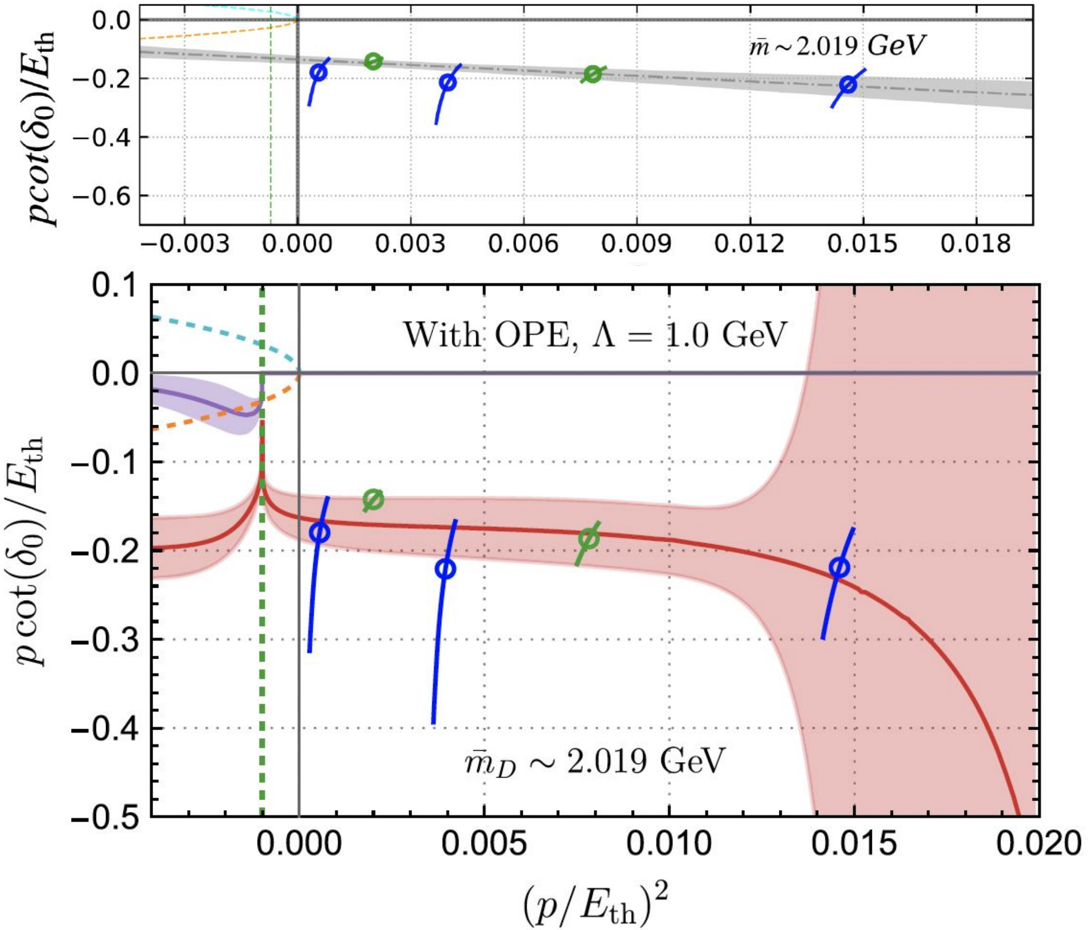


$$O^{DD^*}_{I=1} = \sum_{k,j} A_{k,j}[D(\vec{p}_{1k})D^*_j(\vec{p}_{2k})]_{I=1}$$
$$= \sum_{k,j} A_{k,j}[(\bar{u}\Gamma_1 c)(\vec{p}_{1k})(\bar{d}\Gamma_{2j}c)(\vec{p}_{2k})] + \{u \leftrightarrow d\}.$$

$$O^{MM}: D(0)D^*(0)$$
$$D(1)D^*(-1)|_{l=0}$$
$$D(1)D^*(-1)|_{l=2}$$

➢ Positive energy shifts with respect to the energy thresholds indicates a repulsive interaction near the DD* threshold.

DD* Scattering amplitude: isospin-1

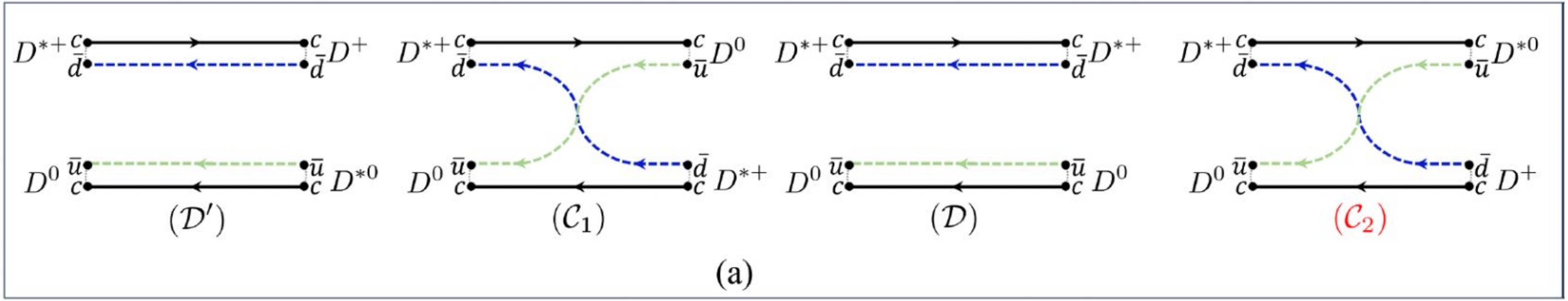


- ➢ No pole in the scattering amplitude near the DD* threshold was found when using Lusher + ERE

- ➢ By additional considering OPE to handle the lhc we did not found a pole in the scattering amplitude for the DD* scattering.

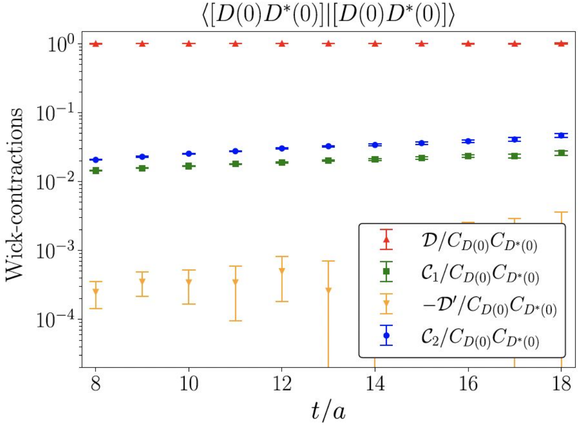
2411.06266

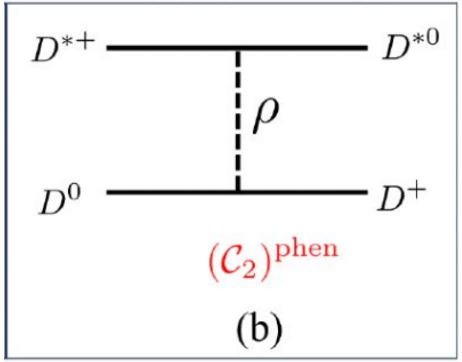
Wick contractions for both isospin channels



$$\langle (DD^*)_I | (DD^*)_I \rangle = \mathcal{D} - \mathcal{C}_1 + (-1)^{I+1}(\mathcal{D}' - \mathcal{C}_2)$$

$$(DD^*)_I = \frac{1}{\sqrt{2}}[D^0 D^{*+} + (-1)^{I+1} D^+ D^{*0}]$$





C_2 Wick contraction responsible for the isospin channels in the tetraquark.

Conclusions

Isospin-0

- ➢ For the relevant isospin-0 Tcc, I implemented meson-meson and additionally diquark-antidiquark interpolators and find that these have some impact on certain eigenenergies.
- ➢ Extraction of the scattering amplitude based on both types of operators.
- ➢ In the end, Tcc is found to be a sub-threshold resonance where the effect of the additional operators renders a Tcc pole slightly closer to the threshold.
- ➢ For Tbb there is a significant energy shift of ~100MeV when diquark-antidiquark operators are additionally considered.

Isospin-1

- ➢ No pole in the scattering amplitude near the DD* threshold was found when using Lusher + ERE neither when using OPE to handle the lhc we did not found a pole in the scattering amplitude for the DD* scattering.
- ➢ C2 is the Wick contraction responsible for the isospin channels in the tetraquark.
- ➢ Rho-exchange is a candidate for distinguish the isospin double charm tetraquark channels.

Thanks!