Can we define climate by means of an ensemble?
A tale of time scales of convergence

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in collaboration with

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ELTE

Outline

Motivation

Climate $\approx$ the statistics of weather

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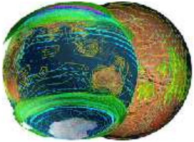
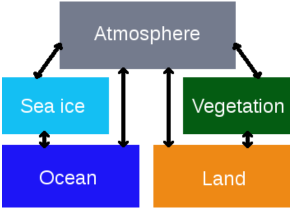
But what is the probability measure?

Outline

A model Earth



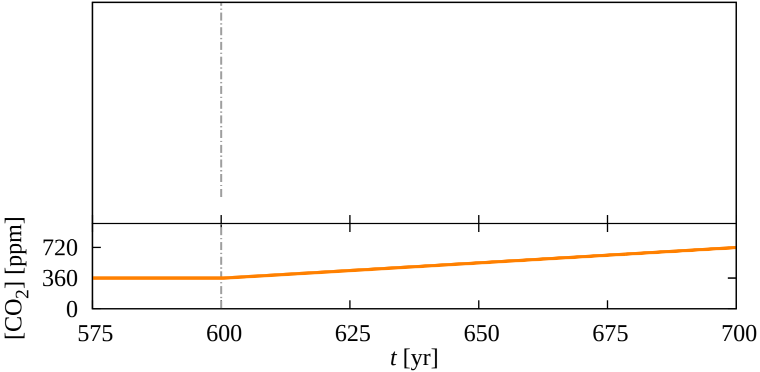
Planet Simulator, University of Hamburg:
an intermediate-complexity climate model



→ Now, for illustrative purposes, it will represent a real world

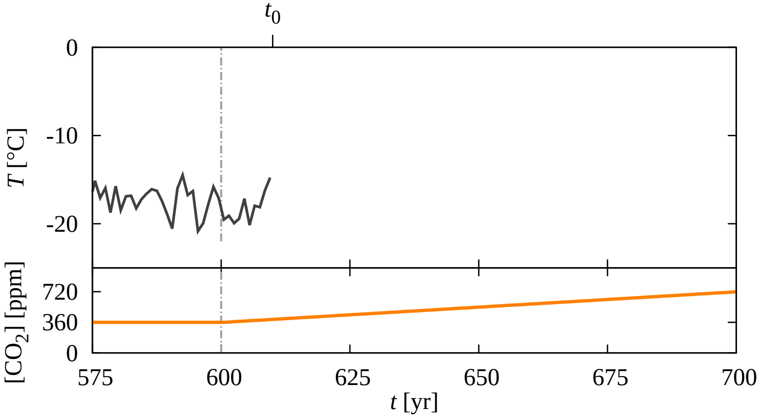
CO_2 forcing

Fixed forcing scenario



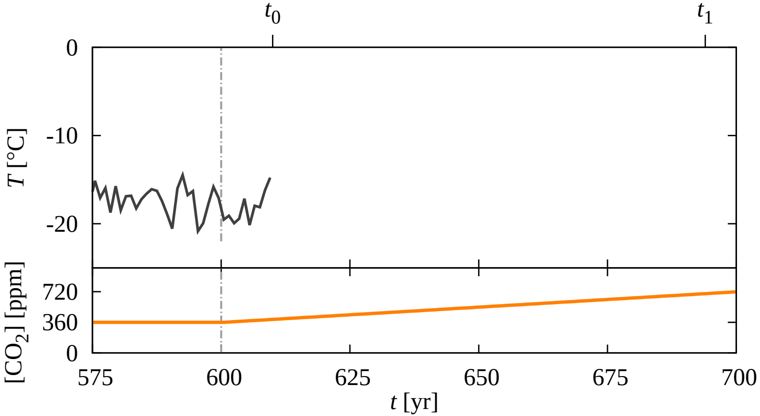
"Instrumental records"

The temperature of a grid point in the Southern Pacific



Prediction?

The temperature of a grid point in the Southern Pacific



Prediction

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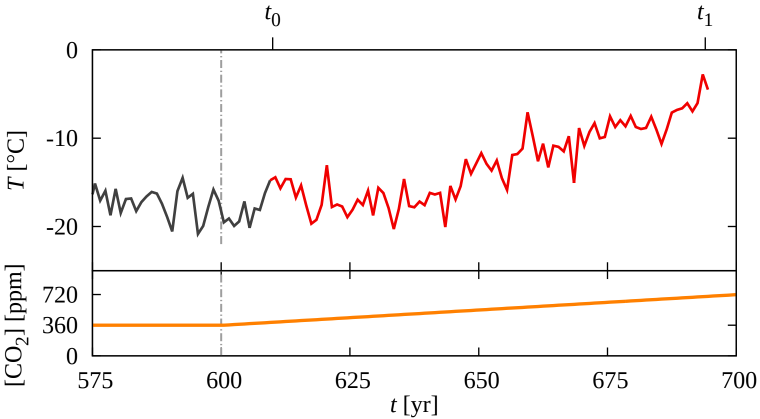
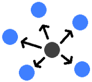
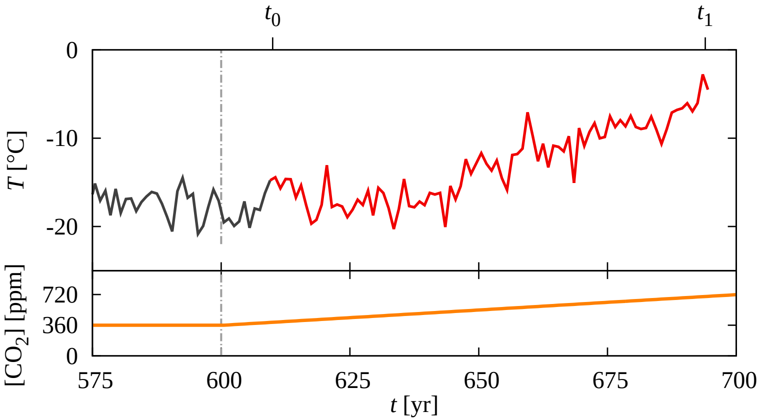


Prediction from perturbed initial conditions





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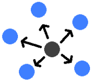
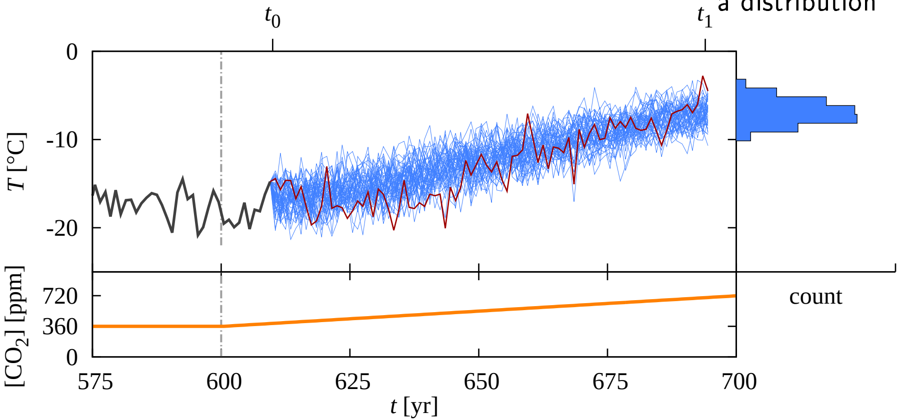


Chaos → a distribution



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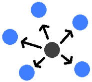
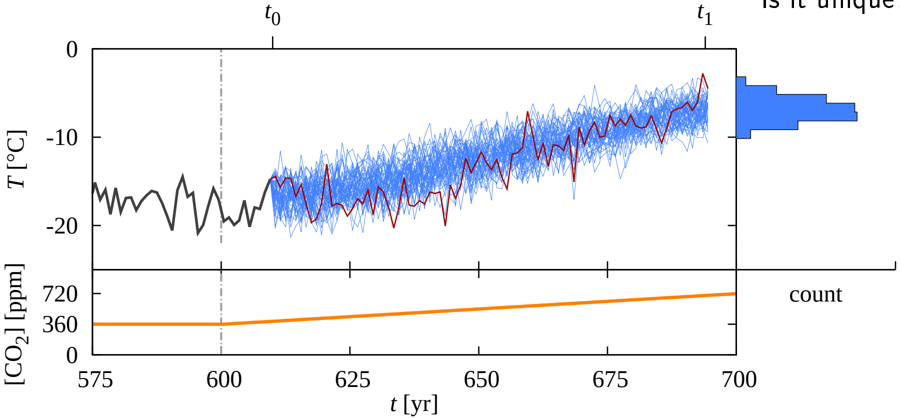
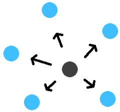
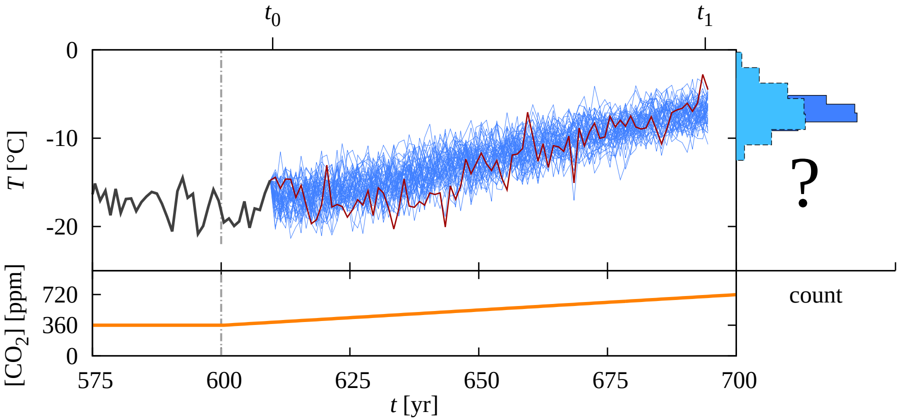


Is it unique?



Prediction from perturbed initial conditions





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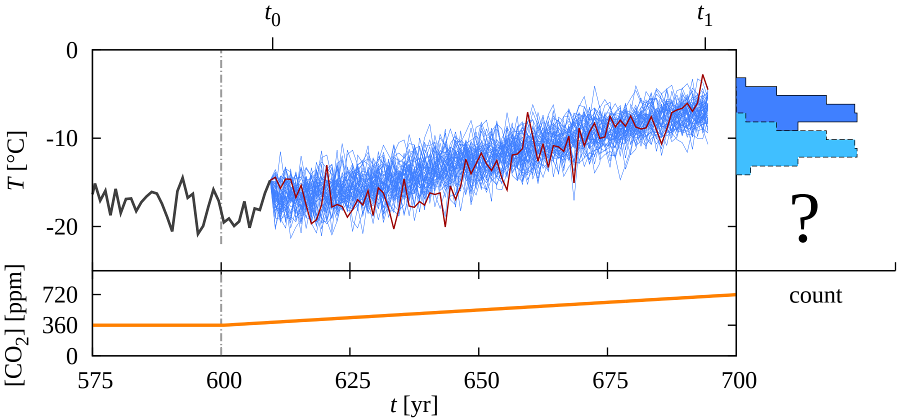

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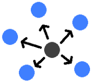

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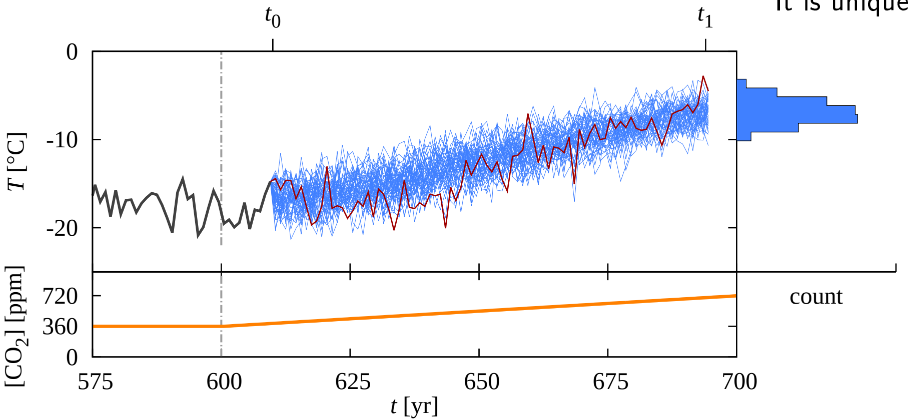


The attractor and its natural probability measure

If $t_0 \to -\infty$ (and $N \to \infty$):

no dependence on the initialization,

natural probability measure of a dynamical (snapshot/pullback*) attractor

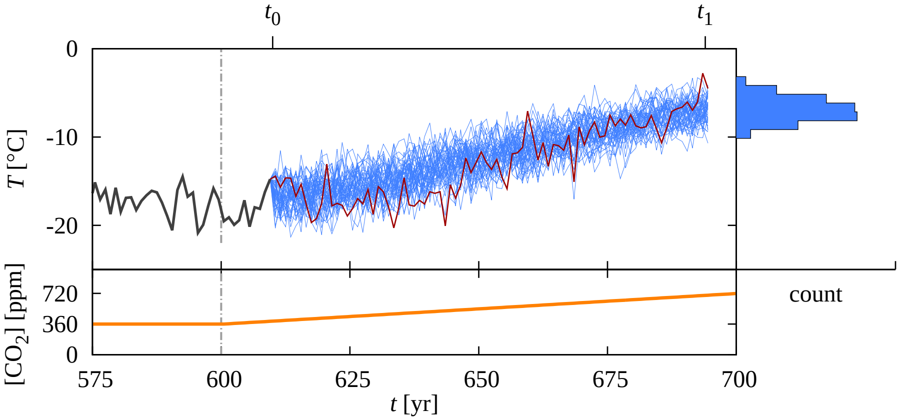
Romeiras, Grebogi and Ott, Phys. Rev. A **41**, 784 (1990)
Ghil, Chekroun and Simonnet, Physica D **237**, 2111 (2008)
Chekroun, Simonnet and Ghil, Physica D **240**, 1685 (2011)

Limit in t_0, but we empirically see a convergence forward in t as well

Drótos, Bódai and Tél, J. Climate **28**, 3275 (2015)
Drótos, Bódai and Tél, Eur. Phys. J. Special Topics **226**, 2031 (2017)

* Non-autonomous dynamical system: $\dot{x} = F(x, t)$

Do we see the natural probability measure for a finite t_0?



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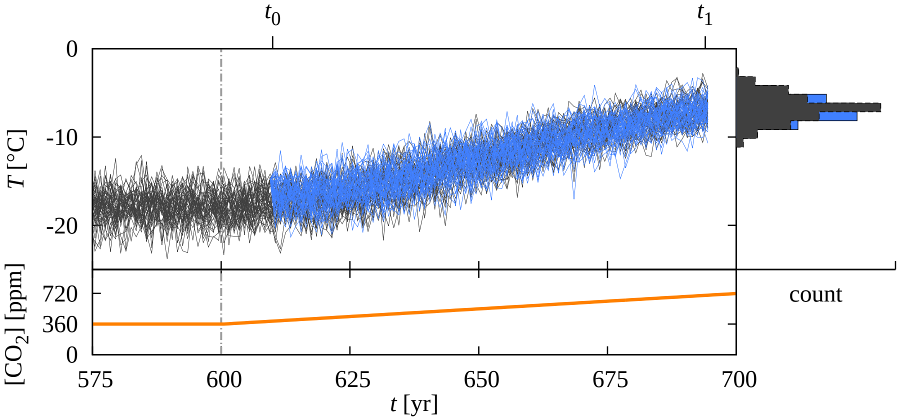
Let us take a reference ensemble (initialized much earlier)



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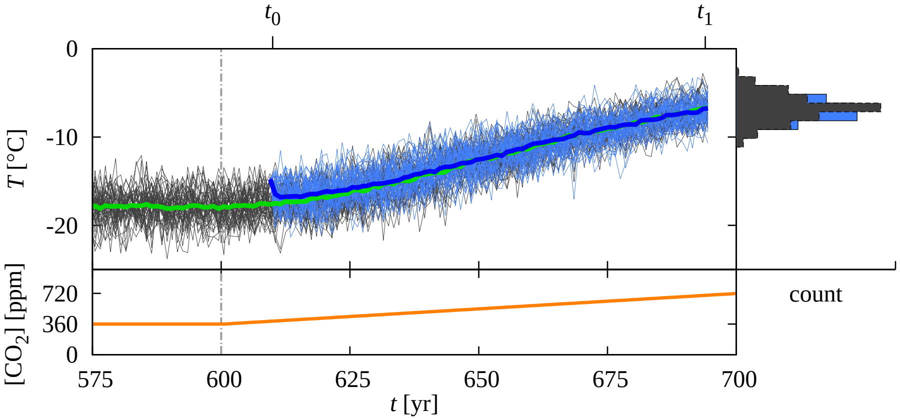

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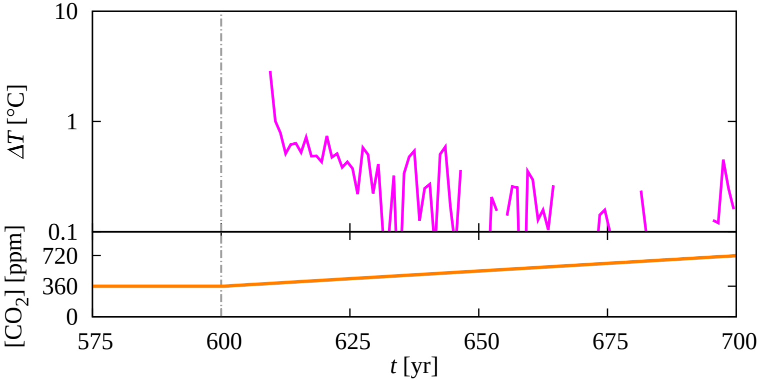
Difference



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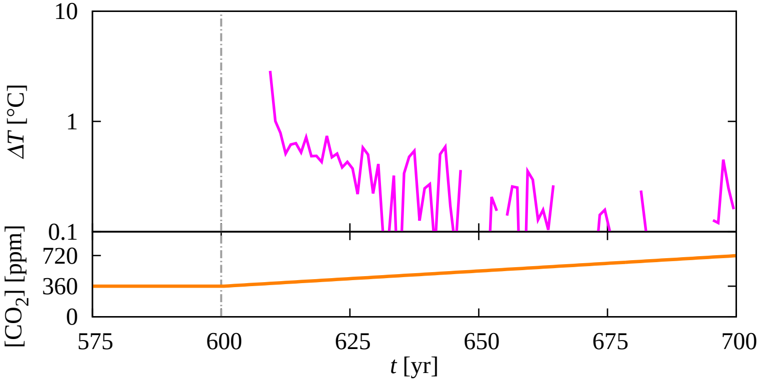
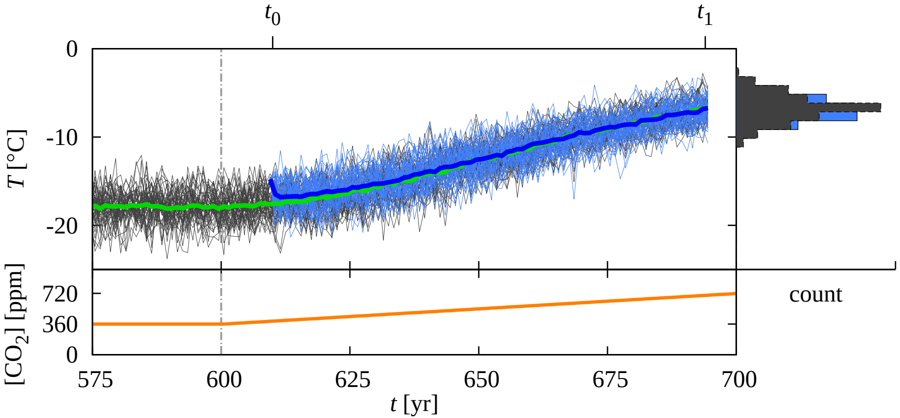


Convergence (unique ensemble spread) in a few decades (exponential-like)

Do we see the natural probability measure for a finite t_0?



Yes, we do, after some convergence time

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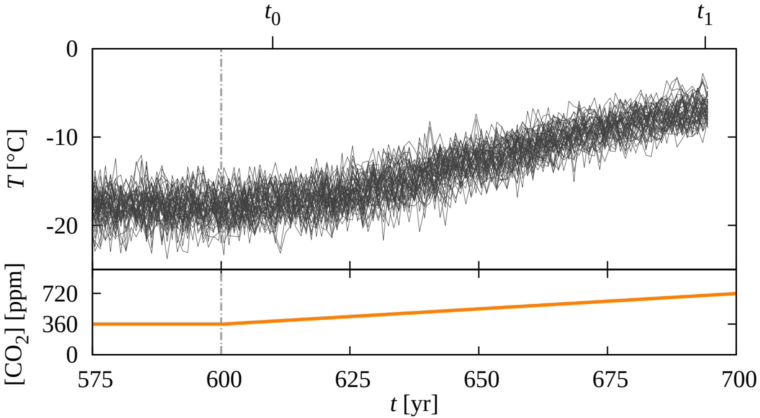

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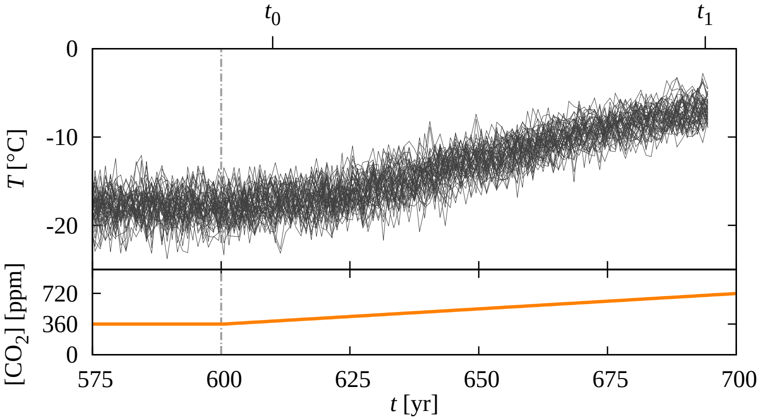

It actually exists at any time, even if it is time dependent

Defining climate

Natural probability measure of the attractor: at any time:

- the *a priori* statistical/probabilistic description: unique, independent of the initialization $\rightarrow$
- characterizes (all possibilities permitted by) the system (incl. forcing), not a particular state (the "weather")

$\rightarrow$ A good candidate for defining climate

Note:

- time dependence (*climate change*) = response to forcing
- traced out by an ensemble after a finite convergence time (numerical simulations)

Problem: slow convergence

- Longest time scales in AOGCMs*: $\mathcal{O}(100)$-$\mathcal{O}(1000)$ yr (deep ocean)
  e.g., Olivié, Peters and Saint-Martin, J. Climate **25**, 7956 (2012)
- Time span of climate projections: $\mathcal{O}(10)$-$\mathcal{O}(100)$ yr

$\rightarrow$ We are not interested in the full variability

* Atmosphere-Ocean General Circulation Models

Outline

A naive approach to defining climate

Refinement

Making the climate conditional

With respect to the targeted time scale: slow vs. fast variables (modes. . .)

Making the climate conditional

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Infinite separation of time scales: one can

- fix a single state (e.g., observations) in the slow variables,
- ensure convergence to the corresponding (conditional) natural probability measure in the fast variables (on the fast time scales)

Making the climate conditional

With respect to the targeted time scale: slow vs. fast variables (modes. . .)

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- ensure convergence to the corresponding (conditional) natural probability measure in the fast variables (on the fast time scales)

Finite but sufficiently large separation: *similar*

Making the climate conditional

Time scales of convergence:

- Autonomous dynamics: eigenvalues $\lambda_i \in \mathbb{C}$ (!) of a $\mathcal{P}$erron–Frobenius operator:

$$\mathcal{P}_{t_0}^{t}(f) = \sum_{i=1}^{N} c_i \lambda_i^{t/T} \varphi_i + r(t)$$

  Györgyi and Szépfalusy, Acta Physica Hungarica **64**, 33 (1988)
  Chekroun, Neelin, Kondrashov, McWilliams and Ghil, PNAS **111**, 1684 (2014)
  Slegers, "Spectral theory for Perron–Frobenius operators", Uppsala Univ. (2019)
  Navarra, Tribbia and Klus, J. Atmos. Sci. **78**, 1227 (2021)

- Nonautonomous dynamics: Froyland et al., Physica D **239**, 1527 (2010)

Thanks to M. Chekroun, G. Froyland, G. Gottwald and Zs. Mihálka.

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→ Conditional climate according to the "slow modes"

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→ Conditional climate according to the "slow modes"
Note: climate can change without forcing

Conclusions

Climate: a probability measure

- obtained after convergence on the fast time scales and
- conditioned on the “slow variables”

Numerically: climate is traced out by an ensemble *after convergence*

Unforced climate changes are possible

Bódai and Tél, Chaos **22**, 023110 (2012)
Drótos, Bódai and Tél, J. Climate **28**, 3275 (2015)
Drótos, Bódai and Tél, Eur. Phys. J. Special Topics **226**, 2031 (2017)
Drótos and Bódai, submitted, `doi:10.5194/egusphere-2025-2030` [preprint] (2025)