The „engineer"'s work in a bank holding company

Building, maintaining and enhancing mathematical models of finance

What is a model?

A quantitative method, system or approach that applies statistical, economic, financial or mathematical theories, techniques and assumptions to process input data into quantitative estimates

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Bólyai Kollégium szakszeminárium

① Quantitative finance

② Little insight to organisational structure

③ Some maths

① High level overview of the potential roles, but in reality these are more versatile and complicat

We all have assumptions on what kind of maths and related skills are needed. Here is a list (minimal)

- Probability theory
- Statistics
- Coding
- Some background in finance
- Excel
- AI

BANKS

HEDGE FUNDS
ASSET MANAGE
MENT COMPANIE

It will be fast (uncomparable to academic pace of daily work)

- require English communication
- be possible to change | internal mobility is supported, sometimes enforced sometimes escape route

- model development, no directs
- AI research for predicting „anything"
- strategist: develops/answers
- review: suggests/questions

(will go to more specific example (of mine)

3/a

Fundamental theorem of asset pricing.

(in the discrete, single-period) market

Two more lists before entering the theorem.

Asset classes

- Equities
- Interest rates
- Foreign eXchange
- Credit

the clean, textbook examples

Types of maths

- Stochastic analysis
- time series
- AI algorithms
  - linear regression
  - PCA
- statistical decision theory

Back to FTAP

Consider n stocks, k states ω_j

$S_t^{(i)}(\omega_j) \in \mathbb{R}^+$ $\quad i = 1, 2, \dots, n$ $\quad S_0^{(i)}(\omega_j) = S_0^{(i)}$

$t = 0, 1$

$j = 1, 2, \dots, k$

and probability (measure), empirical

$p_j = \mathbb{P}(\omega_j)$

A portfolio built from the stocks looks like this

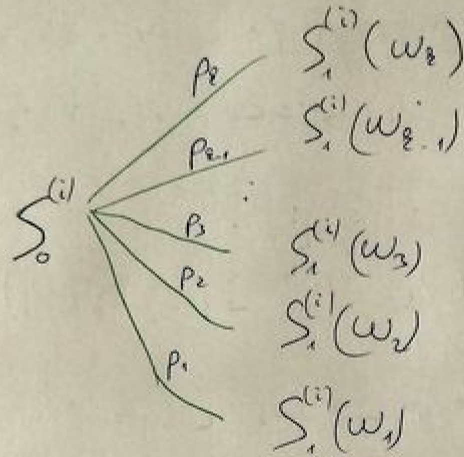
$V_0 \in \mathbb{R}^+$

goes to/comes from the bank

$$V_0 = x_0 + \sum_{i=1}^{n} n_i S_0^{(i)}$$

constant interest rate

$$V_1 = \underbrace{\left(V_0 - \sum_{i=1}^{n} n_i S_0^{(i)}\right)(1+r) + \sum_{i=1}^{n} n_i S_1^{(i)}}_{\text{random variables}}$$



Now cutting the longer story shorter banks, quants want to know the price of assets.

Price of the portfolio is easy: V_0

Any other price would be arbitrage

But there are more complex assets (derivatives)

Any function $f(S^{(i)})$

E.g., a futures contract

Futures: Obligation for the owner to buy 1 stock of type $S_1^{(i)}$

Say for the (strike) price K at maturity T (now $T = 1$)

Associated payoff function

$$f(S) = S - K$$

There is a thing called Risk Neutral Probability measure

typically true for several levels of generalization

- multi period market model
- continuous time market model

characterized by

$$\underbrace{\mathbb{E}_{\tilde{\mathbb{P}}}\left[\frac{S_1^{(i)}}{1+r} - S_0\right]}_{\langle \tilde{\mathbb{P}},\, \Delta\hat{S}^{(i)} \rangle} = 0 \qquad \forall i$$

k-dimensional scalar product

n equations $k-1$ unkowns.

If exist, fair price is given by

$$\text{price} = \mathbb{E}_{\tilde{\mathbb{P}}}\left[\frac{f(S_{(1)})}{1+r}\right]$$

$f\left(S^{(1)}, S^{(2)}, \dots, S^{(n)}\right)$ in general

E.g.,
$$\begin{tikzcd}[row sep=small] & 120 \\ S_0 = 100 \arrow[ur] \arrow[dr] & \\ & 80 \end{tikzcd}$$

$K = 90$

$r = 0$

$$0 \overset{!}{=} \langle \tilde{\mathbb{P}}, \Delta \hat{S} \rangle = \tilde{p}\left(\frac{120}{1+0} - 100\right) + (1-\tilde{p})\left(\frac{80}{1+0} - 100\right)$$

$\tilde{p} = \frac{1}{2}$

$$\mathbb{P}(\omega_1) = \tilde{p}_1 \equiv \tilde{p}$$
$$\mathbb{P}(\omega_2) = \tilde{p}_2 = 1 - \tilde{p}$$

$$\text{price} = \mathbb{E}_{\tilde{\mathbb{P}}}\left(S_{(1)} - K\right) = \frac{1}{2} \cdot 30 + \frac{1}{2}(-10) = 10$$

$\exists$ just one argument why not the empirical, in the example for $f(S) = S$ we would get other than S_0.

What special or fair about this price is that it is <u>arbitrage</u> free.

V_t :
- $V_0 = 0$
- $\mathbb{E}_{\mathbb{P}}[V_1] > 0$
- $V_1(\omega_j) \geqq 0 \quad \forall j$

non-zero probability of profit + zero probability of loss

Theorem (fundamental T. of Asset Pricing)

There exists no arbitrage

$$\Updownarrow$$

There exists risk neutral measure

$$W = \left\{ \sum_{i=1}^{n} n_i \Delta \hat{S}^{(i)} \in \mathbb{R}^k \;\middle|\; (n_1, n_2, \dots, n_n) \in \mathbb{R}^n \right\}$$

$$A^+ = \left\{ X \in \mathbb{R}^k \;\middle|\; X_i \geqq 0,\ X \neq (0, 0, \dots, 0) \right\}$$

$$P = \left\{ X \in \mathbb{R}^k \;\middle|\; 0 < X_i < 1,\ \sum_{i=1}^{k} X_i = 1 \right\}$$

$$W \cap A^+ = \emptyset \iff \mathcal{P} \cap W^+ \neq \emptyset$$

This is generalized as mentioned and it is the backbone of pricing in the financial markets.

Back to the title

Organisational structure

Institutional Securities — capital raising, fin. services — mergers, acquisition, advisory, restructuring, real estate, corporate lending, + FII, + Equities

revenue 20.6 bn

Wealth Management — stock brokerage, investment adv

revenue 17.2 bn

Investment Management — asset management prod. & serv

revenue 2.7 bn

Global footprint

72%	13%	15%
Americas	EMEA	Asia

57.000 employees
2.3 tr clients assets
15.000+ WM financial advisors

RISK

Market
Credit
Liquidity
Operational
Model (MRM)
Regulatory
Reputational

There are models developed for all these

Business Resourse Management
develops pricing models

Risk Methodology
develops risk models (derivatives of prices)

MRM reviews both
- accurateness
- stress testing
- sensitivity testing
- conceptual soundness

interaction with strategists
similar to scientists and journal reviewers

protocols are fixed:
- challenges
- limitations
- annual reviews (recertifications)
- models are grouped by tiers (based on materiality)
- annual simulation exercise by the FED
- batches of new feature of pricing code

3/6 Let's get back to maths once again

CVA team

You would think that market risk is the most important one. It's not allowed.

Credit risk

possibility of default of the counterparty

- current prices are not enough, need to simulate future prices
- We also need to know the probability (density function) of default.

Derivation

MPE_CVA (One sided, other is also computed)

$V^+(t)$ positive part of exposure to a given counterparty

$E_0^Q[\,]$ expectation value under the money market account measure

τ default time of counterparty

$R(t)$ recovery rate of the counterparty

β a type of discounting like $(1+r)$

$$MPE\_CVA = -E_0^Q\left[1_{\tau\leq T}\frac{V^+(\tau)(1-R(\tau))}{\beta(\tau)}\right] =$$

$$= -E_0^Q\int_0^T \frac{V^+(t)(1-R(t))}{\beta(t)}\,dN_t =$$

$$dN_t \equiv 1_{\tau\leq t}$$

$$= -E_0^Q\left[\sum_{i=1}^{n-1}\frac{1}{2}\left(\frac{V^+(t_i)(1-R)}{\beta(t_i)}+\frac{V^+(t_{i+1})(1-R)}{\beta(t_{i+1})}\right)1_{t_i<\tau<t_{i+1}}\right]$$

$$\boxed{= -\sum_{i=0}^{n-1}E_0^Q\left[\frac{1}{2}\left(\frac{V^+(t_i)(1-R)}{\beta(t_i)}+\frac{V^+(t_{i+1})(1-R)}{\beta(t_{i+1})}\right)\Bigg|\,t_i<\tau<t_{i+1}\right]\cdot P_0^Q(t_i<\tau<t_{i+1})}$$

Our models differ from the Front Office

- Need to simulate all assets
- Need to calibrate the simulation
  - Business As Usual
  - Stress
- Need to incorporate collateral rules
- Need to compute default probabilities
  - use Credit Default Swap price
  - use internal / external rating ⟶

Story is endless.