Weak Interactions

Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

October 7, 2020

K_{e4} decay

$$K^+ \to \pi^+ \pi^- e^+ \nu_e$$

$$p = p_1 + p_2 + k_1 + k_2$$

- both currents contribute:
  - K^+ pseudoscalar $\Rightarrow \langle \pi^+ \pi^- | V_+^\mu | K^+ \rangle$ p.vector, $\langle \pi^+ \pi^- | A_+^\mu | K^+ \rangle$ vector

    $\langle \{-\vec{p}_j'\} | V^\mu | \{-\vec{p}_j\} \rangle = \langle \{-\vec{p}_j'\} | P^\dagger P V^\mu P^\dagger P | \{-\vec{p}_j\} \rangle = \mathcal{P}^\mu_\nu \eta' \eta \langle \{\vec{p}_j'\} | V^\mu | \{\vec{p}_j\} \rangle$

    vector if $\eta' \eta = 1$, pseudovector if $\eta' \eta = -1$

    similar argument for axial-vector current
  - 3 indep. hadronic momenta, can build p.vector w/ Levi-Civita tensor

$$\langle \pi^+ \pi^- | A_+^\mu | K^+ \rangle = f_1 (p_1 + p_2)^\mu + f_2 (p_1 - p_2)^\mu + f_3 (p - p_1 - p_2)^\mu$$

$$\langle \pi^+ \pi^- | V_+^\mu | K^+ \rangle = \frac{f_4}{m_K^2} \varepsilon^\mu{}_{\nu\rho\sigma} p^\nu p_1^\rho p_2^\sigma$$

f_j real functions of $p \cdot p_1$, $p \cdot p_2$ and $p_1 \cdot p_2$ of m-dimension -1

- f_3 term contribution is small: using $p - p_1 - p_2 = k_1 + k_2$ and Dirac equation in the lepton matrix element shows $\propto m_\ell$ (f_1 term is $\mathcal{O}(m_K)$)
- f_4 term contribution is small: $\propto m_K (\vec{p}_1 / m_K) \wedge (\vec{p}_2 / m_K)$, suppressed by two powers of momentum
- estimate using PCAC gives $f_{1,2} \approx 1/f_\pi$

Leptonic decays of hyperons

Strangeness-conserving and strangeness-changing currents part of octet

$$(j^\alpha)^i{}_j = \bar{q}^i \mathcal{O}_L^\alpha q_j - \delta^i{}_j \tfrac{1}{3} \textstyle\sum_k \bar{q}^k \mathcal{O}_L^\alpha q_k$$

Making octet nature more explicit

$$j_a^\alpha = \bar{q}\mathcal{O}_L^\alpha t_a q = \bar{q}\gamma^\alpha(1-\gamma^5)t_a q$$

$$j_a^\alpha = (j^\alpha)^i{}_j (t_a)^i{}_j = \mathrm{tr}\, j^\alpha t_a^T = \mathrm{tr}\, j^\alpha t_a^*$$
$$(j^\alpha)^i{}_j = 2\textstyle\sum_a (t_a^*)^i{}_j \bar{q}\mathcal{O}_L^\alpha t_a q = 2\textstyle\sum_a j_a^\alpha (t_a^*)^i{}_j$$

t_a: SU(3) generators in the fundamental representation
$t_a^\dagger = t_a$, $[t_a, t_b] = i f_{abc} t_c$, $2\mathrm{tr}\, t_a t_b = \delta_{ab}$

Hadronic matrix elements (working in exact SU(3) limit)

$$\mathcal{A}_a^\mu(B \to B') = \langle \vec{p}\,'; B' | j_a^\mu(0) | \vec{p}; B \rangle$$

$|\vec{p}; B\rangle = B^a |\vec{p}; a\rangle$ generic octet state, $|\vec{p}; a\rangle$, $a = 1, \ldots, 8$ basis baryons

p, n, etc.

Leptonic decays of hyperons (contd.)

Represent flavour part of $|\vec{p}\,;a\rangle$ in the linear space of traceless 3×3 matrices $|\vec{p}\,;a\rangle \to t_a|\vec{p}\rangle$ (instead of an 8-component complex vector)

- Flavour wave function = traceless 3×3 matrix $B = B^a t_a$
- Hilbert space with positive-definite scalar product $(B', B) \equiv 2\,\mathrm{tr}\,\bar{B}' B$
  $\bar{B} = B^\dagger,\ \bar{B}^a = B^{a\,*}$

$$(B', B) = \bar{B}'^a B^b\, 2\mathrm{tr}\, t_a t_b = \bar{B}'^a B^b \delta_{ab}$$

- under SU(3) transformation $B \to UBU^\dagger$ (adjoint representation)

$$UBU^\dagger = B^a U t_a U^\dagger = t_b (U_{\mathrm{adj}})_{ba} B^a$$

$$UBU^\dagger|_{U\simeq \mathbf{1}} = (\mathbf{1} + i\epsilon_a t_a) B (\mathbf{1} - i\epsilon_b t_b) = B + i\epsilon_a [t_a, B]$$

Matrix elements $\mathcal{A}^\mu_a(B \to B')$ trasform in the adjoint representation under SU(3) transformation of baryon states, find most general object

- linear in B, antilinear in B', linear in the current
- transforming in the adjoint

$$\Longrightarrow \mathcal{A}^\mu_a(B \to B') = C^\mu_1 \cdot 2\,\mathrm{tr}\,\bar{B}' t_a B + C^\mu_2 \cdot 2\,\mathrm{tr}\, B t_a \bar{B}'$$

Leptonic decays of hyperons (contd.)

Use symmetric and antisymmetric combinations

$$\begin{aligned}
\mathcal{A}_a^\mu(B \to B') &= (D^\mu + F^\mu) \cdot 2\,\mathrm{tr}\,\bar{B}' t_a B + (D^\mu - F^\mu) \cdot 2\,\mathrm{tr}\,B t_a \bar{B}' \\
&= D^\mu 2\,\mathrm{tr}\,\bar{B}'\{t_a, B\} + F^\mu 2\,\mathrm{tr}\,\bar{B}'[t_a, B] \\
&= D^\mu d_{abc} B^b B'^c + F^\mu i f_{abc} B^b B'^c \\
&= D^\mu d_{abc} B'^b B^c + F^\mu (-i f_{abc}) B'^b B^c \\
&= D^\mu B'^b (\tilde{T}_a)_{bc} B^c + F^\mu B'^b (T_a)_{bc} B^c
\end{aligned}$$

$(T_a)_{bc} = -i f_{abc}$ (generators in the adjoint representation), $(\tilde{T}_a)_{bc} = d_{abc}$
Flavour structure of matrix elements fully determined, matrix elements parameterised in terms of two unknown objects D^μ and F^μ

- contain both vector and axial vector parts
- do not depend on the baryons involved in the process
- constrained by Lorentz invariance

Leptonic decays of hyperons (contd.)

$$D^\mu = D^\mu_V - D^\mu_A$$
$$D^\mu_V = \bar{u}(p')\left\{f_{D1}(q^2)\gamma^\mu + if_{D2}(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m} + f_{D3}(q^2)\frac{q^\mu}{2m}\right\}u(p)$$
$$D^\mu_A = \bar{u}(p')\left\{g_{D1}(q^2)\gamma^\mu + ig_{D2}(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m} + g_{D3}(q^2)\frac{q^\mu}{2m}\right\}\gamma^5 u(p)$$
$$F^\mu = F^\mu_V - F^\mu_A$$
$$F^\mu_V = \bar{u}(p')\left\{f_{F1}(q^2)\gamma^\mu + if_{F2}(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m} + f_{F3}(q^2)\frac{q^\mu}{2m}\right\}u(p)$$
$$F^\mu_A = \bar{u}(p')\left\{g_{F1}(q^2)\gamma^\mu + ig_{F2}(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m} + g_{F3}(q^2)\frac{q^\mu}{2m}\right\}\gamma^5 u(p)$$

m: octet mass

p, p': initial and final momenta, $q = p - p'$

Small mass differences in the octet $\rightarrow$ small q, can work in static approximation $q = 0$ for matrix elements

$$D^\mu = \bar{u}(p)\gamma^\mu(D_V + D_A\gamma^5)u(p) \qquad F^\mu = \bar{u}(p)\gamma^\mu(F_V + F_A\gamma^5)u(p)$$
$$D_V = f_{D1}(0) \quad D_A = g_{D1}(0) \qquad F_V = f_{F1}(0) \quad F_A = g_{F1}(0)$$

Leptonic decays of hyperons (contd.)

SU(3) symmetry also determines D_V, F_V
Take $j^{\mu}_{Va}(t,\vec{x})$ vector part of the current
Spatial integral of $j^0_{Va}(0,\vec{x})$ equals generator $\mathbf{T}_a$ of SU(3)

$$\begin{aligned}
\int d^3x \langle B'|j^0_{Va}(0,\vec{x})|B\rangle &= \langle B'|\mathbf{T}_a|B\rangle = \bar{B}'^b B^c \langle b|\mathbf{T}_a|c\rangle \\
(2\pi)^3\delta(\vec{q}\,)\langle B'|j^0_{Va}(0)|B\rangle &= (2\pi)^3\delta(\vec{q}\,)2p^0\bar{B}'^b B^c (T_a)_{bc} \\
\langle B'|j^0_{Va}(0)|B\rangle|_{\vec{q}=0} &= 2p^0\bar{B}'^b B^c (T_a)_{bc} \\
\bar{B}'^b B^c (D_V(\tilde{T}_a)_{bc} + F_V(T_a)_{bc})\bar{u}(p)\gamma^0 u(p) &= 2p^0\bar{B}'^b B^c (T_a)_{bc} \\
\bar{B}'^b B^c (D_V(\tilde{T}_a)_{bc} + F_V(T_a)_{bc}) &= \bar{B}'^b B^c (T_a)_{bc} \\
D_V(\tilde{T}_a)_{bc} + F_V(T_a)_{bc} &= (T_a)_{bc}
\end{aligned}$$

$\tilde{T}_a$ symmetric, T_a antisymmetric $\Rightarrow$ linearly independent

$$\Rightarrow D_V = 0,\ F_V = 1$$

Leptonic decays of hyperons (contd.)

Exact SU(3), static approximation:

$$\mathcal{A}^\mu_a(B \to B') = \bar{u}(p)\gamma^\mu \left\{ (T_a)_{bc} + \left[F_A (T_a)_{bc} + D_A (\tilde{T}_a)_{bc} \right] \gamma^5 \right\} u(p) \bar{B}'^b B^c$$
$$= \bar{u}(p)\gamma^\mu \{ [(D_V + F_V) 2 \operatorname{tr} \bar{B}' t_a B + (D_V - F_V) 2 \operatorname{tr} B t_a \bar{B}'] + [(D_A + F_A) 2 \operatorname{tr} \bar{B}' t_a B + (D_A - F_A) 2 \operatorname{tr} B t_a \bar{B}'] \gamma^5 \} u(p)$$

Need the invariants $\operatorname{tr} \bar{B}' t_a B$ and $\operatorname{tr} B t_a \bar{B}'$ for the relevant currents

$$B = \frac{1}{\sqrt{2}} \left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \hline \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{array} \right)$$

p, n, Σ^0, etc. are wave-function components

Assignment of components to $\mathbf{8}_{\mathrm{SU}(3)}$ states from I, Y values

$$\vec{t} = \left(\begin{array}{c|c} \vec{I} & 0 \\ \hline 0 & 0 \end{array} \right) \quad Y = \frac{1}{3} \left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array} \right) \qquad B = \left(\begin{array}{c|c} \Sigma & N \\ \hline \Xi & 0 \end{array} \right) + \frac{1}{\sqrt{6}}\Lambda \left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array} \right)$$

$$[\vec{t}, B] = \left(\begin{array}{c|c} [\vec{I}, \Sigma] & \vec{I} N \\ \hline -\vec{I}^T \Xi & 0 \end{array} \right) \qquad [Y, B] = \left(\begin{array}{c|c} 0 & N \\ \hline -\Xi & 0 \end{array} \right)$$

Leptonic decays of hyperons (contd.)

Exact SU(3), static approximation:

$$\mathcal{A}^\mu_a(B\to B') = \bar{u}(p)\gamma^\mu\left\{(T_a)_{bc} + \left[F_A(T_a)_{bc} + D_A(\tilde{T}_a)_{bc}\right]\gamma^5\right\}u(p)\bar{B}'^b B^c$$
$$= \bar{u}(p)\gamma^\mu\{[(D_V+F_V)2\,\mathrm{tr}\,\bar{B}'t_aB+(D_V-F_V)2\,\mathrm{tr}\,Bt_a\bar{B}']$$
$$+[(D_A+F_A)2\,\mathrm{tr}\,\bar{B}'t_aB+(D_A-F_A)2\,\mathrm{tr}\,Bt_a\bar{B}']\gamma^5\}u(p)$$

Need the invariants $\mathrm{tr}\,\bar{B}'t_aB$ and $\mathrm{tr}\,Bt_a\bar{B}'$ for the relevant currents

$$B = \frac{1}{\sqrt{2}}\left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}\Sigma^0+\frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0+\frac{1}{\sqrt{6}}\Lambda & n \\ \hline \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{array}\right)$$

p, n, Σ^0, etc. are wave-function components
Assignment of components to $\mathbf{8}_{\mathrm{SU}(3)}$ states from I, Y values

$$\vec{t} = \left(\begin{array}{c|c} \vec{I} & 0 \\ \hline 0 & 0 \end{array}\right) \quad Y = \tfrac{1}{3}\left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array}\right) \qquad B = \left(\begin{array}{c|c} \Sigma & N \\ \hline \Xi & 0 \end{array}\right) + \tfrac{1}{\sqrt{6}}\Lambda\left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array}\right)$$

$$[\vec{t}, B] = \left(\begin{array}{c|c} [\vec{I},\Sigma] & \vec{I}N \\ \hline -\vec{I}^{\,T}\Xi & 0 \end{array}\right) \qquad [Y, B] = \left(\begin{array}{c|c} 0 & N \\ \hline -\Xi & 0 \end{array}\right)$$

Leptonic decays of hyperons (contd.)

Exact SU(3), static approximation:

$$\mathcal{A}^\mu_a(B\to B') = \bar{u}(p)\gamma^\mu\left\{(T_a)_{bc} + \left[F_A(T_a)_{bc} + D_A(\tilde{T}_a)_{bc}\right]\gamma^5\right\}u(p)\bar{B}'^b B^c$$
$$= \bar{u}(p)\gamma^\mu\{[(D_V+F_V)2\,\mathrm{tr}\,\bar{B}'t_aB+(D_V-F_V)2\,\mathrm{tr}\,Bt_a\bar{B}']$$
$$+[(D_A+F_A)2\,\mathrm{tr}\,\bar{B}'t_aB+(D_A-F_A)2\,\mathrm{tr}\,Bt_a\bar{B}']\gamma^5\}u(p)$$

Need the invariants $\mathrm{tr}\,\bar{B}'t_aB$ and $\mathrm{tr}\,Bt_a\bar{B}'$ for the relevant currents

$$B = \frac{1}{\sqrt{2}}\left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}\Sigma^0+\frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0+\frac{1}{\sqrt{6}}\Lambda & n \\ \hline \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{array}\right)$$

p, n, Σ^0, etc. are wave-function components
Assignment of components to $\mathbf{8}_{\mathrm{SU}(3)}$ states from I, Y values

$$\vec{t} = \left(\begin{array}{c|c} \vec{I} & 0 \\ \hline 0 & 0 \end{array}\right) \quad Y = \tfrac{1}{3}\left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array}\right) \qquad B = \left(\begin{array}{c|c} \Sigma & N \\ \hline \Xi & 0 \end{array}\right) + \tfrac{1}{\sqrt{6}}\Lambda\left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array}\right)$$

$$[\vec{t}, B] = \left(\begin{array}{c|c} [\vec{I},\Sigma] & \vec{I}N \\ \hline -\vec{I}^{\,T}\Xi & 0 \end{array}\right) \qquad [Y, B] = \left(\begin{array}{c|c} 0 & N \\ \hline -\Xi & 0 \end{array}\right)$$

Leptonic decays of hyperons (contd.)

Exact SU(3), static approximation:

$$\mathcal{A}^\mu_a(B\to B') = \bar{u}(p)\gamma^\mu\left\{(T_a)_{bc} + \left[F_A(T_a)_{bc} + D_A(\tilde{T}_a)_{bc}\right]\gamma^5\right\}u(p)\bar{B}'^b B^c$$
$$= \bar{u}(p)\gamma^\mu\{[(D_V+F_V)2\,\mathrm{tr}\,\bar{B}'t_aB+(D_V-F_V)2\,\mathrm{tr}\,Bt_a\bar{B}']$$
$$+[(D_A+F_A)2\,\mathrm{tr}\,\bar{B}'t_aB+(D_A-F_A)2\,\mathrm{tr}\,Bt_a\bar{B}']\gamma^5\}u(p)$$

Need the invariants $\mathrm{tr}\,\bar{B}'t_aB$ and $\mathrm{tr}\,Bt_a\bar{B}'$ for the relevant currents

$$B = \frac{1}{\sqrt{2}}\left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \hline \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{array}\right)$$

p, n, Σ^0, etc. are wave-function components

Assignment of components to $\mathbf{8}_{\mathrm{SU}(3)}$ states from I, Y values

$$\vec{t} = \left(\begin{array}{c|c} \vec{I} & 0 \\ \hline 0 & 0 \end{array}\right) \quad Y = \tfrac{1}{3}\left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array}\right) \qquad B = \left(\begin{array}{c|c} \Sigma & N \\ \hline \Xi & 0 \end{array}\right) + \tfrac{1}{\sqrt{6}}\Lambda\left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array}\right)$$

$$[\vec{t}, B] = \left(\begin{array}{c|c} [\vec{I},\Sigma] & \vec{I}N \\ \hline -\vec{I}^{\,T}\Xi & 0 \end{array}\right) \qquad [Y, B] = \left(\begin{array}{c|c} 0 & N \\ \hline -\Xi & 0 \end{array}\right)$$

Leptonic decays of hyperons (contd.)

Exact SU(3), static approximation:

$$\mathcal{A}^\mu_a(B \to B') = \bar{u}(p)\gamma^\mu \left\{ (T_a)_{bc} + \left[F_A (T_a)_{bc} + D_A (\tilde{T}_a)_{bc} \right] \gamma^5 \right\} u(p) \bar{B}'^b B^c$$
$$= \bar{u}(p)\gamma^\mu \{ [(D_V + F_V) 2 \operatorname{tr} \bar{B}' t_a B + (D_V - F_V) 2 \operatorname{tr} B t_a \bar{B}'] + [(D_A + F_A) 2 \operatorname{tr} \bar{B}' t_a B + (D_A - F_A) 2 \operatorname{tr} B t_a \bar{B}'] \gamma^5 \} u(p)$$

Need the invariants $\operatorname{tr} \bar{B}' t_a B$ and $\operatorname{tr} B t_a \bar{B}'$ for the relevant currents

$$B = \frac{1}{\sqrt{2}} \left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \hline \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{array} \right)$$

p, n, Σ^0, etc. are wave-function components

Assignment of components to $\mathbf{8}_{\mathrm{SU}(3)}$ states from I, Y values

$$\vec{t} = \left(\begin{array}{c|c} \vec{I} & 0 \\ \hline 0 & 0 \end{array} \right) \quad Y = \frac{1}{3} \left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array} \right) \qquad B = \left(\begin{array}{c|c} \Sigma & N \\ \hline \Xi & 0 \end{array} \right) + \frac{1}{\sqrt{6}}\Lambda \left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -2 \end{array} \right)$$

$$[\vec{t}, B] = \left(\begin{array}{c|c} [\vec{I}, \Sigma] & \vec{I} N \\ \hline -\vec{I}^T \Xi & 0 \end{array} \right) \qquad [Y, B] = \left(\begin{array}{c|c} 0 & N \\ \hline -\Xi & 0 \end{array} \right)$$

Leptonic decays of hyperons (contd.)

Currents of interest

$$\bar{u}\mathcal{O}_L^\mu d = j_1^\mu + i j_2^\mu = j_{1+i2}^\mu$$
$$\bar{u}\mathcal{O}_L^\mu s = j_4^\mu + i j_5^\mu = j_{4+i5}^\mu$$

...and their h.c.

$$t_{1+i2} = t_1 + i t_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad t_{4+i5} = t_4 + i t_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$2\,\mathrm{tr}\,\bar{B}' t_{1+i2} B = \left(\tfrac{1}{\sqrt{2}}\bar{\Sigma}^0 + \tfrac{1}{\sqrt{6}}\bar{\Lambda}\right)\Sigma^- + \bar{\Sigma}^+\left(-\tfrac{1}{\sqrt{2}}\Sigma^0 + \tfrac{1}{\sqrt{6}}\Lambda\right) + \bar{p}n\,,$$
$$2\,\mathrm{tr}\,B t_{1+i2}\bar{B}' = \bar{\Sigma}^+\left(\tfrac{1}{\sqrt{2}}\Sigma^0 + \tfrac{1}{\sqrt{6}}\Lambda\right) + \left(-\tfrac{1}{\sqrt{2}}\bar{\Sigma}^0 + \tfrac{1}{\sqrt{6}}\bar{\Lambda}\right)\Sigma^- + \bar{\Xi}^0\Xi^-$$
$$2\,\mathrm{tr}\,\bar{B}' t_{4+i5} B = \left(\tfrac{1}{\sqrt{2}}\bar{\Sigma}^0 + \tfrac{1}{\sqrt{6}}\bar{\Lambda}\right)\Xi^- + \bar{\Sigma}^+\Xi^0 + \bar{p}\left(-\sqrt{\tfrac{2}{3}}\Lambda\right)$$
$$2\,\mathrm{tr}\,B t_{4+i5}\bar{B}' = \bar{p}\left(\tfrac{1}{\sqrt{2}}\Sigma^0 + \tfrac{1}{\sqrt{6}}\Lambda\right) + \bar{n}\Sigma^- + \left(-\sqrt{\tfrac{2}{3}}\bar{\Lambda}\right)\Xi^-$$

Leptonic decays of hyperons (contd.)

Coefficients of $\bar{u}\gamma^\mu u$ ($D = D_V$, $F = F_V$) and $\bar{u}\gamma^\mu\gamma^5 u$ ($D = D_A$, $F = F_A$)

Strangeness-conserving processes: including $\cos\theta_C$

$n \to p$:	$\cos\theta_C(D+F)$
$\Xi^- \to \Xi^0$:	$\cos\theta_C(D-F)$
$\Lambda \to \Sigma^+$:	$\cos\theta_C\sqrt{\frac{2}{3}}D$
$\Sigma^- \to \Lambda$:	$\cos\theta_C\sqrt{\frac{2}{3}}D$
$\Sigma^0 \to \Sigma^+$:	$-\cos\theta_C\sqrt{2}F$
$\Sigma^- \to \Sigma^0$:	$\cos\theta_C\sqrt{2}F$

Strangeness-changing processes: including $\sin\theta_C$

$\Lambda \to p$:	$-\sin\theta_C\frac{1}{\sqrt{6}}(D+3F)$
$\Xi^- \to \Lambda$:	$-\sin\theta_C\frac{1}{\sqrt{6}}(D-3F)$
$\Sigma^0 \to p$:	$\sin\theta_C\frac{1}{\sqrt{2}}(D-F)$
$\Xi^- \to \Sigma^0$:	$\sin\theta_C\frac{1}{\sqrt{2}}(D+F)$
$\Sigma^- \to n$:	$\sin\theta_C(D-F)$
$\Xi^- \to \Sigma^+$:	$\sin\theta_C(D+F)$

Not all relevant for actual processes in real, SU(3) non-symmetric world

E.g.: $\Lambda \to \Sigma$ forbidden since $m_\Sigma > m_\Lambda$

Leptonic decays of hyperons (contd.)

Including γ-matrices, using $D_V = 0$, $F_V = 1 \Rightarrow$ hadronic matrix elements

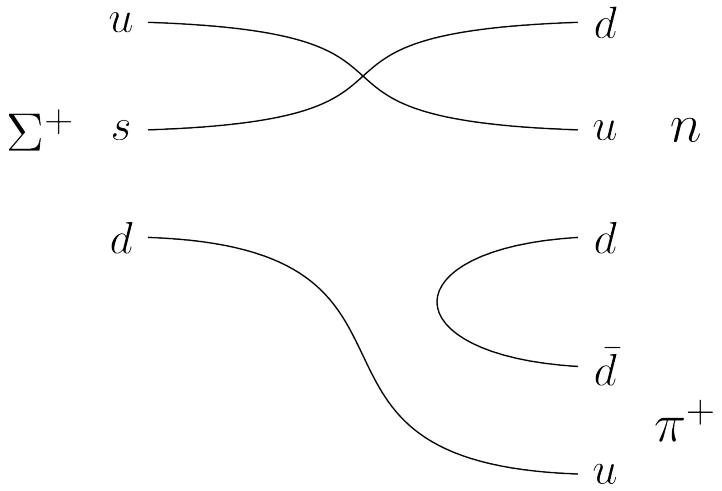
To be sandwiched between initial and final hadron bispinors

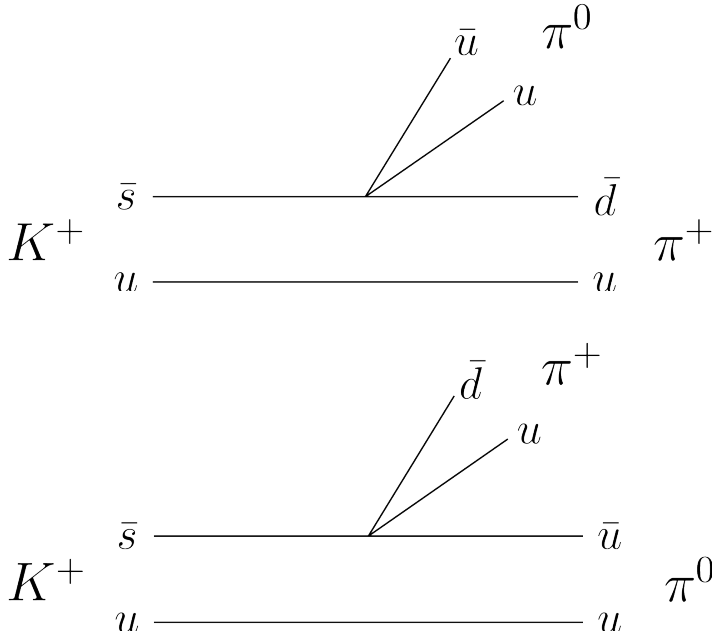
$$
\begin{aligned}
n \to p\, e^-\, \bar{\nu}_e : &\quad \cos\theta_C[\gamma^\mu + (D_A + F_A)\gamma^\mu\gamma^5] \\
\Sigma^\pm \to \Lambda\, e^\pm\, \nu_e(\bar{\nu}_e) : &\quad \cos\theta_C D_A \gamma^\mu\gamma^5 \\
\Lambda \to p\, e^-\, \bar{\nu}_e : &\quad -\sqrt{\tfrac{3}{2}}\sin\theta_C[\gamma^\mu + (F_A + \tfrac{1}{3}D_A)\gamma^\mu\gamma^5] \\
\Sigma^- \to n\, e^-\, \bar{\nu}_e : &\quad -\sin\theta_C[\gamma^\mu + (F_A - D_A)\gamma^\mu\gamma^5] \\
\Xi^- \to \Lambda\, e^-\, \bar{\nu}_e : &\quad \sqrt{\tfrac{3}{2}}\sin\theta_C[\gamma^\mu + (F_A - \tfrac{1}{3}D_A)\gamma^\mu\gamma^5] \\
\Xi^- \to \Sigma^0\, e^-\, \bar{\nu}_e : &\quad \tfrac{1}{\sqrt{2}}\sin\theta_C[\gamma^\mu + (F_A + D_A)\gamma^\mu\gamma^5] \\
\Xi^0 \to \Sigma^+\, e^-\, \bar{\nu}_e : &\quad \tfrac{1}{\sqrt{2}}\sin\theta_C[\gamma^\mu + (F_A + D_A)\gamma^\mu\gamma^5]
\end{aligned}
$$

Decay rates parameterised by D_A, F_A and θ_C

- neutron β decay fixes $D_A + F_A = g_A = 1.25$
- best fit $D_A = 0.80$, $F_A = 0.45$, $\sin\theta_C = 0.23$ ($= 0.21$ from K-decay)

Strangeness-changing non-leptonic interactions



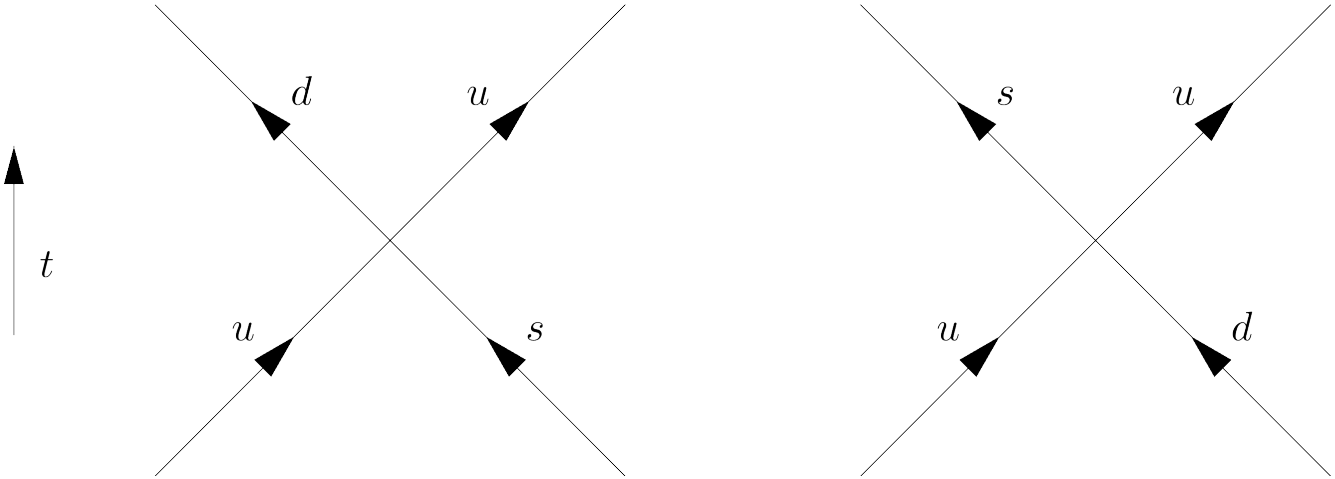


- Responsible for $\Sigma \to N\pi$, $K \to 2\pi, 3\pi, \ldots$
- Involve product of two hadronic currents
- Restrict to lightest quarks, relevant part of $\mathcal{L}_W$

$$-\frac{G}{\sqrt{2}}\bar{d}'\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d' = -\frac{G}{\sqrt{2}}[\cos^2\theta_C\bar{d}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d + \sin^2\theta_C\bar{s}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}s$$
$$+\sin\theta_C\cos\theta_C\left(\bar{d}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}s + \bar{s}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d\right)]$$

$$d' = \cos\theta_C d + \sin\theta_C s$$

Strangeness-changing non-leptonic interactions

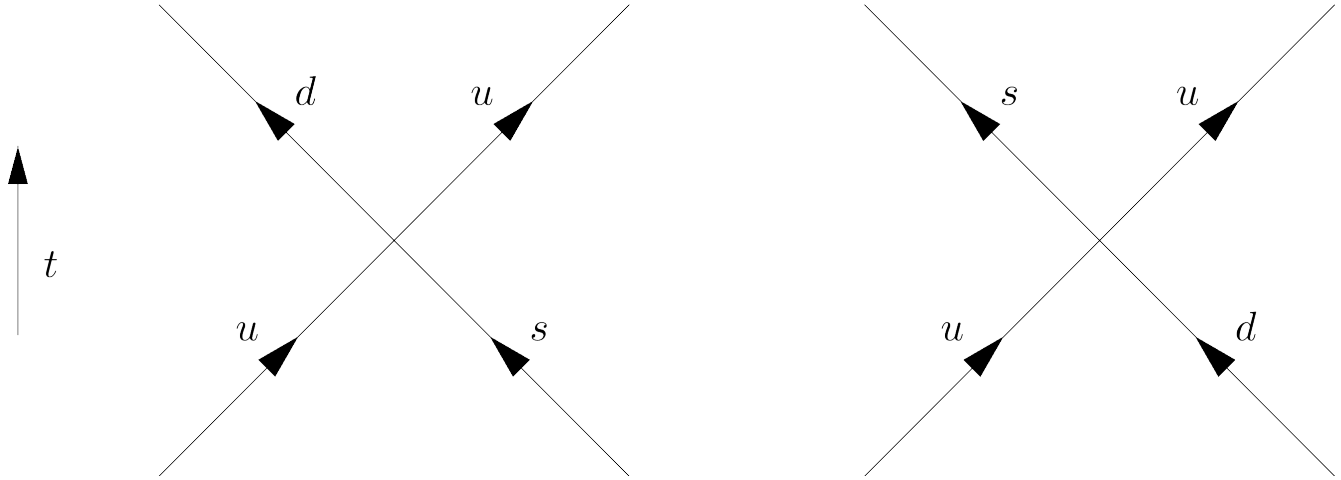


- Responsible for $\Sigma \to N\pi$, $K \to 2\pi, 3\pi, \ldots$
- Involve product of two hadronic currents
- Restrict to lightest quarks, relevant part of $\mathcal{L}_W$

$$-\frac{G}{\sqrt{2}}\bar{d}'\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d' = -\frac{G}{\sqrt{2}}[\cos^2\theta_C\bar{d}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d + \sin^2\theta_C\bar{s}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}s$$
$$+\sin\theta_C\cos\theta_C\left(\bar{d}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}s + \bar{s}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d\right)]$$

$$d' = \cos\theta_C d + \sin\theta_C s$$

Strangeness-changing non-leptonic interactions



- Responsible for $\Sigma \to N\pi$, $K \to 2\pi, 3\pi, \ldots$
- Involve product of two hadronic currents
- Restrict to lightest quarks, relevant part of $\mathcal{L}_W$

$$-\frac{G}{\sqrt{2}}\bar{d}'\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d' = -\frac{G}{\sqrt{2}}[\cos^2\theta_C\bar{d}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d + \sin^2\theta_C\bar{s}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}s$$
$$+\sin\theta_C\cos\theta_C\left(\bar{d}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}s + \bar{s}\mathcal{O}_L^\alpha u\bar{u}\mathcal{O}_{L\alpha}d\right)]$$

$$d' = \cos\theta_C d + \sin\theta_C s$$

Strangeness-changing non-leptonic interactions (contd.)

Recalling $\mathcal{O}_L^\alpha = 2\gamma^\alpha P_L$ with $P_L = \frac{1-\gamma^5}{2}$, interesting part

$$\mathcal{L}_{\rm SCNL} = -2\sqrt{2}G\sin\theta_C\cos\theta_C\left(\bar{s}_L\gamma^\alpha u_L\bar{u}_L\gamma_\alpha d_L + \bar{d}_L\gamma^\alpha u_L\bar{u}_L\gamma_\alpha s_L\right)$$

- strangeness-changing, realises effectively $s \to d$ but not through a flavour-changing neutral current
- strangeness either decreased or increased by 1
  $\Rightarrow$ selection rule $|\Delta S| = 1$
- isospin: $\Delta I_3 = \pm\frac{1}{2}$,
  - $\bar{s}_L\gamma^\alpha u_L$ and $\bar{u}_L\gamma_\alpha s_L$ part of isodoublets $I = \frac{1}{2}$
  - $\bar{u}_L\gamma_\alpha d_L$ and h.c. contain $\frac{1}{2}\otimes\frac{1}{2} = 0\oplus 1$
  - $\frac{1}{2}\otimes(0\oplus 1) = \frac{1}{2}\oplus(\frac{1}{2}\oplus\frac{3}{2})$

  $\Rightarrow$ selection rule $|\Delta I| = \frac{1}{2}, \frac{3}{2}$

Suppression of $\Delta I = \frac{3}{2}$ transitions

Take the second term (first term analogous)

$$2\bar{d}_L\gamma^\alpha u_L\bar{u}_L\gamma_\alpha s_L = (\bar{d}_L\gamma^\alpha u_L\bar{u}_L\gamma_\alpha s_L - \bar{u}_L\gamma^\alpha u_L\bar{d}_L\gamma_\alpha s_L)$$
$$+ (\bar{d}_L\gamma^\alpha u_L\bar{u}_L\gamma_\alpha s_L + \bar{u}_L\gamma^\alpha u_L\bar{d}_L\gamma_\alpha s_L) = M_+ + M_-$$
$$M_\pm = (\bar{d}_L \otimes \bar{u}_L \pm \bar{u}_L \otimes \bar{d}_L)_{ai,bj}\,(\gamma^\alpha u_L \otimes \gamma_\alpha s_L)_{ai,bj}$$

Lorentz indices $a, b = 1, \ldots, 4$, colour indices $i, j = 1, 2, 3$

- Second factor has $I = \frac{1}{2}$
- First factor (anti)symmetric product of isodoublet $I = \frac{1}{2}$, fundamental ($\mathbf{3}$) colour representations
  - symmetric (+): isotriplet $I = 1$, colour $\mathbf{6}$ representation
  - antisymmetric (–): isosinglet $I = 0$, colour $\bar{\mathbf{3}}$ representation

Overall $\Rightarrow$

- antisymmetric $I_3 = M_- + \mathrm{h.c.}$ mediates $|\Delta I| = \frac{1}{2}$ transitions
- symmetric $I_6 = M_+ + \mathrm{h.c.}$ mediates $|\Delta I| = \frac{1}{2}, \frac{3}{2}$ transitions

$$\mathcal{L}_{\mathrm{bare}} = -\sqrt{2}G\sin\theta_C\cos\theta_C\,(I_3 + I_6)$$

Suppression of $\Delta I = \frac{3}{2}$ transitions (contd.)

Experimental fact: $|\Delta I| = \frac{1}{2}$ transitions are around one order of magnitude enhanced (in amplitude) wrt $|\Delta I| = \frac{3}{2}$ transitions

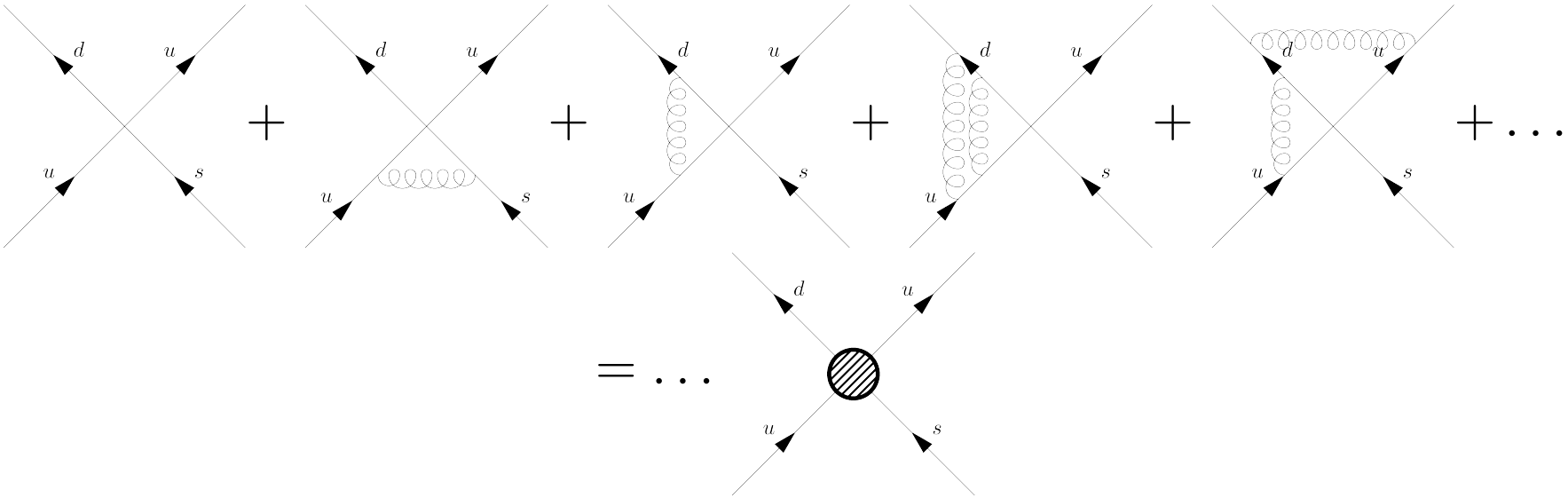
- no particular apparent difference between $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ parts
- reason must be dynamical, enhancement and suppression result from interplay of weak and other interactions
- "bare" Lagrangian before further interactions "dress" it

What is the effect of strong interactions?

- strong interactions $\approx$ exchange of gluons between quarks
  - soft gluon exchange, with low transferred momentum $q^2 < \mu^2$
  - hard gluon exchange, with high transferred momentum $q^2 > \mu^2$
- accurate perturbative treatment requires *running coupling constant*

  Depends on energy scale of the process
- in QCD due to *asymptotic freedom* the running coupling constant becomes small at high energy, while it is large at low energies
- high-energy processes (like hard gluon exchange) can be dealt with perturbatively, low-energy ones (like soft gluon exchange) cannot
- separation scale $\mu \sim 1/L_{\text{confinement}} \approx 100\ \text{MeV} \div 1\ \text{GeV}$

Suppression of $\Delta I = \frac{3}{2}$ transitions (contd.)



In the context of weak hadronic interactions

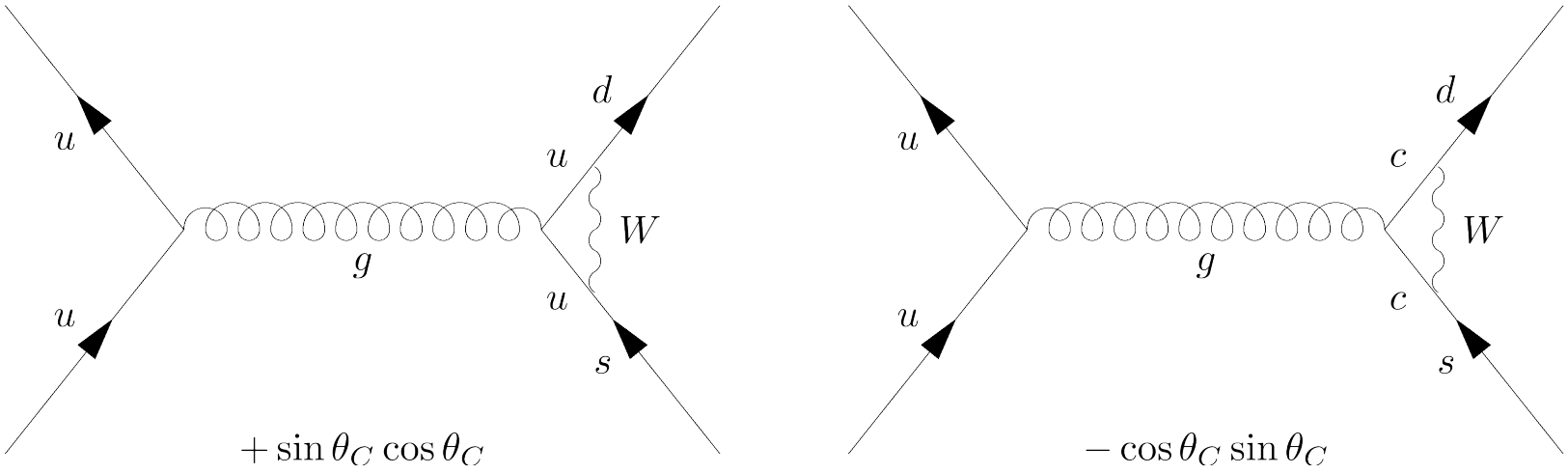
- effect of soft gluon exchange included in hadronic wave functions
- effect of hard gluons can be studied explicitly in perturbation theory

First type of diagrams: add g to W exchanges between different q lines

- low-energy limit $\rightarrow$ local four-fermion interaction
- effectively reduce to original vertex $I_{3,6}$ receive different contributions

$$I_3 + I_6 \rightarrow a_3 I_3 + a_6 I_6$$

Suppression of $\Delta I = \frac{3}{2}$ transitions (contd.)



Second type of diagrams: add g exchanges to emission and subsequent reabsorption of W in the same q line

- after emitting W, s turn into u or c but with opposite coupling
- would cancel exactly if $m_u = m_c$, but $m_u \ll m_c$ so they do not
- new effective four-fermion interaction (also including more g)

$$I_R = -(\bar{d}_L \gamma^\alpha \lambda^a s_L)(\bar{u}_R \gamma_\alpha \lambda^a u_R + \bar{d}_R \gamma_\alpha \lambda^a d_R)\,,$$

λ^a: Gell-Mann matrices, $a = 1, \ldots, 8$

Suppression of $\Delta I = \frac{3}{2}$ transitions (contd.)

$$I_R = -(\bar{d}_L \gamma^\alpha \lambda^a s_L)(\bar{u}_R \gamma_\alpha \lambda^a u_R + \bar{d}_R \gamma_\alpha \lambda^a d_R)\,,$$

- first factor $I = \frac{1}{2}$, second factor $I = 0 \Rightarrow \Delta I = \frac{1}{2}$
- involves colour and most importantly R-component of the quark fields

I_L with $u_R, d_R \to u_L, d_L$ also present but small overall coupling, can be neglected compared to I_3

Effective, "dressed" Lagrangian

$$\mathcal{L}_{\text{dressed}} = -\sqrt{2} G \sin\theta_C \cos\theta_C \left(a_3 I_3 + a_6 I_6 + a_R I_R\right)$$

- perturbative calculation: $a_3 \simeq 3$, $a_6 \simeq 0.6$, $a_R \simeq 0.12$
- $\Delta I = \frac{3}{2}$ transitions mediated only by $I_6 \Rightarrow$ suppression...
- ...but degree of suppression not enough to agree with experiments
- issues involving good understanding of hadronic interactions more complicated to study theoretically

Non-leptonic decays of kaons: neutral kaons

Main non-leptonic kaon decays: $K \to 2\pi, 3\pi$

$$K^0, \bar{K}^0 \to \pi^+\pi^-, \pi^0\pi^0 \qquad K^0, \bar{K}^0 \to \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0$$
$$K^\pm \to \pi^\pm\pi^0 \qquad K^\pm \to \pi^\pm\pi^+\pi^-, \pi^\pm\pi^0\pi^0$$

$K^0(d\bar{s})$, $\bar{K}^0(-s\bar{d})$: S eigenstates but not CP (or C) eigenstates

Produced in strong interactions with definite S and indefinite CP, decay via weak interactions which conserve CP (in two-family approx.) but not S $\Rightarrow$ convenient to use linear combinations of $K^0, \bar{K}^0$ that are CP rather than S eigenstates to study their weak decays

Physically allowed: K^0 and $\bar{K}^0$ differ only in S for which there is no superselection rule, mixed by a second-order weak interaction

$P|K^0\rangle = -|K^0\rangle$, $P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$, choose $C|K^0\rangle = |\bar{K}^0\rangle$, $C|\bar{K}^0\rangle = |K^0\rangle$

$$|K_1^0\rangle = \frac{K^0 - \bar{K}^0}{\sqrt{2}} \qquad CP|K_1^0\rangle = |K_1^0\rangle$$
$$|K_2^0\rangle = \frac{K^0 + \bar{K}^0}{\sqrt{2}} \qquad CP|K_2^0\rangle = -|K_2^0\rangle$$

Non-leptonic decays of kaons: neutral kaons (contd.)

$2, 3\pi$ states with definite orbital angular momentum are CP eigenstates

In the decay of neutral K

- 2π system produced with $\ell = 0$ hence always $CP = 1$
  - $\pi^+\pi^-$ system has $CP = 1$ independently of ℓ
  - $\pi^0\pi^0$ system has $CP = (-1)^\ell$
- 3π system: two orbital angular momenta, ℓ ($\pi^+\pi^-$ or one pair of π^0s) and L (motion of third π wrt CM of other two), with $\ell = L$ ($J_K = 0$)
  - $\pi^+\pi^-\pi^0$: charged pair CP-invariant,

$$CP|\pi^+\pi^-\pi^0\rangle = -(-1)^L|\pi^+\pi^-\pi^0\rangle = (-1)^{\ell+1}|\pi^+\pi^-\pi^0\rangle$$

  - $\pi^0\pi^0\pi^0$:

$$CP|\pi^0\pi^0\pi^0\rangle = (-1)^3(-1)^{\ell+L}|\pi^0\pi^0\pi^0\rangle = -|\pi^0\pi^0\pi^0\rangle$$

In summary

- $CP = 1$ for two-pion final states, and for three-pion $\pi^+\pi^-\pi^0$ if ℓ of the charged pair is odd (process suppressed for phase-space reasons)
- $CP = -1$ for three-pion $\pi^0\pi^0\pi^0$ states and for three-pion $\pi^+\pi^-\pi^0$ if ℓ of charged pair is even

References

- R. L. Garwin, L. M. Lederman, M. Weinrich, Phys.Rev. 105 (1957) 1415
- S. Fubini, G. Furlan, Physics Physique Fizika 1 (1965) 229