Weak Interactions

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Strangeness-changing leptonic decays of hadrons (contd.)

SU(3) flavour symmetry relates S-conserving and S-changing processes

Octet of currents $j^\alpha = j^\alpha_a t_a$

$$(j^\alpha)^i{}_j = \bar{q}^i \mathcal{O}^\alpha_L q_j - \frac{1}{3}\sum_m \bar{q}^m \mathcal{O}^\alpha_L q_m \qquad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

- charged weak currents belong to this octet

$$(j^\alpha)^1{}_2 = \bar{u}\mathcal{O}^\alpha_L d \quad (j^\alpha)^2{}_1 = \bar{d}\mathcal{O}^\alpha_L u \quad (j^\alpha)^1{}_3 = \bar{u}\mathcal{O}^\alpha_L s \quad (j^\alpha)^3{}_1 = \bar{s}\mathcal{O}^\alpha_L u$$

- two independent flavour-diagonal, neutral currents

$$\frac{1}{\sqrt{2}}\left(\bar{u}\mathcal{O}^\alpha_L u - \bar{d}\mathcal{O}^\alpha_L d\right) \qquad \frac{1}{\sqrt{6}}\left(\bar{u}\mathcal{O}^\alpha_L u + \bar{d}\mathcal{O}^\alpha_L d - 2\bar{s}\mathcal{O}^\alpha_L s\right)$$

  first one part of an isovector with $\bar{u}\mathcal{O}^\alpha_L d$, $\bar{d}\mathcal{O}^\alpha_L u$, second one isoscalar
- remaining currents are the FCNC $\bar{d}\mathcal{O}^\alpha_L s$, $\bar{s}\mathcal{O}^\alpha_L d$, complete the isospin doublets of the strangeness-changing currents (not in $\mathcal{L}_{\rm weak}$)

Weak int. not SU(3) invariant $\Rightarrow$ different roles played by different $(j^\alpha)^i{}_j$

Strong int. (approx.) SU(3) invariant $\Rightarrow$ relations among decay amplitudes

$K_{\ell 2}$ decays

Purely leptonic decay of K

$$K^+ \to \ell^+ \nu_\ell$$

- analogues of the charged pion decays $\pi^+ \to \ell^+ \nu_\ell$
- would have exactly the same amplitude in the SU(3)-symmetric limit
- only axial current contributions: for kaons of momentum p

$$H^\alpha = \langle 0|\bar{s}\mathcal{O}_L^\alpha u|K^+\rangle = i\sqrt{2} f_K p^\alpha \,,$$

f_K: kaon decay constant (real, m-dimension 1)
In the SU(3)-symmetric limit $f_K = f_\pi$, $m_K = m_\pi$

From momentum conservation $p = p_\ell + p_\nu$

$$\begin{aligned}\mathcal{M}_{\rm fi} &= -\frac{G}{\sqrt{2}} \sin\theta_C \, i\sqrt{2} f_K p^\alpha \, \bar{u}_{\nu_\ell} \mathcal{O}_L^\alpha v_\ell = -iG \sin\theta_C f_K \, \bar{u}_{\nu_\ell} \not{p} (1-\gamma^5) v_\ell \\ &= -iG \sin\theta_C f_K \, \bar{u}_{\nu_\ell} (\not{p}_\nu + \not{p}_\ell)(1-\gamma^5) v_\ell = -iG \sin\theta_C f_K \, \bar{u}_{\nu_\ell} (1+\gamma^5) \not{p}_\ell v_\ell \\ &= iG \sin\theta_C f_K \, m_\ell \bar{u}_{\nu_\ell} (1+\gamma^5) v_\ell\end{aligned}$$

Entirely analogous to charged pion decay after $\cos\theta_C \to \sin\theta_C$, $f_\pi \to f_K$

$K_{\ell 2}$ decays (contd.)

Following the same steps

$$\Gamma = \frac{G^2 \sin^2\theta_C f_K^2}{8\pi} m_K m_\ell^2 \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2$$

From $\sin\theta_C \simeq 0.21$, $m_\pi = 140\ \mathrm{MeV}$, $m_K = 497\ \mathrm{MeV}$, $m_\mu = 106\ \mathrm{MeV}$, and

$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) \simeq \Gamma(\pi^+) = \left(2.6 \cdot 10^{-8} s\right)^{-1}$$
$$\Gamma(K^+ \to \mu^+ \nu_\mu) \simeq 0.63\,\Gamma(K^+) = 0.63\left(1.2 \cdot 10^{-8} s\right)^{-1}$$

$$\Longrightarrow \frac{f_\pi}{f_K} = 1.3$$

Charged kaon decays: 63% $\mu\nu_\mu$, 28% hadrons + significant contributions from K_{e3} and $K_{\mu 3}$

$\frac{m_K}{m_\pi} \simeq 3.5 \Rightarrow$ ratio $\frac{\Gamma(K^+ \to e^+ \nu_\mu)}{\Gamma(K^+ \to \mu^+ \nu_\mu)}$ closer to asymptotic limit $(\frac{m_e}{m_\mu})^2 = 2 \cdot 10^{-5}$

$K_{\ell 3}$ decays

$K^+ \to \ell^+ \nu_\ell \pi^0$, $K^0 \to \ell^+ \nu_\ell \pi^-$ similar to pion β decay

Momentum conservation $p_K = p_\pi + p_\ell + p_\nu$

Set $p = p_K + p_\pi$, $q = p_K - p_\pi$

$$H_\alpha^{(+)} = \langle \pi^0 | \bar{s} \mathcal{O}_{L\alpha} u | K^+ \rangle = f_+^{(+)}(q^2) p_\alpha + f_-^{(+)}(q^2) q_\alpha$$
$$H_\alpha^{(0)} = \langle \pi^- | \bar{s} \mathcal{O}_{L\alpha} u | K^0 \rangle = f_+^{(0)}(q^2) p_\alpha + f_-^{(0)}(q^2) q_\alpha$$

Only vector current contributions, $f_\pm^{(+,0)}$ real (in CP-symmetric case)

$$\mathcal{M}_{\rm fi}^{(+,0)} = -\frac{G}{\sqrt{2}} \sin\theta_c [f_+^{(+,0)}(q^2) p_\alpha + f_-^{(+,0)}(q^2) q_\alpha] \bar{u}_{\nu_\ell} \gamma^\alpha (1 - \gamma^5) v_\ell$$

$f_-^{(+,0)}$ term $\propto m_\ell$ (plug $q = p_\ell + p_\nu$ into leptonic matrix element), negligible in K_{e3} decays

Approximation $q^2 \simeq 0$ less accurate than in π β-decay but still reasonable

$K_{\ell 3}$ decays (contd.)

In the SU(3)-symmetric limit $f_+^{(+,0)}(0)$ determined from symmetry
Relevant weak current related to SU(3) generator

$$\int d^3x \, \left(\bar{s}\mathcal{O}_L^0 u\right)(0,\vec{x}) = V_-$$

Matrix elements

$$(2\pi)^3\delta^{(3)}(\vec{q}\,)\langle f|\left(\bar{s}\mathcal{O}_L^0 u\right)(0)|i\rangle = \langle f|V_-|i\rangle = 2p^0(2\pi)^3\delta^{(3)}(\vec{q}\,)\langle\langle f|V_-|i\rangle\rangle$$

$\langle\langle\ldots\rangle\rangle$: only flavour part of wave functions, $\vec{q}=0$

$$f_+^{(+)}(0) = \langle\langle\pi^0|V_-|K^+\rangle\rangle \qquad f_+^{(0)}(0) = \langle\langle\pi^-|V_-|K^0\rangle\rangle$$

Meson octet: $\begin{cases} |K^+\rangle\rangle = \bar{s}u & |K^0\rangle\rangle = \bar{s}d \\ |\pi^+\rangle\rangle = \bar{d}u & |\pi^0\rangle\rangle = \frac{\bar{d}d-\bar{u}u}{\sqrt{2}} \qquad |\pi^-\rangle\rangle = -\bar{u}d \end{cases}$

From commutation relations $[I_3, V_-] = -\frac{1}{2}V_-$, $[Y, V_-] = -V_-$

$$|K^0\rangle\rangle = I_-|K^+\rangle\rangle \qquad |\pi^+\rangle\rangle = -W_-|K^+\rangle\rangle$$

$$\sqrt{2}|\pi^0\rangle\rangle = I_-|\pi^+\rangle\rangle \qquad \sqrt{2}|\pi^-\rangle\rangle = I_-|\pi^0\rangle\rangle \qquad \sqrt{2}|\pi^0\rangle\rangle = I_+|\pi^-\rangle\rangle$$

$K_{\ell 3}$ decays (contd.)

π^- and K^0 are $V_3 = -\frac{1}{2}$ and $V_3 = \frac{1}{2}$ members of V-spin doublet $V = \frac{1}{2}$
$\Rightarrow f_+^{(0)}(0)$ from "weak charge" of V-spin (similar to neutron β-decay)

$$f_+^{(0)}(0) = \sqrt{V(V+1) - V_3(V_3-1)}|_{V=V_3=\frac{1}{2}} = 1$$

Using $I_+|\pi^-\rangle = \sqrt{2}|\pi^0\rangle$ and $[I_-, V_-] = 0$

$$\langle\langle\pi^0|V_-|K^+\rangle\rangle = \frac{1}{\sqrt{2}}\langle\langle\pi^-|I_-V_-|K^+\rangle\rangle = \frac{1}{\sqrt{2}}\langle\langle\pi^-|V_-I_-|K^+\rangle\rangle = \frac{1}{\sqrt{2}}\langle\langle\pi^-|V_-|K^0\rangle\rangle$$

$$\Longrightarrow \quad f_+^{(+)}(0) \underset{\text{isospin limit}}{=} \frac{1}{\sqrt{2}} f_+^{(0)}(0) \underset{\text{SU(3) limit}}{=} \frac{1}{\sqrt{2}}$$

- first relation exact in isospin limit, so same degree of accuracy as isospin symmetry
- *Ademollo-Gatto theorem*: corrections to $f_+^{(0)}(0)$ due to SU(3) symmetry breaking quadratic in the symmetry breaking parameter $\delta_{\mathrm{SU(3)}}$ (i.e., $m_s - \frac{m_u+m_d}{2}$), $f_+^{(0)}(0) = 1$ reasonable approximation

$K_{\ell 3}$ decays (contd.)

- In the SU(3)-symmetric limit $f_-^{(+,0)}(q^2) = 0$ due to current conservation (extends to the octet of currents) but no theorem preventing large corrections due to SU(3) breaking
- Using PCAC hypothesis $\Rightarrow$ *Callan-Treiman relation*

$$f_+(m_K^2) + f_-(m_K^2) = \frac{f_K}{f_\pi}$$

  - relates $K_{\ell 2}$ and $K_{\ell 3}$ decays
  - involves form factors at unphysical arguments since

$$m_\ell^2 \leq q^2 \leq (m_K - m_\pi)^2 \,,$$

    needs extrapolation from experimental data
  - good agreement with experimental results
- for K_{e3}^+ it is possible to obtain a reasonably accurate theoretical prediction since the $f_-^{(+)}$ term can be neglected

$K_{\ell 3}$ decays (contd.)

K^+_{e3} decay: neglect $f_-^{(+)}$, $f_+^{(+)}(q^2)$ fitted to experimental results

$$f_+^{(+)}(q^2) = f_+^{(+)}(0)\left(1 + \lambda_+^{(+)}\frac{q^2}{m_{\pi^+}^2}\right)$$

- $f_+^{(+)}(0) \simeq \frac{1}{\sqrt{2}}$ from SU(3) symmetry
- fit to experimental results gives $\lambda_+^{(+)} \simeq 0.03$
- ultrarelativistic electron, its mass can be neglected

Decay width

$$\Gamma = \frac{G^2 \sin\theta_C^2 \left(f_+^{(+)}(0)\right)^2}{12\pi^3} m_K^5 \left(\frac{m_{\pi^0}}{m_K}\right)^4 \left\{1.62 + \lambda_+^{(+)} 5.988\right\}$$

$$= \frac{G^2 \sin\theta_C^2 \left(f_+^{(+)}(0)\right)^2}{768\pi^3} m_K^5 \left\{0.58 + \lambda_+^{(+)} 2.1\right\}$$

Comparison with experiments allows to determine $\sin\theta_C$

Intermezzo: Ademollo-Gatto theorem

Proof as for isospin corrections, using V-spin instead of I-spin

$$\underbrace{[V_+, V_-] = 2V_3 = I_3 + \tfrac{3}{2}Y}_{\substack{\text{generators in SU(3) limit,}\\ \text{obey SU(3) comm. rel.}}} = \underbrace{Q + Y}_{\substack{\text{exactly conserved also}\\ \text{away from SU(3) limit,}\\ \text{unchanged}}}$$

$$q + y = 2\langle\langle h_{V\,V_3}|V_3|h_{V\,V_3}\rangle\rangle = \sum_n |\langle\langle n|V_-|h_{V\,V_3}\rangle\rangle|^2 - |\langle\langle n|V_+|h_{V\,V_3}\rangle\rangle|^2$$

- $|K^0\rangle\rangle = |h_{V,V}\rangle\rangle$ with $V = V_3 = \frac{1}{2}$
- π^- only **8** state with V_- matrix element with $K^0 \neq 0$ in SU(3) limit, $|\langle\langle\pi^-|V_-|K^0\rangle\rangle|^2|_{\text{SU(3) limit}} = 1$

$$\begin{aligned} 1 &= |\langle\langle\pi^-|V_-|K^0\rangle\rangle|^2 + \sum_n{}' |\langle\langle n|V_-|K^0\rangle\rangle|^2 - |\langle\langle n|V_+|K^0\rangle\rangle|^2 \\ &= |\langle\langle\pi^-|V_-|K^0\rangle\rangle|^2 + \mathcal{O}(\delta^2_{\mathrm{SU(3)}}) \end{aligned}$$

$$f_+^{(0)}(0) = \lim_{|\vec{p}_K|=|\vec{p}_\pi|\to\infty} \langle\langle\pi^-|V_-|K^0\rangle\rangle = f_+^{(0)}(0)|_{\text{SU(3) limit}} + \mathcal{O}(\delta^2_{\mathrm{SU(3)}})$$

References

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