

Weak Interactions

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Neutron beta decay: decay rate

To leading order (ignoring $\mathcal{O}(\Delta)$ in matrix elements): set $q^2 = 0$, keep only term $\propto f_1(0) = g_V = 1$ and $g_1(0) = g_A$

$$\mathcal{M}_{\text{fi}} = -\frac{G}{\sqrt{2}} \cos \theta_C \bar{u}_p \gamma^\mu (1 - \alpha \gamma^5) u_n \bar{u}_e \gamma_\mu (1 - \gamma^5) v_{\nu_e}$$

$$\alpha = g_1(0)/f_1(0) = g_A/g_V$$

Neutron and the electron polarisation vectors

$$s_n = (0, \vec{\eta}_n) \quad s_e = \left(\frac{\vec{\eta}_e \cdot \vec{k}_e}{m_e}, \vec{\eta}_e + \frac{\vec{k}_e (\vec{k}_e \cdot \vec{\eta}_e)}{m_e(E_e + m_e)} \right)$$

Amplitude square:

$$|\mathcal{M}_{\text{fi}}|^2 = \frac{G^2 \cos^2 \theta_C}{2} \text{tr} \gamma^\mu (1 - \alpha \gamma^5) (\not{p} + m_n) \frac{1 + \gamma^5 \not{s}_n}{2} \gamma^\nu (1 - \alpha \gamma^5) (\not{p}' + m_p) \\ \times \text{tr} \gamma_\mu (1 - \gamma^5) \not{k}_{(\nu)} \gamma_\nu (1 - \gamma^5) (\not{k}_e + m_e) \frac{1 + \gamma^5 \not{s}_e}{2}$$

Sum over proton spin, antineutrino helicity fixed

Neutron beta decay: decay rate (contd.)

Leptonic trace

$$\begin{aligned} & \text{tr } \gamma_\mu (1 - \gamma^5) \not{k}_{(\nu)} \gamma_\nu (1 - \gamma^5) (\not{k}_e + m_e) \frac{1 + \gamma^5 \not{s}_e}{2} = \text{tr} (1 + \gamma^5) \gamma_\mu \not{k}_{(\nu)} \gamma_\nu \not{k}_e \\ &= 4 \left\{ k_{(\nu)\mu} \not{k}_{e\mu} + k_{(\nu)\nu} \not{k}_{e\mu} - \eta_{\mu\nu} k_{(\nu)} \cdot \not{k}_e - i \epsilon_{\mu\alpha\nu\beta} k_{(\nu)}^\alpha \not{k}_e^\beta \right\} \\ & \quad \not{k}_e = k_e - m_e s_e \\ & \quad E_{e, \bar{\nu}_e}, |\vec{p}_{e, \bar{\nu}_e}|, m_e \text{ of order } \Delta = m_n - m_p, \text{ trace of order } \Delta^2 \end{aligned}$$

Hadronic trace

$$\begin{aligned} & \text{tr } \gamma^\mu (1 - \alpha \gamma^5) (\not{p} + m_n) (1 + \gamma^5 \not{s}_n) \gamma^\nu (1 - \alpha \gamma^5) (\not{p}' + m_p) \\ &= \text{tr } \gamma^\mu [(1 + \alpha^2) \not{p} - 2\alpha m_n \not{s}_n] \gamma^\nu \not{p}' - \text{tr } \gamma^5 \gamma^\mu [(1 + \alpha^2) m_n \not{s}_n - 2\alpha \not{p}] \gamma^\nu \not{p}' \\ & \quad + (1 - \alpha^2) m_n m_p \text{tr } \gamma^\mu \gamma^\nu + (1 - \alpha^2) m_p \text{tr } \gamma^5 \gamma^\mu \not{p} \not{s}_n \gamma^\nu \end{aligned}$$

Retain only leading order \Rightarrow set $m_n = m_p = m$ and $p' = p$

$$\begin{aligned} & \text{tr } \gamma^\mu (1 - \alpha \gamma^5) (\not{p} + m_n) (1 + \gamma^5 \not{s}_n) \gamma^\nu (1 - \alpha \gamma^5) (\not{p}' + m_p) \rightarrow \\ &= 8 \left\{ (1 + \alpha^2) p^\mu p^\nu - \alpha^2 m^2 \eta^{\mu\nu} - \alpha m (s_n^\mu p^\nu + s_n^\nu p^\mu) + i \alpha^2 m \epsilon^{\mu\alpha'\nu\beta'} s_{n\alpha'} p_{\beta'} \right\} \end{aligned}$$

Neutron beta decay: decay rate (contd.)

Contracting traces and including all factors

$$\begin{aligned} |\mathcal{M}_{\text{fi}}|^2 &= \frac{G^2 \cos^2 \theta_C}{2} 4^2 \left\{ (1 + \alpha^2) p^\mu p^\nu - \alpha^2 m^2 \eta^{\mu\nu} - \alpha m (s_n^\mu p^\nu + s_n^\nu p^\mu) \right. \\ &\quad \left. + i \alpha^2 m \epsilon^{\mu\alpha'\nu\beta'} s_{n\alpha'} p_{\beta'} \right\} \\ &\times \left\{ k_{(\nu)\mu} \tilde{k}_{e\mu} + k_{(\nu)\nu} \tilde{k}_{e\mu} - \eta_{\mu\nu} k_{(\nu)} \cdot \tilde{k}_e - i \epsilon_{\mu\alpha\nu\beta} k_{(\nu)}^\alpha \tilde{k}_e^\beta \right\} \\ &= 8 G^2 \cos^2 \theta_C \left\{ 2(1 + \alpha^2) p \cdot k_{(\nu)} p \cdot \tilde{k}_e - (1 - \alpha^2) m^2 k_{(\nu)} \cdot \tilde{k}_e \right. \\ &\quad \left. + 2m[(\alpha^2 - \alpha) s_n \cdot \tilde{k}_e p \cdot k_{(\nu)} - (\alpha^2 + \alpha) s_n \cdot k_{(\nu)} p \cdot \tilde{k}_e] \right\} \end{aligned}$$

Reminder:

$$p = (m, \vec{0})$$

$$s_n = (0, \vec{\eta}_n)$$

$$\tilde{k}_e = k_e - m_e s_e$$

$$s_e = \left(\frac{\vec{\eta}_e \cdot \vec{k}_e}{m_e}, \vec{\eta}_e + \frac{\vec{k}_e (\vec{k}_e \cdot \vec{\eta}_e)}{m_e (E_e + m_e)} \right)$$

Unpolarised neutron, no electron spin measurement

Sum over e spin, average over n spin $\Rightarrow s_n \rightarrow 0$, $\tilde{k}_{(e)} \rightarrow k_{(e)}$, $\times \frac{4}{2} = 2$ in $d\Gamma$

$$\begin{aligned} d\Gamma &= \frac{8G^2 \cos^2 \theta_C}{m} \{2(1 + \alpha^2)m^2 E_e E_\nu - (1 - \alpha^2)m^2(E_e E_\nu - \vec{k}_{(\nu)} \cdot \vec{k}_{(e)})\} d^{(3)}\Phi \\ &= 8G^2 \cos^2 \theta_C m E_e E_\nu \{(1 + 3\alpha^2) + (1 - \alpha^2)\beta_e \cos \theta\} d^{(3)}\Phi \end{aligned}$$

β_e : electron velocity, θ : relative angle between e and $\bar{\nu}_e$ trajectories

Phase space element neglecting proton recoil $E_p - m_p = \frac{\vec{p}'^2}{E_p + m_p} = o(\Delta)$

$$\begin{aligned} d^{(3)}\Phi &= \frac{1}{8(2\pi)^5} \frac{d^3 k_e}{E_e} \frac{d^3 k_\nu}{E_\nu} \frac{d^3 p'}{E_p} \delta^{(4)}(p - p' - k_{(\nu)} - k_{(e)}) \\ &= \frac{1}{8(2\pi)^5} \frac{d^3 k_e}{E_e} \frac{d^3 k_\nu}{E_\nu} \frac{1}{E_p} \delta(m_n - E_p - E_\nu - E_e) \quad (E_p^2 = m_p^2 + (\vec{k}_\nu + \vec{k}_e)^2) \\ &= \frac{1}{8(2\pi)^5} \frac{d^3 k_e}{E_e} \frac{d^3 k_\nu}{E_\nu} \frac{1}{E_p} \left(1 + \frac{\partial E_p}{\partial E_\nu}\right)^{-1} \delta(E_\nu - E_\nu^*) \\ \frac{\partial E_p}{\partial E_\nu} &= \frac{1}{E_p} (E_\nu + |\vec{k}_e| \cos \theta) \ll 1 \quad E_\nu + E_e = m_n - m_p - (E_p - m_p) = \Delta + o(\Delta) \end{aligned}$$

$$\begin{aligned} d^{(3)}\Phi &\simeq \frac{1}{8(2\pi)^5} \frac{d^3 k_e}{E_e} \frac{d^3 k_\nu}{E_\nu} \frac{1}{m} \delta(E_\nu + E_e - \Delta) \\ &= \frac{1}{8(2\pi)^5 m E_e E_\nu} dE_e E_e \sqrt{E_e^2 - m_e^2} d\Omega_{(e)} dE_\nu E_\nu^2 d\Omega_{(\nu)} \delta(E_\nu + E_e - \Delta) \end{aligned}$$

Unpolarised neutron,no electron spin measurement(contd.)

Angular correlation $C(\cos \theta)$ between e and $\bar{\nu}_e$ from $d\Gamma$

Measured indirectly from proton recoil

$$C(\cos \theta) = \frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta} = 1 + \frac{1-\alpha^2}{1+3\alpha^2} \beta_e \cos \theta$$

Electron energy distribution

$$\begin{aligned} d\Gamma &= 8G^2 \cos^2 \theta_C (1 + 3\alpha^2) \frac{1}{8(2\pi)^5} (4\pi)^2 dE_e E_e \sqrt{E_e^2 - m_e^2} (\Delta - E_e)^2 \\ &= \frac{G^2 \cos^2 \theta_C}{2\pi^3} (1 + 3\alpha^2) dE_e E_e \sqrt{E_e^2 - m_e^2} (\Delta - E_e)^2 \end{aligned}$$

Total decay width $\Gamma = \int_{m_e}^{\Delta} dE_e \frac{d\Gamma}{dE_e}$

$$\boxed{\Gamma = \frac{G^2 \cos^2 \theta_C}{2\pi^3} (1 + 3\alpha^2) \Delta^5 I\left(\frac{m_e}{\Delta}\right)}$$

$$I(z) = \int_z^1 dx x \sqrt{x^2 - z^2} (1 - x)^2 = \frac{2}{3} \left\{ -\frac{1}{5} (1 - z^2)^{\frac{5}{2}} + \frac{3}{8} z^4 \operatorname{arccosh} \frac{1}{z} + \frac{1}{4} \sqrt{1 - z^2} [1 - \frac{5}{2} z^2] \right\}$$

$$\boxed{\Gamma = 0.47 \frac{G^2 \cos^2 \theta_C}{60\pi^3} (1 + 3\alpha^2) \Delta^5}$$

$$m_e = 0.51 \text{ MeV}, \Delta = 1.29 \text{ MeV}, z = 0.395 \rightarrow \frac{I(z)}{I(0)} = 0.47, I(0) = \frac{1}{30}$$

Unpolarised neutron, electron spin measurement

Average over neutron spin $\Rightarrow s_n \rightarrow 0, \times \frac{1}{2} = 1$ in $d\Gamma$

$$\begin{aligned} d\Gamma &= \frac{4G^2 \cos^2 \theta_C}{m} \left\{ 2(1 + \alpha^2) p \cdot k_{(\nu)} p \cdot \tilde{k}_{(e)} - (1 - \alpha^2) m^2 k_{(\nu)} \cdot \tilde{k}_{(e)} \right\} d\Phi^{(3)} \\ &= \frac{4G^2 \cos^2 \theta_C}{m} \left\{ 2(1 + \alpha^2) m^2 E_\nu \tilde{k}_{(e)}^0 - (1 - \alpha^2) m^2 (E_\nu \tilde{k}_{(e)}^0 - \vec{k}_\nu \cdot \vec{\tilde{k}}_e) \right\} d\Phi^{(3)} \\ &= 4G^2 \cos^2 \theta_C m E_\nu E_e \left\{ (1 + 3\alpha^2) \frac{\tilde{k}_{(e)}^0}{E_e} + (1 - \alpha^2) \frac{\vec{k}_\nu}{E_\nu} \cdot \frac{\vec{\tilde{k}}_e}{E_e} \right\} d\Phi^{(3)} \end{aligned}$$

Integrating over \vec{k}_ν , we read off the electron longitudinal polarisation P_e :

- taking the relative angle between electron and neutrino trajectories as polar angle, second term in braces drops
- longitudinal polarisation means $\vec{\eta}_e = \hat{k}_e$

$$p(\vec{k}_e, \vec{\eta}_e) = \frac{d\Gamma(\vec{k}_e, \vec{\eta}_e)}{\Gamma(|\vec{k}_e|)} = \frac{\tilde{k}_{(e)}^0}{E_e} = 1 - \frac{\vec{\eta}_e \cdot \vec{k}_e}{E_e} \Rightarrow P_e = -\frac{|\vec{k}_e|}{E_e} = -\beta_e$$

$$\frac{1}{2} P_e = \text{tr } \rho \hat{k}_e \cdot \vec{S} = \sum_{s=\pm\frac{1}{2}} \frac{1}{2} (1 - (2s) \frac{\hat{k}_e \cdot \vec{k}_e}{E_e}) \langle s; \hat{k}_e | \hat{k}_e \cdot \vec{S} | s; \hat{k}_e \rangle = -\frac{1}{2} \frac{|\vec{k}_e|}{E_e}$$

Polarised neutron, no electron spin measurement

Sum over electron spin $\Rightarrow s_e \rightarrow 0, \times 2$ in $d\Gamma$

$$\begin{aligned} d\Gamma &= \frac{8G^2 \cos^2 \theta_C}{m} \left\{ 2(1 + \alpha^2) p \cdot k_{(\nu)} p \cdot k_{(e)} - (1 - \alpha^2) m^2 k_{(\nu)} \cdot k_{(e)} \right. \\ &\quad \left. + 2m[(\alpha^2 - \alpha)s_n \cdot k_{(e)} p \cdot k_{(\nu)} - (\alpha^2 + \alpha)s_n \cdot k_{(\nu)} p \cdot k_{(e)}] \right\} d^{(3)}\Phi \\ &= 8G^2 \cos^2 \theta_C m E_e E_\nu \left\{ 2(1 + \alpha^2) - (1 - \alpha^2)(1 - \beta_e \cos \theta) \right. \\ &\quad \left. - 2[(\alpha^2 - \alpha)\vec{\eta}_n \cdot \frac{\vec{k}_e}{E_e} - (\alpha^2 + \alpha)\vec{\eta}_n \cdot \frac{\vec{k}_\nu}{E_\nu}] \right\} d^{(3)}\Phi \end{aligned}$$

Integrating over \vec{k}_ν or \vec{k}_e leaves only \vec{k}_e or \vec{k}_ν

$$\frac{d\Gamma}{\Gamma_E} = \begin{cases} 1 - \frac{2(\alpha^2 - \alpha)}{1 + 3\alpha^2} \frac{\vec{\eta}_n \cdot \vec{k}_e}{E_e} & \text{(electron correlation)} \\ 1 + \frac{2(\alpha^2 + \alpha)}{1 + 3\alpha^2} \frac{\vec{\eta}_n \cdot \vec{k}_\nu}{E_\nu} & \text{(neutrino correlation)} \end{cases}$$

$$\Gamma_E(E_{e,\nu}) = \int d\Omega_p^{(\nu,e)} \frac{d\Gamma}{d\Omega_p^{(\nu,e)}}$$

Summary of free neutron decay

$d\Gamma$ in the static approximation ($q^2 \simeq 0$), no e spin measurement

$$d\Gamma = \frac{G^2 \cos^2 \theta_C}{2\pi^3} (1 + 3\alpha^2) \times \left\{ 1 + \frac{1 - \alpha^2}{1 + 3\alpha^2} \vec{\beta}_e \cdot \vec{n}_\nu - 2 \left[\frac{\alpha^2 - \alpha}{1 + 3\alpha^2} \vec{\beta}_e \cdot \vec{\eta}_n - \frac{\alpha^2 + \alpha}{1 + 3\alpha^2} \vec{n}_\nu \cdot \vec{\eta}_n \right] \right\} \times \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi} dE_e E_e \sqrt{E_e^2 - m_e^2} (\Delta - E_e)^2$$

- Angular correlation from coefficients of
 - $\vec{\beta}_e \cdot \vec{n}_\nu$: between the momenta of electron and neutrino
 - $\vec{\beta}_e \cdot \vec{\eta}_n$: between neutron polarisation and electron momentum
 - $\vec{n}_\nu \cdot \vec{\eta}_n$: between neutron polarisation and neutrino momentum
- Axial charge g_A from $\alpha = g_A/g_V \simeq g_A$ (since $g_V \simeq 1$)
 - $|\alpha|$ from e - ν ang. correlation [from p recoil $\frac{\vec{p}_p^2}{2m_p} = E_\nu^2 + \vec{k}_e^2 + 2E_\nu |\vec{k}_e| \cos \theta$]
 - sign of α from angular correlation of the electron with the neutron spin in polarised neutron decay
- From α^2 , τ_n , and G from μ decay $\rightarrow |\cos \theta_C|$

Summary of free neutron decay (contd.)

Electron energy spectrum given by Fermi function (up to constant factors)

$$F(x, W_0) = x\sqrt{x^2 - 1}(W_0 - x)^2 \quad x = \frac{E_e}{m_e} \quad W_0 = \frac{\Delta}{m_e}$$

Spectrum for free n decay, first approximation for nuclear β^\mp decays

Including effects from $f_2(0)$

$$F(x, W_0) \rightarrow F(x, W_0)(1 \pm \varepsilon x) \quad \text{for } \beta^\mp \text{ decays}$$

Hard to detect in n decay (V^μ dominated by weak charge), become leading V^μ contribution for transitions between different isomultiplets ($f_1(0) = 0$)

Example: $(^{12}\text{B}, ^{12}\text{C}^*, ^{12}\text{N})_{I=1} \rightarrow (^{12}\text{C})_{I=0}$ (β^- , γ , β^+)

- $^{12}\text{C}^* \rightarrow ^{12}\text{C}$ is a magnetic dipole transition governed by f_2^{em}
- since $\Delta I = 1$ only isovector current contributes to f_2^{em}
- isovector EM current in same isotriplet as charge weak currents governing β^\mp decays of $(^{12}\text{B}$ and $^{12}\text{N}) \Rightarrow f_2$ from f_2^{em}
- f_2^{em} from $^{12}\text{C}^* \rightarrow ^{12}\text{C}$

\Rightarrow prediction for ε , agrees with experiments

Fermi and Gamow-Teller nuclear transitions

NR limit

$$\langle p | V^\mu | n \rangle = g_V \bar{u}_p \gamma^\mu u_n \simeq \delta^\mu_0 g_V \bar{u}_p^\dagger u_n \quad \Rightarrow \text{only } \mu = 0$$

$$\langle p | A^\mu | n \rangle = g_A \bar{u}_p \gamma^\mu \gamma^5 u_n \simeq \sum_{j=1}^3 \delta^\mu_j g_A \bar{u}_p^\dagger \sigma_j u_n \quad \Rightarrow \text{only } \mu = 1, 2, 3$$

Beta decay of nucleus $N \rightarrow N'$

$$\langle N' | V^0 | N \rangle = g_V \langle N' | \sum_i \tau_+^{(i)} | N \rangle \equiv g_V \langle 1 \rangle$$

$$\langle N' | \vec{A} | N \rangle = g_A \langle N' | \sum_i \tau_+^{(i)} \vec{\sigma}^{(i)} | N \rangle \equiv g_A \langle \vec{\sigma} \rangle$$

i runs over n in N , that can undergo β decay turning in one of the p in N'
 $\tau_+^{(i)}$ has non-zero matrix elements for i th n in N turning into i th p in N'
antisymmetry of initial and final state is used

- $\langle 1 \rangle \neq 0$ and $\langle \vec{\sigma} \rangle = 0$: *Fermi transitions*
- $\langle 1 \rangle = 0$ and $\langle \vec{\sigma} \rangle \neq 0$: *Gamow-Teller transitions*
- both terms are nonzero: mixed transitions

Fermi and Gamow-Teller nuclear transitions (contd.)

Set

$$X^\mu \equiv \left(\langle N' | V^0 | N \rangle, -\langle N' | \vec{A} | N \rangle \right)$$

Decay amplitude squared

$$|\mathcal{M}_{\text{fi}}|^2 \propto X^\mu X^{\nu*} \left(k_{(e)\mu} k_{(\nu)\nu} + k_{(e)\nu} k_{(\nu)\mu} - \eta_{\mu\nu} k_{(e)} \cdot k_{(\nu)} - i \epsilon_{\mu\alpha\nu\beta} k_{(\nu)}^\alpha k_{(e)}^\beta \right)$$

For e - ν correlation, only constant term and term $\propto \cos \theta_{e\nu}$

Terms depending on the orientation of ν and e w.r.t. other directions drop after integration over overall orientation of final products

$$\begin{aligned} |\mathcal{M}_{\text{fi}}|^2|_{\text{relevant}} &\propto X^0 X^{*0} 2E_e E_\nu + X^i X^{j*} (\vec{k}_e{}_i \vec{k}_\nu{}_j + \vec{k}_\nu{}_i \vec{k}_e{}_j) \\ &\quad - X \cdot X^* (E_e E_\nu - \vec{k}_e \cdot \vec{k}_\nu) - i(X^0 X^{j*} - X^j X^{0*}) \varepsilon_{0jkI} \vec{k}_\nu{}^k \vec{k}_e{}^I \\ &= [2(g_V |\langle 1 \rangle|)^2 - (g_V \langle 1 \rangle)^2 + (g_A |\langle \vec{\sigma} \rangle|)^2] E_e E_\nu \\ &\quad + (X^i X^{j*} + X^j X^{i*}) \vec{k}_e{}_i \vec{k}_\nu{}_j + X \cdot X^* \vec{k}_e \cdot \vec{k}_\nu \\ &\quad + i(X^0 \vec{X}^* - \vec{X} X^{0*}) \cdot \vec{k}_\nu \wedge \vec{k}_e \end{aligned}$$

Fermi and Gamow-Teller nuclear transitions (contd.)

Sum over final spins, average over the polarisation of initial nucleus \Rightarrow
result must be invariant under rotation of spins

$$X^i X^{*j} \xrightarrow{\text{average}} \frac{1}{3} \delta^{ij} \vec{X} \cdot \vec{X}^*$$

$$\begin{aligned} |\mathcal{M}_{\text{fi}}|^2|_{\text{relevant}} &\propto [(g_V\langle 1 \rangle)^2 + (g_A\langle \vec{\sigma} \rangle)^2] E_e E_\nu + [(g_V\langle 1 \rangle)^2 - \frac{1}{3}(g_A\langle \vec{\sigma} \rangle)^2] \vec{k}_e \cdot \vec{k}_\nu \\ &= [(g_V\langle 1 \rangle)^2 + (g_A\langle \vec{\sigma} \rangle)^2] E_e E_\nu \left\{ 1 + \frac{(g_V\langle 1 \rangle)^2 - \frac{1}{3}(g_A\langle \vec{\sigma} \rangle)^2}{(g_V\langle 1 \rangle)^2 + (g_A\langle \vec{\sigma} \rangle)^2} \beta_e \cos \theta \right\} \\ &\propto 1 - \xi \beta_e \cos \theta \end{aligned}$$

- Fermi transition: e - ν correlation coefficient $\xi = -1$
- Gamow-Teller transition: e - ν correlation coefficient $\xi = \frac{1}{3}$
- compare with free neutron: $\xi = \frac{\alpha^2 - 1}{3\alpha^2 + 1} \simeq 0.1$

Hyperon decays

β decay of Σ^\pm (hyperons = strange baryons)

$$\begin{aligned}\Sigma^+ &\rightarrow \Lambda e^+ \nu_e & \Sigma^- &\rightarrow \Lambda e^- \bar{\nu}_e \\ [u &\rightarrow d e^+ \nu_e] & [d &\rightarrow u e^- \bar{\nu}_e]\end{aligned}$$

$$m_{\Sigma^+} = 1.1894 \text{ GeV} \quad m_{\Sigma^-} = 1.1974 \text{ GeV} \quad m_\Lambda = 1.1157 \text{ GeV}$$

$$[\Sigma^+ = (uus) \quad \Sigma^- = (dds) \quad \Lambda = (uds)]$$

$$\Delta_+ = m_{\Sigma^+} - m_\Lambda = 73.7 \text{ MeV} \quad \Delta_- = m_{\Sigma^-} - m_\Lambda = 81.7 \text{ MeV}$$

Decay into μ forbidden

Strangeness-conserving processes, relevant currents: $\bar{d}\mathcal{O}_L^\mu u$, $\bar{u}\mathcal{O}_L^\mu d$

$$\langle \Lambda | V_-^\mu | \Sigma^+ \rangle = \langle \Lambda | \bar{d} \gamma^\mu u | \Sigma^+ \rangle = \bar{u}_\Lambda \left(f_1 \gamma^\mu + i \frac{f_2}{2M} \sigma^{\mu\nu} q_\nu + f_3 \frac{q^\mu}{2M} \right) u_{\Sigma^+}$$

$$\langle \Lambda | A_-^\mu | \Sigma^+ \rangle = \langle \Lambda | \bar{d} \gamma^\mu \gamma^5 u | \Sigma^+ \rangle = \bar{u}_\Lambda \left(g_1 \gamma^\mu + i \frac{g_2}{2M} \sigma^{\mu\nu} q_\nu + g_3 \frac{q^\mu}{2M} \right) \gamma^5 u_{\Sigma^+}$$

$$2M = m_\Lambda + m_{\Sigma^+}$$

Hyperon decays (contd.)

Δ_{\pm} small \Rightarrow set $q^2 = 0$

In the isospin limit:

- $f_1(0)|_{\text{iso}} = 0$ ($I_{\Sigma} = 1 \neq I_{\Lambda} = 0$) so $f_1(0) = \mathcal{O}(\Delta_+ - \Delta_-)$
- from current conservation

$$0 = \bar{u}_{\Lambda} \left(f_1(q^2)|_{\text{iso}} q + f_3(q^2)|_{\text{iso}} \frac{q^2}{2M} \right) u_{\Sigma^+} = \bar{u}_{\Lambda} \left(f_1(q^2)|_{\text{iso}} \Delta_+ + f_3(q^2)|_{\text{iso}} \frac{q^2}{2M} \right) u_{\Sigma^+}$$
$$\Rightarrow \frac{f_3(0)}{(2M)} = \mathcal{O}(\Delta_+) \quad \text{if } \frac{f_1(q^2)}{q^2} \Big|_{q \approx 0} = \mathcal{O}(1)$$

- f_2 related to $f_{2\text{em}}$ in the EM decay $\Sigma^0 \rightarrow \Lambda \gamma \Rightarrow$ some $\mathcal{O}(1)$ number
 \Rightarrow vector current suppressed w.r.t. axial current as long as $g_1(0) \neq 0$
- same matrix elements for Σ^+ and Σ^- , adapt result from n β -decay
 $\Rightarrow \Gamma_{\pm} \propto \Delta_{\pm}^5$ ratio determined by available phase space

$$\frac{\Gamma(\Sigma^+ \rightarrow \Lambda e^+ \nu_e)}{\Gamma(\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e)} = \left(\frac{\Delta_+}{\Delta_-} \right)^5 \simeq 0.6$$

Strangeness-changing leptonic decays of hadrons

Most leptonic decays of strange particles are strangeness-changing

Relevant interactions: $(\bar{u}\mathcal{O}_L^\alpha s)$ $(\bar{\ell}\mathcal{O}_{L\alpha}\nu_\ell)$, $(\bar{s}\mathcal{O}_L^\alpha u)$ $(\bar{\nu}_\ell\mathcal{O}_{L\alpha}\ell)$

Lightest hadrons with strangeness-changing weak decays: strange octet mesons – kaons, strange octet baryons + decuplet Ω baryon

Notation for kaon decays

$$K_{\mu 2}^+ : K^+ \rightarrow \mu^+ \nu_\mu \quad K_{\mu 4}^+ : K^+ \rightarrow \mu^+ \nu_\mu \pi^+ \pi^-$$

$$K_{\mu 3}^+ : K^+ \rightarrow \mu^+ \nu_\mu \pi^0 \quad K_{\mu 4}'^+ : K^+ \rightarrow \mu^+ \nu_\mu \pi^0 \pi^0$$

$$K_{\mu 3}^0 : K^0 \rightarrow \mu^+ \nu_\mu \pi^- \quad K_{\mu 4}^0 : K^0 \rightarrow \mu^+ \nu_\mu \pi^0 \pi^-$$

$K_{\mu 2}^-$, $K_{\mu 3}^-$ etc. for $K^- \rightarrow \mu^- + \dots$; $K_{e 2}^\pm$, $K_{e 3}^\pm$ etc. for e^\pm in final state

$K_{\ell 2}^-$, $K_{\ell 3}^-$: lepton type in the final state is summed over

No special notation for hyperon decays

$$\Lambda \rightarrow p \ell^- \bar{\nu}_\ell$$

$$\Sigma^- \rightarrow n \ell^- \bar{\nu}_\ell$$

$$\Xi^- \rightarrow \Lambda \ell^- \bar{\nu}_\ell$$

$$\Xi^- \rightarrow \Sigma^0 \ell^- \bar{\nu}_\ell$$

$$\Xi^0 \rightarrow \Sigma^+ \ell^- \bar{\nu}_\ell$$

$$\Omega^- \rightarrow \Xi^0 \ell^- \bar{\nu}_\ell$$

Strangeness-changing leptonic decays of hadrons (contd.)

General form of decay amplitude

$$\mathcal{M}_{\text{fi}} = -\frac{G}{\sqrt{2}} \sin \theta_C H_\alpha L^\alpha$$

$$H^\alpha = \langle f | \bar{u} \mathcal{O}_L^\alpha s | i \rangle \quad \text{or} \quad H^\alpha = \langle f | \bar{s} \mathcal{O}_L^\alpha u | i \rangle$$

$$L^\alpha = \bar{u}_\ell \mathcal{O}_L^\alpha v_{\nu_\ell} \quad \text{or} \quad L^\alpha = \bar{u}_{\nu_\ell} \mathcal{O}_L^\alpha v_\ell$$

Selection rules:

- $\bar{u} \mathcal{O}_L^\alpha s$ and $\bar{s} \mathcal{O}_L^\alpha u$ change strangeness by $+1$ and -1

\implies

$$|\Delta S| = 1$$

- they change electric charge and strangeness by the same amount

\implies

$$\Delta Q = \Delta S$$

- $I_u = \frac{1}{2}$, $I_s = 0$, currents belong to isodoublets

\implies

$$|\Delta I| = \frac{1}{2}$$

Processes violating selection rules not strictly forbidden (can take place in higher orders of perturbation theory) but very strongly suppressed

References

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