Weak Interactions

Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

September 24, 2020

Intermezzo: corrections to the isospin limit

Comm. relation of charges (= integrated ch. currents) in the isospin limit

$$\underbrace{[I_+, I_-] = 2I_3}_{\substack{\text{generators in iso limit,}\\ \text{obey SU(2) comm. rel.}}} \quad = \quad \underbrace{2Q - Y}_{\substack{\text{exactly conserved}\\ \text{also away from iso limit,}\\ \text{unchanged}}}$$

$$\begin{aligned} 2q - y &= 2\langle h_{i,i_3}|I_3|h_{i,i_3}\rangle = \langle h_{i,i_3}|[I_+, I_-]|h_{i,i_3}\rangle \\ &= \textstyle\sum_n \langle h_{i,i_3}|I_+|n\rangle\langle n|I_-|h_{i,i_3}\rangle - \langle h_{i,i_3}|I_-|n\rangle\langle n|I_+|h_{i,i_3}\rangle \\ &= |\langle h_{i,i_3-1}|I_-|h_{i,i_3}\rangle|^2 - |\langle h_{i,i_3+1}|I_+|h_{i,i_3}\rangle|^2 + \mathcal{O}(\delta^2) \end{aligned}$$

For $i_3 = i$ (e.g., $\pi^+ \to \pi^0$)

$$\left|\langle h_{i,i-1}|I_-|h_{i,i}\rangle\right|^2 = 2q - y + \mathcal{O}(\delta^2) = \left|\langle h_{i,i-1}|I_-|h_{i,i}\rangle|_{\text{iso}}\right|^2 + \mathcal{O}(\delta^2)$$

Vector form factor from infinite-momentum limit of charge matrix element

$$(2\pi)^3\delta^{(3)}(\vec{q})f_+(0) = \lim_{|\vec{p_1}|=|\vec{p_2}|\to\infty} \frac{1}{p_1^0+p_2^0}\langle h_{i,i-1}|I_-|h_{i,i}\rangle|_{\vec{q}=0}$$

[Fubini & Furlan (1965)]

Neutron beta decay

Basic process behind all nuclear beta decays

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

Fundamental process at quark level: $n(udd) \to p(uud) \Rightarrow d \to u$

$$d \longrightarrow u + e^- + \bar{\nu}_e$$

Relevant term in the Lagrangian

$$-\frac{G}{\sqrt{2}} \cos\theta_C (\bar{u}\mathcal{O}_L^\alpha d)(\bar{e}\mathcal{O}_{L\alpha}\nu_e)$$

Decay amplitude

$$\mathcal{M}_{\rm fi} = -\frac{G}{\sqrt{2}} \cos\theta_C H^\alpha \bar{u}_e(p_{(e)})\gamma_\alpha(1-\gamma^5)v_e(p_{(\nu)})$$

Hadronic matrix element receives both V and A contributions

$$H^\alpha = \langle p|(\bar{u}\mathcal{O}_L^\alpha d)(0)|n\rangle = V_+^\alpha + A_+^\alpha$$

$$V_+^\alpha = \langle p|(\bar{u}\gamma^\alpha d)(0)|n\rangle \qquad A_+^\alpha = \langle p|(\bar{u}\gamma^\alpha\gamma^5 d)(0)|n\rangle$$

Neutron beta decay (contd.)

Use symmetries to constrain matrix elements – from Lorentz invariance:

$$\langle p|V_+^\mu|n\rangle = \bar{u}_p(p_p,s_p)\left(f_1(q^2)\gamma^\mu + f_2(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2m} + f_3(q^2)\frac{q^\mu}{2m}\right)u_n(p_n,s_n)$$
$$= \bar{u}_p(p_p,s_p)M^\mu(q)u_n(p_n,s_n)$$
$$\langle p|A_+^\mu|n\rangle = \bar{u}_p(p_p,s_p)\left(g_1(q^2)\gamma^\mu + g_2(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2m} + g_3(q^2)\frac{q^\mu}{2m}\right)\gamma^5 u_n(p_n,s_n)$$
$$= \bar{u}_p(p_p,s_p)M_5^\mu(q)u_n(p_n,s_n)$$

Most general linearly independent structures one can build out of $\mathrm{D} = (\frac{1}{2},0)\oplus(0,\frac{1}{2})$ and its conjugate $\bar{\mathrm{D}}$ transforming like a vector/axial vector

$q = p_n - p_p$, $m = \frac{m_p+m_n}{2}$; f_i, g_i: real dimensionless functions (*form factors*)

Can be determined experimentally via $\nu_e/\bar{\nu}_e - n/p$ scattering – same matrix el.

$$\langle p|V_{\mathrm{em}}^\mu|p\rangle = \bar{u}_p(p'_p,s'_p)\left(f_{p1}(q^2)\gamma^\mu + f_{p2}(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2m_p} + f_{p3}(q^2)\frac{q^\mu}{2m_p}\right)u_p(p_p,s_p)$$
$$\langle n|V_{\mathrm{em}}^\mu|n\rangle = \bar{u}_n(p'_n,s'_n)\left(f_{n1}(q^2)\gamma^\mu + f_{n2}(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2m_n} + f_{n3}(q^2)\frac{q^\mu}{2m_n}\right)u_n(p_n,s_n)$$

f_{pi}, f_{ni}: electromagnetic form factors of proton and neutron

Neutron beta decay: vector matrix element

Further constraint from conserved em current $\Rightarrow$ CVC in the iso limit: transverse matrix elements $q_\mu V^\mu_{\rm em} = 0$, $q_\mu V^\mu_+|_{\rm iso} = 0$

f_1 term: $\bar{u}(p',s')\not{q}u(p,s) = \bar{u}(p',s')(\not{p}' - \not{p})u(p,s) = \bar{u}(p',s')(m-m)u(p,s) = 0$

f_2 term: $\sigma^{\mu\nu}$ antisymmetric $\rightarrow q_\mu q_\nu \sigma^{\mu\nu} = 0$

$$\Longrightarrow f_{p3}(q^2) = f_{n3}(q^2) = 0 \qquad f_3(q^2)|_{\rm iso} = 0$$

Alternatively use behaviour of vector current under G-parity, $G = Ce^{i\pi I_2}$, to drop f_3 in the iso limit

Use $\langle I\, I_3{+}1|V^\mu_+|I\, I_3\rangle = \sqrt{I(I{+}1) - I_3(I_3{+}1)}\left[\langle I\, I_3{+}1|V^\mu_{\rm em}|I\, I_3{+}1\rangle - \langle I\, I_3|V^\mu_{\rm em}|I\, I_3\rangle\right]$ for the nucleon isodoublet ($I = \frac{1}{2}$) in the isospin limit

$$\langle p|V^\mu_+|n\rangle = \langle p|V^\mu_{\rm em}|p\rangle - \langle n|V^\mu_{\rm em}|n\rangle$$
$$f_i(q^2)|_{\rm iso} = f_{p\,i}(q^2)|_{\rm iso} - f_{n\,i}(q^2)|_{\rm iso} \quad i = 1, 2, 3$$

Neutron beta decay: vector matrix element (contd.)

$f_1(0)$: weak charge

In the limit $q \to 0$

$$\begin{aligned}
\langle p|V^0_{\rm em}(0)|p\rangle|_{\vec{q}=0} &= f_{p1}(0)\bar{u}_p(p,s'_p)\gamma^0 u_p(p,s_p) = f_{p1}(0)2p^0\delta_{s'_p,s_p} \\
&= 2p^0 Q_p \delta_{s'_p,s_p} = 2p^0\delta_{s'_p,s_p} \\
\langle n|V^0_{\rm em}(0)|n\rangle|_{\vec{q}=0} &= f_{n1}(0)\bar{u}_n(p,s'_n)\gamma^0 u_n(p,s_n) = f_{n1}(0)2p^0\delta_{s'_n,s_n} \\
&= 2p^0 Q_n \delta_{s'_n,s_n} = 0
\end{aligned}$$

$$f_1(0) = f_{p1}(0) - f_{n1}(0) = Q_p - Q_n = 1$$

Alternatively: $n \to p$ is $\Delta I = 0$, $\Delta S = 0$ transition, governed by weak charge $Q_W(\frac{1}{2}, -\frac{1}{2}) = 1$ at $\vec{q} = 0$

$$\begin{aligned}
\langle p|V^0_+(0)|n\rangle|_{\vec{q}=0} &= f_1(0)\bar{u}_p(p,s_p)\gamma^0 u_n(p,s_n) = f_1(0)2p^0\delta_{s_p,s_n} \\
&= 2p^0\delta_{s_p,s_n}\sqrt{\tfrac{3}{4}+\tfrac{1}{4}} = 2p^0\delta_{s_p,s_n} \Rightarrow f_1(0) = 1
\end{aligned}$$

Neutron beta decay: vector matrix element (contd.)

$f_2(0)$: weak magnetism

Form factors $f_{p,n2}(0)$ at $q = 0$: *anomalous magnetic moments*, related to response of particles to external static magnetic field; from experiment

$$f_{p2}(0) = 1.79 \qquad f_{n2}(0) = -1.91 \Longrightarrow f_2(0) = 3.7$$

f_3: "effective scalar"

$$q_\mu \bar{u}_\nu \gamma^\mu (1-\gamma^5) u_e = \bar{u}_\nu (\not{q}_\nu + \not{q}_e)(1-\gamma^5) u_e = \bar{u}_\nu (1+\gamma^5) \not{q}_e u_e = m_e \bar{u}_\nu (1+\gamma^5) u_e$$

In summary: to leading order in q and isospin breaking

$$\langle p | V_+^\mu | n \rangle = \bar{u}_p(p_p, s_p) \left(\gamma^\mu + i\sigma^{\mu\nu} q_\nu \frac{f_2(0)}{2m} \right) u_n(p_n, s_n)$$

Near the isospin limit, expand in iso-breaking $\Delta = m_n - m_p$

- $q^2 = (p_{(e)} + p_{(\nu)})^2 \geq m_e^2$ and $q^2 = m_n^2 + m_p^2 - 2m_n E_p \leq (m_n - m_p)^2 = \Delta^2$, so $q = \mathcal{O}(\Delta)$
- $f_i = f_i^{(0)} + \Delta f_i^{(1)} + \ldots$; from current conservation $f_3^{(0)}(q^2) = 0$ so $f_3 = \mathcal{O}(\Delta)$
- f_1 term is $\mathcal{O}(1)$, $f_2 q$ term is $\mathcal{O}(\Delta)$, and $f_3 q$ term is $\mathcal{O}(\Delta^2)$

Working to order Δ, drop f_3 and deviations from $f_{1,2}(0)$

Neutron beta decay: axial matrix element

Term $\sigma^{\mu\nu} q_\nu \gamma^5$ ("weak electrism") forbidden in the isospin limit, can be dropped when working at order Δ together with q dependence of $g_{1,3}$

Forbidden by G-parity: G-even while A_μ is G-odd

$$\langle p|A_+^\mu|n\rangle = \bar{u}_p(p_p, s_p)\left(\gamma^\mu g_1(0) + q^\mu \frac{g_3(0)}{2m}\right)\gamma^5 u_n(p_n, s_n)$$

PCAC hyp. $-iq_\mu \langle p|A_+^\mu(0)|n\rangle = f_\pi m_\pi^2 \langle p|\phi_+(0)|n\rangle$, pole structure implies

$$\langle p|\phi_+(0)|n\rangle = 2i\frac{g_{\pi NN}(q^2)}{m_\pi^2 - q^2}\bar{u}_p(p_p, s_p)\gamma^5 u_n(p_n, s_n)$$

$g_{\pi NN}(q^2)$: pion-nucleon-nucleon vertex function, regular at $q^2 = m_\pi^2$

Off-shell pole corresponding to (unphysical) $n\pi^+ \to p$

$$(2\pi)^4\delta^{(4)}(p_p - p_\pi - p_n) i\mathcal{M}_{n\,\pi^+\to p} = {}_{\rm out}\langle p|n\pi^+\rangle_{\rm in} = i\int d^4x e^{-ip_\pi\cdot x}(\Box_x + m_\pi^2)\,{}_{\rm out}\langle p|\frac{\phi_+(x)}{\sqrt{2}}|n\rangle_{\rm in}$$

$$= -i(p_\pi^2 - m^2)\int d^4x e^{-i(p_\pi + p_n - p_p)\cdot x}\,{}_{\rm out}\langle p|\frac{\phi_+(0)}{\sqrt{2}}|n\rangle_{\rm in}$$

$$\underbrace{i\mathcal{M}_{n\,\pi^+\to p}}_{\propto g_{\pi NN}} = {}_{\rm out}\langle p|\frac{\phi_+(0)}{\sqrt{2}}|n\rangle_{\rm in}\frac{p_\pi^2 - m_\pi^2}{i}$$

Neutron beta decay: axial matrix element (contd.)

Combining structure of A^{μ}_{+} matrix element with pole structure

$$-\bar{u}_p(p_p,s_p)\left(\not{q}g_1(q^2)+q^2\frac{g_3(q^2)}{2m}\right)\gamma^5 u_n(p_n,s_n)$$
$$=2f_\pi m_\pi^2\frac{g_{\pi NN}(q^2)}{m_\pi^2-q^2}\bar{u}_p(p_p,s_p)\gamma^5 u_n(p_n,s_n)$$

$$(m_p+m_n)g_1(q^2)-q^2\frac{g_3(q^2)}{2m}=2f_\pi m_\pi^2\frac{g_{\pi NN}(q^2)}{m_\pi^2-q^2}$$

At $q=0$ $(m_p=m_n=m)$

$$mg_1(0)=f_\pi g_{\pi NN}(0)$$

Relation between experimentally accessible quantities:

- $g_1(0)\simeq 1.267$ obtained from measurements of neutron β decay
- assuming pion-exchange dominance, $g=g_{\pi NN}(m_\pi^2)$ of pion-nucleon coupling from nucleon-nucleon scattering experiments, $g\simeq 13.169$

Assuming further $g_{\pi NN}(m_\pi^2)\simeq g_{\pi NN}(0)\Rightarrow$ *Goldberger-Treiman relation*

$$mg_1(0)=f_\pi g$$

Satisfied within $2\div 3\%$

Conserved axial current in the massless limit

If $m_\pi \to 0 \Rightarrow \partial_\mu A_+^\mu = 0 \Rightarrow 2mg_1(q^2) - q^2 \frac{g_3(q^2)}{2m} = 0 \Rightarrow \boxed{\frac{g_3(q^2)}{2m} = \frac{2mg_1(q^2)}{q^2}}$

Pole at $q = 0$ in g_3 from χSSB indicates presence of massless particles (πs)

Phenomenological origin of the pole: "blob"-like coupling of nucleons (non-elementary) to leptonic weak current, include

- pointlike n-p-leptons four-fermion interaction
- pointlike n-p-π interaction followed by pion decaying into lepton pair
- ...

$$\mathcal{L}^{\text{eff}} = -\frac{G\cos\theta}{\sqrt{2}}\left[\bar{N}\tau_+\gamma^\mu(1-g_A\gamma^5)N\,\bar{e}\gamma_\mu(1-\gamma^5)\nu + \text{h.c.}\right] + ig\bar{N}\tau_a\gamma^5 N\phi_a$$

N: nucleon doublet, $g_A = g_1(0)$

$$g_1 \text{ term}: i\underbrace{\frac{G}{\sqrt{2}}(-g_A)u_p\gamma^\mu\gamma^5 u_n}_{NN\ell\nu_\ell \text{ coupling}}$$

$$g_3 \text{ term}: i\underbrace{(ig\sqrt{2})u_p\gamma^5 u_n}_{NN\pi \text{ coupling}}\ \underbrace{\frac{i}{q^2-m_\pi^2}}_{\pi \text{ propagator}}\ \underbrace{\frac{-iG}{\sqrt{2}}(i\sqrt{2}f_\pi q_\mu)}_{\pi\ell\nu_\ell \text{ coupling}} = \frac{2f_\pi g q_\mu}{m_\pi^2-q^2}\frac{-iG}{\sqrt{2}}$$

References

- R. L. Garwin, L. M. Lederman, M. Weinrich, Phys.Rev. 105 (1957) 1415
- S. Fubini, G. Furlan, Physics Physique Fizika 1 (1965) 229