Weak Interactions

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SU(5) GUT: Higgs mechanism

Two-step mechanism:

- First step: break $\mathrm{SU}(5) \to G_{\mathrm{SM}}$
  Using appropriate potential for adjoint (**24**) Higgs field H
  $$\langle H \rangle = v\lambda^{24}$$
  $\Rightarrow$ gives mass to X and Y (recall $[\lambda^{24}, G_{\mathrm{SM}}] = 0$)
- Second step: break $G_{\mathrm{SM}} \to \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$ as before
- Result: Standard Model particles plus new bosons

$\mathrm{SU}(5)$ covariant derivative

$$D_\mu = \partial_\mu + ig_5 \sum_{a=1}^{24} A_\mu^a \frac{\lambda^a}{2}$$

SM covariant derivative

$$D_\mu = \partial_\mu + ig_3 \sum_{a=1}^{8} G_\mu^a \frac{\lambda^a_{\mathrm{GM}}}{2} + ig \sum_{a=1}^{3} W_\mu^a \frac{\tau^a}{2} + ig' B_\mu \frac{Y}{2}$$

Comparing the two equations (recall $Y = -\sqrt{5/3}\lambda^{24}$)

$$g_3 = g = g_5 \qquad g' = -\sqrt{\tfrac{3}{5}} g_5 \Longrightarrow \tan\theta_W = \frac{g'}{g} = -\sqrt{\tfrac{3}{5}}$$

$\mathrm{SU}(5)$ GUT: Higgs mechanism (contd.)

From comparison:

- Using $\sin^2 x = \frac{\tan^2 x}{1+\tan^2 x} \Rightarrow \sin^2\theta_W = 0.375$, differs from $\sin^2\theta_W^{\mathrm{exp}} = 0.212023$
- $g_3 = g$ (unified strong and weak coupling) known to be not true experimentally at low energies

Couplings effectively "run" with energy, i.e., are energy dependent

However: identification of the two couplings should be done at some high energy scale M_X of the order of the masses of the new bosons, i.e., the scale at which the full $\mathrm{SU}(5)$ symmetry breaks down

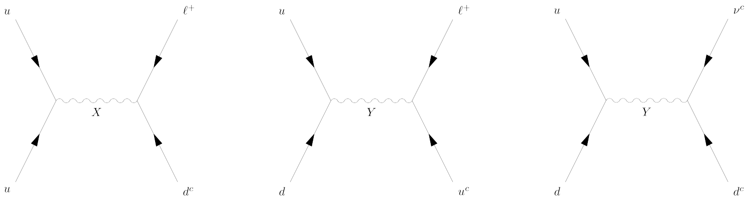
Running of θ_W in the unified theory (for three Standard Model families)

$$\sin^2\theta_W(\mu) = \frac{3}{8} - \frac{55}{24\pi}\alpha(\mu)\ln\frac{M_x}{\mu}$$

$\alpha(\mu)$: running QCD coupling
μ: energy scale of the relevant process

Imposing $\sin^2\theta_W = 0.22 - 0.23$ at low energy (i.e., at $\mu \sim M_Z$)
$\Rightarrow \theta_W(\mu)$ reaches grand unification value at $\mu = M_X \sim 4\cdot 10^4\,\mathrm{GeV}$

SU(5) GUT: proton decay



Drawback: $B - L$ still conserved but B, L separately not conserved anymore

- $p \to e^+\pi^0$ ($\Delta B = -1$, $\Delta L = -1$) allowed
- $n \to e^-\pi^+$ ($\Delta B = -1$, $\Delta L = 1$) still forbidden

New bosons mediate proton decay at tree level: expected since leptons and quarks are in the same multiplet (it is like $u \to d$ via W boson)

$\Delta B \neq 0$ processes suppressed due to $M_{X,Y}$ large

SU(5) GUT model predicts $\tau_p \sim \frac{M_X^4}{m_p^5}$

From tree-level X-exchange $\Gamma \propto \left(\frac{g^2}{M_X^2}\right)^2$, dimensional analysis $\Rightarrow \tau \propto \frac{M_X^4}{m_p^5}$

$\mathrm{SU}(5)$ GUT: proton decay

So far no proton decay observed experimentally

Current bound on τ_p from null result: $\tau_p > 10^{34}\,\mathrm{years}$

- τ_p predicted by $\mathrm{SU}(5)$ GUT model $\tau_p \sim \frac{M_X^4}{m_p^5} \approx 10^{30} \div 10^{31}\,\mathrm{years} \Rightarrow$ experimentally disproved
- τ_p very sensitive to M_X, which is constrained by low-energy phenomenology (e.g., $\sin^2\theta_W$)
- non-minimal models (supersymmetric or not) exist, most of which predict proton decay; so far, no proton decay and no experimentally vindicated GUT model

THE END.