Weak Interactions

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Neutrinos (contd.)

Neutrino field coupled by the charged weak interaction with P_-, relevant component

$$P_-\nu(x) = \frac{1-\gamma^5}{2}\int d\Omega_p \sum_{h=R,L}\left\{b_h(\vec{p})u_h(\vec{p})e^{-ip\cdot x} + d_h(\vec{p})^\dagger v_h(\vec{p})e^{ip\cdot x}\right\}$$
$$= \int d\Omega_p \left\{b_L(\vec{p})u_L(\vec{p})e^{-ip\cdot x} + d_R(\vec{p})^\dagger v_R(\vec{p})e^{ip\cdot x}\right\} \equiv \nu_L(x)$$

- $\nu_L(x) = P_-\nu(x)$ annihilates a left-handed ν, creates a right-handed $\bar{\nu}$
- $\bar{\nu}_L(x) = (\nu_L(x))^\dagger\gamma^0 = \bar{\nu}P_+$ creates a left-handed ν, annihilates a right-handed $\bar{\nu}$

$\nu_R(x) = P_+\nu(x)$ never appears in the weak Lagrangian $\Rightarrow$
massless ν can only be left-handed and $\bar{\nu}$ can only be right-handed

Massless right-handed neutrinos and left-handed antineutrinos can exist but are *sterile*, not interacting with anything

P, C, and CP

Under P, C, vector $V^\alpha \equiv \bar{f}\gamma^\alpha f$ and axial-vector $A^\alpha \equiv \bar{f}\gamma^\alpha\gamma^5 f$,

$$V^\alpha \underset{P}{\longrightarrow} \mathcal{P}^\alpha{}_\beta V^\beta \qquad V^\alpha \underset{C}{\longrightarrow} -V^{\alpha\dagger}$$

$$A^\alpha \underset{P}{\longrightarrow} -\mathcal{P}^\alpha{}_\beta A^\beta \qquad A^\alpha \underset{C}{\longrightarrow} A^{\alpha\dagger}$$

$$\mathcal{P}^\alpha{}_\beta = \mathrm{diag}(1, -1, -1, -1)$$

$V - A$ structure of charged interaction

$$\mathcal{L} = (V^{\alpha\dagger} - A^{\alpha\dagger})(V_\alpha - A_\alpha)$$

Under P and C

$$\mathcal{L} \underset{P}{\longrightarrow} (V^{\alpha\dagger} + A^{\alpha\dagger})(V_\alpha + A_\alpha) \qquad \mathcal{L} \underset{C}{\longrightarrow} (V^{\alpha\dagger} + A^{\alpha\dagger})(V_\alpha + A_\alpha)$$

- P and C are broken

  Ex.: left-handed ν transformed by P into right-handed ν, and by C into left-handed $\bar{\nu}$
- $J_l^\alpha = V_\alpha - A_\alpha$ exactly $\Rightarrow$ CP good symmetry in the leptonic sector...

P, C, and CP (contd.)

...but not in the hadronic sector

$$J_h^\alpha = \bar{d}'\,\mathcal{O}_L^\alpha\,u + \bar{s}'\,\mathcal{O}_L^\alpha\,c + \bar{b}'\,\mathcal{O}_L^\alpha\,t = \sum_{q_1=u,c,t}\sum_{q_2=d,s,b}(V_{\rm CKM})_{q_1q_2}\,\bar{q}_2\,\mathcal{O}_L^\alpha\,q_1$$

$$= \sum_{q_1=u,c,t}\sum_{q_2=d,s,b}(V_{\rm CKM})_{q_1q_2}(V^\alpha_{q_2q_1} - A^\alpha_{q_2q_1}) \equiv J_h^\alpha(V_{\rm CKM})\,.$$

Under CP

$$J_h^\alpha \xrightarrow[CP]{} -\mathcal{P}^\alpha{}_\beta \sum_{q_1=u,c,t}\sum_{q_2=d,s,b}(V_{\rm CKM})_{q_1q_2}(V^\beta_{q_2q_1} - A^\beta_{q_2q_1})^\dagger = -\mathcal{P}^\alpha{}_\beta J_h^\beta(V^*_{\rm CKM})^\dagger$$

- Two families: redefining phase of fermion fields one makes $V_{\rm CKM}$ real $\Rightarrow$ CP is a symmetry
- Three families: one ineliminable phase factor $\Rightarrow$ CP violation possible

Similar CP-violating phase can appear in the lepton sector, assuming a nontrivial mixing matrix, if neutrinos are not massless (or mass-degenerate)

P, C, and CP (contd.)

Even if P, C and CP are broken, $\Theta = CPT$ is conserved: for any local Poincaré-invariant QFT, Θ good (antiunitary) symmetry (CPT theorem)

$\Rightarrow$ particles and antiparticles have the same mass and lifetime

$$\langle \bar{\alpha}; \vec{p}\,', -s' | P^2 | \bar{\alpha}; \vec{p}, -s \rangle = \langle \alpha; \vec{p}\,', s' | \Theta^\dagger P^2 \Theta | \alpha; \vec{p}, s \rangle = \langle \alpha; \vec{p}\,', s' | P^2 | \alpha; \vec{p}, s \rangle$$
$$\Rightarrow m_{\bar{\alpha}} = m_\alpha$$

particle with quantum numbers α (antiparticle $\bar{\alpha}$), momentum $\vec{p}$ and spin component $s \Rightarrow$ $\Theta|\alpha; \vec{p}, s\rangle = |\bar{\alpha}; \vec{p}, -s\rangle$ (with the appropriate choice of phases)

Baryon, lepton and lepton family number

Structure of charged currents + quark mixing + barring lepton mixing

- quark/lepton flavour not conserved
- quark/baryon/lepton number conserved
- "quark family" not conserved number
- lepton family number conserved

$L_\ell(\ell^-, \nu_\ell) = 1$, $L_\ell(\ell^+, \bar{\nu}_\ell) = -1$, $L_\ell = 0$ otherwise, $L_\ell(\{X_1, \ldots, X_n\}) = \sum_{i=1}^n L_\ell(X_i)$

leptonic mixing matrix nontrivial, small violations are present

L_ℓ conservation forbids otherwise allowed processes

$$\mu^- \to e^- \gamma \qquad \mu^- \to e^- e^+ e^- \qquad {}^Z_A\mathrm{N} + \nu_\mu \to {}^{Z+1}_A\mathrm{N} + e^-$$

${}^Z_A\mathrm{N}$: nucleus with atomic number Z and mass number A

Allowed processes: ${}^Z_A\mathrm{N} + \nu_e \to {}^{Z+1}_A\mathrm{N} + e^-$, ${}^Z_A\mathrm{N} + \bar{\nu}_e \to {}^{Z-1}_A\mathrm{N} + e^+$

L_ℓ conservation also forbids the $0\nu\beta\beta$ decay

$${}^Z_A\mathrm{N} \to {}^{Z+2}_A\mathrm{N} + 2e^-$$

$0\nu\beta\beta$ allowed if $\nu = \bar{\nu}$ massive truly neutral (*Majorana fermions*)

(If massless, the two helicity states still are different particles independently of L_ℓ conservation)

Decay of unstable particles

Definitions:

- *decay rate/(total) decay width* Γ: decay probability per unit time (in any allowed final state)
- for a large sample one expects $N(t) = N(0)e^{-\frac{t}{\tau}}$, *lifetime* $\tau = 1/\Gamma$
- *partial width* Γ_i: decay probability per unit time in *channel* i (decay mode=specific set of products)
- *branching ratio/fraction* Γ_i/Γ: relative decay probability in channel i
- *differential decay rate/width*: decay probability per unit time (possibly in given channel) with definite momenta and/or spins

Decay of unstable particle, momentum p^μ, into n particles, momenta p_i^μ

$$d\Gamma^{(n)} = \frac{|\mathcal{M}_{\rm fi}|^2}{2p^0}\, d\Phi^{(n)}$$

$$d\Phi^{(n)} = (2\pi)^4 \delta^{(4)}(p - \textstyle\sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2p_i^0}\,,$$

$d\Phi^{(n)}$: n-particle phase space element

$\mathcal{M}_{\rm fi}$: matrix element of the decay operator between initial and final states

Decay amplitude

Full development of formal theory of decay not needed: weak interactions are weak, first-order perturbative approximation will (almost always) suffice

$$(2\pi)^4\delta^{(4)}(P_f - P_i)\mathcal{M}_{\rm fi} = -\int dx^0\,\langle f|H_W^{\rm int}(x^0)|i\rangle$$

$|i,f\rangle$: initial and final free-particle states (relativistic normalisation)

$H_W^{\rm int}(x^0)$: weak interaction Hamiltonian in the interaction picture

$$H_W^{\rm int}(x^0) = \int d^3x\,\mathscr{H}_W^{\rm int}(f_j(x),\bar{f}_j(x)) = -\int d^3x\,\mathcal{L}_W^{\rm int}(f_j(x),\bar{f}_j(x))$$

no derivative couplings

$\{f_j,\bar{f}_j\}$: free fermion fields (fields in the interaction representation)

Decay amplitude:

$$\mathcal{M}_{\rm fi} = \langle f|\mathcal{L}_W^{\rm int}(f_j(0),\bar{f}_j(0))|i\rangle$$

translation invariance used to integrate over x

Feynman rules

Decay amplitude/matrix elements evaluated via Feynman diagrams

$\Rightarrow$ interaction vertex couples two currents/four Fermi fields $-\frac{G}{\sqrt{2}} j_1^\alpha j_{2\alpha}$

1. for each vertex, draw a dot and include a factor $-\frac{G}{\sqrt{2}}$

$\Rightarrow$ currents of general form $g_{ab}\bar{f}_a \mathcal{O}^\alpha f_b$ with couplings g_{ab} (e.g., V_{CKM} matrix elements) and $\mathcal{O}^\alpha$ a combination of gamma matrices

$\Rightarrow$ include Dirac bispinors $\bar{w}_a$ and w_b corresponding to the fields $\bar{f}_a$ and f_b creating or destroying particles in the initial and final states

2. $u_s(\vec{p})$: initial state particle/oriented ext. line flowing into vertex
3. $\bar{u}_s(\vec{p})$: final state particle/oriented ext. line flowing out of vertex
4. $\bar{v}_s(\vec{p})$: initial state antiparticle/oriented ext. line flowing out of vertex
5. $v_s(\vec{p})$: final state antiparticle/oriented ext. line flowing into vertex

$\Rightarrow$ remaining fermion fields contracted with each other

6. fermion propagators connecting vertices/oriented int. lines (from vertex containing $\bar{f}$ to vertex containing f)

Feynman rules (contd.)

⇒ Lorentz indices of bispinors and propagators must be contracted according to the structure of the currents

6. contract bispinors, propagators and vertex factors $g_{ab}\mathcal{O}^{\alpha}$ along each uninterrupted fermion line, moving from the end backwards
   (for single vertex) contract bispinor corresponding to the same current with the appropriate factor $g_{ab}\mathcal{O}^{\alpha}$ (e.g., $\mathcal{O}^{\alpha}_{L}$ for charged currents) $\rightarrow$ get bilinears of the type $g_{ab}\bar{w}_a\mathcal{O}^{\alpha}w_b$
7. contract Lorentz indices of currents coupled at a vertex

⇒ Standard practice:

8. impose conservation of momentum at each vertex
9. integrate over internal (propagator) momenta with measure $\frac{d^4q}{(2\pi)^4}$
10. include minus sign for each fermionic loop, and each fermionic line crossing the diagram *from top to bottom*
11. include numerical factors counting equivalent diagrams

CPT and lifetime of antiparticles

CPT-invariance $\Rightarrow \tau_\alpha = \tau_{\bar{\alpha}}$

In the rest frame of unstable particle, decay governed by $\mathscr{H}^{\rm int}$,

$$\Gamma = \frac{1}{2m}\sum_n \int d\Phi^{(n)}\, |\mathcal{M}_{\rm i\to n}|^2 = \frac{1}{2m}\sum_n \int d\Phi^{(n)}\, \left|\langle n|\mathscr{H}^{\rm int}(0)|i,s\rangle\right|^2$$

$$\bar{\Gamma} = \frac{1}{2\bar{m}}\sum_n \int d\Phi^{(n)}\, |\mathcal{M}_{\bar{\imath}\to \rm n}|^2 = \frac{1}{2\bar{m}}\sum_n \int d\Phi^{(n)}\, \left|\langle n|\mathscr{H}^{\rm int}(0)|\bar{\imath},s\rangle\right|^2$$

Using $\Theta = CPT$ invariance of the complete set $\{|n\rangle\}$ and of $\mathscr{H}^{\rm int}$

$$\bar{\Gamma} = \frac{1}{2\bar{m}}\sum_n \int d\Phi^{(n)}\, \left|\langle n|\Theta^\dagger\mathscr{H}^{\rm int}(0)\Theta|i,-s\rangle\right|^2$$

$$= \frac{1}{2\bar{m}}\sum_n \int d\Phi^{(n)}\, \left|\langle n|\mathscr{H}^{\rm int}(0)|i,-s\rangle\right|^2 = \Gamma$$

We used also $m = \bar{m}$ and rotation invariance (which implies that Γ is independent of the initial polarisation)

References