Weak Interactions

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November 18, 2020

The Standard Model: Summary

Electroweak sector of the Standard Model with three fermion generations

- gauge group $G = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- $3 \times 15 = 45$ Weyl fermion fields
- 3 massive ($W^{\pm}$ and Z) and 1 massless (γ) vector particles
- 1 Higgs scalar
- 18 free parameters
  - 2 gauge couplings $e, \sin\theta_W$
  - 3 lepton, 6 quark masses (Yukawa couplings)
  - 3 Cabibbo angles, 1 Kobayashi-Maskawa phase
  - W-boson and Higgs-boson masses m_W, m_η (corresponding to vev v and mass parameter μ)
  - Higgs self-coupling λ

Strong sector: Quantum Chromodynamics (QCD), gauge group $\mathrm{SU}(3)_C$ (colour SU(3)), full gauge group $G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

- quarks: $\mathrm{SU}(3)_C$ fund. triplet, other matter particles: colour singlets
- include eight massless gluon (gauge boson) fields of $\mathrm{SU}(3)_C$ group
- 1 more parameter: dimensionless strong fine structure constant α_S

Beyond the Standard Model

Reasons to extend the Standard Model:

- theoretical/aesthetical (but **no** compelling experimental reason):
  - unification of electroweak and strong interactions: Grand Unification Theories (GUTs)
  - hierarchy problem/fine-tuning/naturalness: supersymmetric extensions, extra dimensions,...
  - quantisation of gravity
- experimental:
  - neutrino oscillations $\Rightarrow$ massive neutrinos
  - dark matter

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Neutrino oscillations

History of neutrino masses is long and complicated:

- Pauli proposal (1930) and Fermi theory (1933): neutrino must be very light, but no particular reason to believe it is massless
- Two-component neutrino hypothesis (1956-57): neutrino assumed to have definity chirality/handedness, requires masslessness
- Around the same time: ideas about massive neutrinos and oscillations between types (Pontecorvo)

Late '60s: R. Davies *et al.* detect solar neutrinos (Homestake experiment)

- Experimental technique (Pontecorvo): use neutrino-capture reaction,

$$\nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}$$

- Measured flux significantly lower than theoretical prediction (Bahcall *et al.*) $\Rightarrow$ *solar anomaly*, confirmed by others (KamiokaNDE, . . .)
- Most natural explanation: right amount of ν_e produced in the Sun, but turn into different flavour along the way and escape detection (only ν_e could be detected)

Neutrino oscillations (contd.)

More neutrino anomalies:

• In atmospheric neutrinos:

$$\begin{aligned} &\pi^+ \to \mu^+ \, \nu_\mu && \pi^- \to \mu^- \, \bar{\nu}_\mu \\ &\qquad \hookrightarrow e^+ \, \nu_e \, \bar{\nu}_\mu && \qquad \hookrightarrow e^- \, \bar{\nu}_e \, \nu_\mu \end{aligned}$$

- expected muonic to electronic flux ratio: $2:1$
- measured ratio sensitive to direction in which flux is measured
  - fluxes coming from above show expected ratio
  - fluxes coming from below (after crossing Earth) show ratio $1:1$

• In electronic antineutrinos from nuclear reactors (KamLAND, . . .): flux depends on distance from the reactor

All three anomalies can be explained by neutrino oscillations

Neutrino oscillations (contd.)

Quantum mechanical description of neutrino oscillations:

- consider two families, assume neutrinos masses $m_{a,b}$
- no reason to assume that weak flavour eigenstates (neutrino states coupled directly to e, μ, τ) are also mass eigenstates (recall discussion of Yukawa couplings)
- weak flavour eigenstates are in general linear superpositions of mass eigenstates

1. Neutrinos produced in a weak process has definite flavour (i.e., lepton family number) as it comes together with a charged lepton
2. As they propagate in space, the evolution of the quantum state is determined by their content in mass eigenstates (tiny chance of interacting along the way $\approx$ free temporal evolution)
3. Neutrinos detected in states that are again flavour eigenstates (detection is signalled by production of charged lepton)

Neutrino oscillations (contd.)

Let $|\ell_{1,2}\rangle$ be flavour eigenstates, $|a, b\rangle$ mass eigenstates with masses $m_{a,b}$

Most general parameterisation of flavour eigenstates

Two families $\Rightarrow$ extra phases can be reabsorbed by redefining phases of eigenstates

$$
\begin{aligned}
|\ell_1\rangle &= \cos\theta|a\rangle + \sin\theta|b\rangle \\
|\ell_2\rangle &= -\sin\theta|a\rangle + \cos\theta|b\rangle
\end{aligned}
$$

θ: mixing angle

$|\psi(t)\rangle = e^{-iH_{\text{free}}t}|\psi(0)\rangle$, assume $|\psi(0)\rangle = |\ell_1\rangle$ with definite $\vec{p}$

$$
|\psi(t)\rangle = \cos\theta e^{-iE_a t}|a\rangle + \sin\theta e^{-iE_b t}|b\rangle \qquad E_{a,b} = \sqrt{\vec{p}^{\,2} + m_{a,b}^2}
$$

Probability to detect flavour ℓ_1 at time t

$$
\begin{aligned}
|\langle\ell_1|\psi(t)\rangle|^2 &= |\cos\theta e^{-iE_a t}\langle\ell_1|a\rangle + \sin\theta e^{-iE_b t}\langle\ell_1|b\rangle|^2 \\
&= |\cos^2\theta e^{-iE_a t} + \sin^2\theta e^{-iE_b t}|^2 \\
&= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos(E_a - E_b)t
\end{aligned}
$$

Neutrino oscillations (contd.)

For small $m_{a,b}$ neutrinos are produced ultrarelativistic, $|\vec{p}\,| \gg m_{a,b} \Rightarrow$

$$E_a - E_b = \frac{E_a^2 - E_b^2}{E_a + E_b} = \frac{m_a^2 - m_b^2}{E_a + E_b} \simeq \frac{m_a^2 - m_b^2}{2|\vec{p}\,|} = \frac{\Delta m^2}{2|\vec{p}\,|}$$

Distance covered from production process $x \simeq t$

Neutrino flux $\Phi_1(x(t)) \propto |\langle \ell_1 | \psi(t)\rangle|^2 \Rightarrow$

$$\Phi_1(x) = A + B \cos \frac{\Delta m^2}{2|\vec{p}\,|} x$$

$$\frac{A}{B} = \frac{\cos^4\theta + \sin^4\theta}{2\cos^2\theta \sin^2\theta} = \frac{1 + \cos^2 2\theta}{1 - \cos^2 2\theta}$$

Neutrino oscillation can explain anomalies, but requires non-degeneracy of neutrino masses $\Rightarrow$ at least one of the neutrinos must be massive

Oscillations have been observed experimentally:

- must abandon assumption of massless neutrinos
- lepton flavour (family number) not conserved anymore

Neutrino oscillations (contd.)

Generalisation to three families:

- mixing matrix parameterised in terms of three angle and one ineliminable phase (other phases unphysical)
- three mass-square differences

Experimental results:

$$\Delta m_{21}^2 = 7.55^{+0.20}_{-0.16} \cdot 10^{-5}\mathrm{eV}^2 \qquad |\Delta m_{31}^2| = \begin{cases} 2.50 \pm 0.03 \cdot 10^{-3}\mathrm{eV}^2 & \text{(NO)} \\ 2.42^{+0.03}_{-0.04} \cdot 10^{-3}\mathrm{eV}^2 & \text{(IO)} \end{cases}$$

$$\sin^2\theta_{12} = 3.20^{+0.20}_{-0.16} \cdot 10^{-1} \qquad \sin^2\theta_{23} = \begin{cases} 5.47^{+0.20}_{-0.30} \cdot 10^{-1} & \text{(NO)} \\ 5.51^{+0.18}_{-0.30} \cdot 10^{-1} & \text{(IO)} \end{cases}$$

$$\sin^2\theta_{13} = \begin{cases} 2.160^{+0.083}_{-0.069} \cdot 10^{-2} & \text{(NO)} \\ 2.220^{+0.074}_{-0.076} \cdot 10^{-2} & \text{(IO)} \end{cases} \qquad \frac{\delta_{CP}}{\pi} = \begin{cases} 1.32^{+0.21}_{-0.15} & \text{(NO)} \\ 1.56^{+0.13}_{-0.15} & \text{(IO)} \end{cases}$$

NO $=$ "normal ordering": $\Delta m_{32}^2 > 0 \Rightarrow m_1 < m_2 \ll m_3$
IO $=$ "inverted ordering": $\Delta m_{32}^2 < 0 \Rightarrow m_3 \ll m_1 < m_2$

Which ordering is realised is not determined by current experiments

Neutrino masses

How to modify the Standard Model to account for neutrino masses?

Simplest possibility: add Yukawa coupling of neutrinos to Higgs field

$$f^{(\nu)}_{AB}(\bar{\bar{\ell}}_A)_L\tilde{\phi}(\nu_B)_R$$

$(\nu_A)_R$: right-handed (more precisely: negative chirality) neutrino fields
$f^{(\nu)}_{AB}$ not necessarily diagonal, neutrino mixing allowed

RH fields needed for *Dirac mass term* $\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L$, generated from the above after spontaneous symmetry breaking

RH neutrino not coupled to any of the other particles in the Standard Model: *sterile* neutrino

Neutrino masses (contd.)

Mass matrix:

$$M_{AB}^{(\nu)} = \frac{v}{\sqrt{2}} f_{AB}^{(\nu)} = S^{(\nu)\dagger} M_{\mathrm{diag}}^{(\nu)} T^{(\nu)}$$

Leptonic current $\bar{\nu}_A \mathcal{O}_L^\alpha \ell_A$ in terms of definite-mass charged lepton fields $\ell_A \Rightarrow$ neutrino fields ν_A have definite lepton family number (by definition)
Definite-mass neutrino fields obtained by means of unitary transformation

$$\nu_L = S^{(\nu)} \nu_L^{(\mathrm{mass})} \qquad \nu_R = T^{(\nu)} \nu_R^{(\mathrm{mass})}$$

$S^{(\nu)} \equiv U_{\mathrm{PMNS}}$: Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

- relates mass and lepton-family (left-handed) eigenstates
- 3×3 unitary matrix, parameterised (up to unphysical phases) in terms of three angles and one phase
- lepton number still conserved, lepton family number not anymore

RH neutrino field ν_R is $\mathrm{SU}(2)_L$ singlet with vanishing $\mathrm{U}(1)_Y$ charge

$$-y(\ell) + y(\tilde{\phi}) = 1 - 1 = 0$$

$\Rightarrow$ invariant under whole gauge group G, no problems with the anomaly

Majorana neutrinos and the see-saw mechanism

Dirac mass term: simple solution, but

- introduces essentially unobservable particles
- no chance of explaining why neutrino masses are so small

ν_R (with definite flavour) is a G-singlet, truly neutral fermion field
$\Rightarrow$ can take *Majorana mass term* (possibly in addition to Dirac mass term)

Majorana's neutrality condition: $\boxed{(\nu_R)^c = \nu_R}$

$$(\nu_R)^c \equiv C\bar{\nu}_R^T \quad C = -i\gamma^2\gamma^0 \Rightarrow (\nu_R)^c = -i\gamma^2\nu_R^*$$

Majorana mass term

$$\mathcal{L}_{\mathrm{Maj}} = \tfrac{1}{2} m_M \bar{\nu_R}(\nu_R)^c + \mathrm{c.c.}$$

More generally: $\bar{\nu_{\ell R}} M_{\ell\ell'}(\nu_{\ell' R})^c$, can be diagonalised yielding Majorana terms

- violates lepton number but no other symmetry
- experimental signature: neutrinoless double-beta decay processes (unobserved so far)
- $(\nu_R)^c$ is a LH field $\Rightarrow$ Majorana ν ($= \bar{\nu}$) appears with both chiralities

Majorana neutrinos and the see-saw mechanism (contd.)

Putting together Dirac and Majorana mass terms

$$\mathcal{L} = \tfrac{1}{2}\bar{\nu}^c M \nu \qquad \nu = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \qquad M = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}$$

ν: doublet of LH fields

Diagonalise M, eigenvalues $m_\pm = \frac{1}{2}\left(m_M \pm \sqrt{m_M^2 + 4m_D^2}\right)$

m_M: origin of L-breaking, naturally assumed large – scale of new physics?

In the limit $m_M \gg m_D$, eigenvalue and definite-mass fields

$$m_+ \simeq m_M \qquad\qquad m_- \simeq -\frac{m_D^2}{m_M}$$

$$N \simeq (\nu_R)^c + \frac{m_D}{m_M}\nu_L \qquad\qquad \nu \simeq \nu_L - \frac{m_D}{m_M}(\nu_R)^c$$

Sign of m_- not problematic, can be changed by redefining $\psi \to \gamma^5\psi$

- N: large mass, small coupling to active neutrino field $\nu_L \Rightarrow$ heavy neutrino weakly interacting with other matter
- $\nu \simeq \nu_L$: naturally small mass: reasonable expectation is m_D of same order of corresponding charged lepton, m_D/m_M leads to strong suppression (*see-saw mechanism*)

Grand Unification Theories

Standard Model gauge group: $G_{\rm SM} = {\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm U}(1)_y$

Different coupling for each factor, not truly unified, i.e., containing a single coupling constant governing all the types of interactions

Some find it an unsatisfactory aspect: look for further unification

Basic idea: find bigger gauge group G with a single coupling constant, get Standard Model gauge group $G_{\rm SM}$ via symmetry breaking

Minimal possibility: use group ${\rm SU}(5)$

- contains $G_{\rm SM}$
- rank 4, same rank as $G_{\rm SM} \Rightarrow$ four commuting generators that can be identified with t_3, t_8, T_3, Y
- only rank-4 group admitting complex representations (required by chiral structure) which can accommodate matter spectrum of SM (including electric charge) without introducing new matter

Grand Unification Theories (contd.)

$\mathrm{SU}(5)$: 24-dimensional Lie group of 5-dim unitary unimodular matrices

- simple group $\Rightarrow$ single coupling constant
- besides known gauge bosons, $24 - (8+3+1) = 12$ new ones

Among diagonal generators there is λ^{24}

$$\lambda^{24} = \frac{1}{\sqrt{15}}\mathrm{diag}(2,2,2,-3-3)$$

$\lambda^{24} \sim$ hypercharges of SM particles up to common normalisation:

- First three entries $\propto$ hypercharge Y of $d_L^c \sim d_R^*$
  $\psi_L^c = C\bar{\psi}_R^T = -i\gamma^2\gamma^0\bar{\psi}_R^T = -i\gamma^2\psi_R^*$
- Last two entries $\propto$ hypercharge Y of ℓ_L

Embed $\mathrm{SU}(3)_c$ and $\mathrm{SU}(2)_L$ factors of G_{SM} in upper and lower corners

$$\begin{pmatrix} \mathrm{SU}(3)_c & 0 \\ 0 & \mathrm{SU}(2)_L \end{pmatrix} \quad \Rightarrow \quad [\lambda^{24}, G_{\mathrm{SM}}] = 0$$

Needed:

- group representations for matter particles
- suitable symmetry-breaking pattern

SU(5) GUT: matter multiplets

$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L$$

$\bar{\mathbf{5}}_F$ (antifundamental):
use for three colours of negatively-charged quark and the leptons of one SM generation
Representations of $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \subset G_{\mathrm{SM}}$:

- First three components: $(\bar{\mathbf{3}}, \mathbf{1})$
- Last two components: $(\mathbf{1}, \mathbf{2})$

$$\begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_L$$

$\mathbf{10}$ (antisymmetric part of $\mathbf{5}_F \otimes \mathbf{5}_F = \mathbf{10} \oplus \mathbf{15}$):
use for remaining matter fields

- Top-left block: $(\bar{\mathbf{3}}, \mathbf{1})$ (antisymmetric part of colour $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$, and $\mathrm{SU}(2)_L$ singlet)
- Top-right block: $(\mathbf{3}, \mathbf{2})$
- Bottom-right block: $(\mathbf{1}, \mathbf{1})$ (antisym. part of $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$ of $\mathrm{SU}(2)_L$), corresponds to e_R

$$\begin{pmatrix} d_{1R} \\ d_{2R} \\ d_{3R} \\ e_L^{-c} \\ -\nu_{eL}^c \end{pmatrix}$$

$\mathbf{5}_F$ (fundamental): would contain RH fields, not used

SU(5) GUT: matter multiplets (contd.)

- $\bar{\mathbf{5}}_F$ and **10** fit precisely one SM generation
- number of generations unexplained in this framework

SM charges:

$$T_3 = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \tau^3 \end{pmatrix} \qquad Q = T_3 + \frac{Y}{2} = T_3 + c\frac{\lambda^{24}}{2} \qquad c = -\sqrt{\frac{5}{3}}$$

$$Y(\bar{\mathbf{5}}_F) = (+\tfrac{2}{3}, +\tfrac{2}{3}, +\tfrac{2}{3}, -1, -1) \qquad Q(\bar{\mathbf{5}}_F) = (+\tfrac{1}{3}, +\tfrac{1}{3}, +\tfrac{1}{3}, -1, 0)$$

$$Y(\mathbf{10}) = \begin{pmatrix} 0 & -\frac{4}{3} & -\frac{4}{3} & +\frac{1}{3} & +\frac{1}{3} \\ & 0 & -\frac{4}{3} & +\frac{1}{3} & +\frac{1}{3} \\ & & 0 & +\frac{1}{3} & +\frac{1}{3} \\ & & & 0 & 2 \\ & & & & 0 \end{pmatrix} \qquad Q(\mathbf{10}) = \begin{pmatrix} 0 & -\frac{2}{3} & -\frac{2}{3} & +\frac{2}{3} & -\frac{1}{3} \\ & 0 & -\frac{2}{3} & +\frac{2}{3} & -\frac{1}{3} \\ & & 0 & +\frac{2}{3} & -\frac{1}{3} \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}$$

$$Y(\mathbf{10}) = Y_{\text{row}}(\mathbf{5}) + Y_{\text{column}}(\mathbf{5}) = -(Y_{\text{row}}(\bar{\mathbf{5}}) + Y_{\text{column}}(\bar{\mathbf{5}}))$$
$$Q(\mathbf{10}) = Q_{\text{row}}(\mathbf{5}) + Q_{\text{column}}(\mathbf{5}) = -(Q_{\text{row}}(\bar{\mathbf{5}}) + Q_{\text{column}}(\bar{\mathbf{5}}))$$

Y, Q match SM + correct representations of $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L$

SU(5) GUT: gauge bosons

Gauge bosons transform in **24** representation (adjoint)
Can be organised as follows ($\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{1} \oplus \mathbf{24}$)

$$A_\mu = \left(\begin{array}{ccc|cc} \multicolumn{3}{c|}{G^i{}_{j\mu} + \frac{2}{\sqrt{30}} B_\mu \delta^i{}_j} & \begin{matrix} X^{1c}_\mu \\ X^{2c}_\mu \\ X^{3c}_\mu \end{matrix} & \begin{matrix} Y^{1c}_\mu \\ Y^{2c}_\mu \\ Y^{3c}_\mu \end{matrix} \\ \hline X^1_\mu & X^2_\mu & X^3_\mu & \frac{1}{\sqrt{2}} W^3_\mu - \sqrt{\frac{3}{10}} B_\mu & W^+_\mu \\ Y^1_\mu & Y^2_\mu & Y^3_\mu & W^-_\mu & -\frac{1}{\sqrt{2}} W^3_\mu - \sqrt{\frac{3}{10}} B_\mu \end{array}\right)$$

$G^i_j \sim (\mathbf{8}, \mathbf{1})$: gluons
$W^{\pm,3} \sim (\mathbf{1}, \mathbf{3})$: intermediate vector bosons
$B \sim (\mathbf{1}, \mathbf{1})$: hypercharge generator (mixes with W^3 to yield Z^0 and γ)
$X, Y \sim (\bar{\mathbf{3}}, \mathbf{2})$: 12 new gauge bosons (notice $X^c, Y^c \sim (\mathbf{3}, \bar{\mathbf{2}})$)

Electric charges:

$$Q^\dagger A_\mu Q = (Q(\bar{\mathbf{5}}) + Q(\mathbf{5})) A_\mu$$

X, Y have fractional charges $Q_X = -\frac{1}{3} - 1 = -\frac{4}{3}$, $Q_Y = -\frac{1}{3} + 0 = -\frac{1}{3}$