Weak Interactions

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Third quark family: b quark

$\boxed{\textbf{\textit{b} (bottom) quark}}$ Discovered @Fermilab in 1977 by Lederman *et al.*:

- proton beam against platinum target, process $p\,p \to \ell^+\,\ell^-\,X$
- look mostly for $\ell = \mu$: large penetrating power, survive filtering required to remove all uninteresting hadrons from final products
- n. of events vs. $\mu^+\mu^-$ invariant mass shows narrow bump (resonance) at $\sqrt{s} = 9.46\,\mathrm{GeV}$, identified as Υ = bound state of a new quark
- $s_\Upsilon = 1$, $m_\Upsilon = 9.46\,\mathrm{GeV}$, $\Gamma_\Upsilon = 44.3\,\mathrm{keV} \to \tau_\Upsilon = 1.2 \cdot 10^{-20}\,s$
- Υ: ground state of *bottomonium*, $b\bar{b}$ bound state, can also be seen in lepton collider via $e^+\,e^- \to \Upsilon \to \mathrm{hadrons}$ ($m_b = 4.2\,\mathrm{GeV}$, $q_b = -\frac{1}{3}$)
- narrow Γ_Υ from small Υ coupling to hadrons (OZI rule), but stronger than EM coupling $\Rightarrow$ intermediate τ_Υ between hadronic and EM
- lightest mesons with nonzero "bottomness": B mesons $B^0 = d\bar{b}$, $\bar{B}^0 = b\bar{d}$, $B^+ = u\bar{b}$, $B^- = b\bar{u}$, $m_B \simeq 5.3\,\mathrm{GeV}$
- $B^0 - \bar{B}^0$ similar to $K^0 - \bar{K}^0$ system $\to$ "bottomness" oscillations

Third quark family: t quark

$\boxed{t \textbf{ (top) quark}}$ Discovered @Fermilab in 1995 by CDF and DØ exp.:

- $m_t = 173\,\mathrm{GeV}$ — heaviest known elementary particle, $q_t = \frac{2}{3}$
- no top-antitop bound states have been observed: top quark decays in about $10^{-25}\,s$, not enough time to hadronise
- third quark family suggested already in 1973 by Kobayashi and Maskawa $\Rightarrow$ minimal number of families allowing for CP violating effects in the weak Lagrangian

Existence of three and only three families of leptons with light neutrinos has been extablished experimentally at LEP

CKM matrix

Assuming n families of quarks and universality of charged weak interactions

$$J_h^\alpha = \sum_{f,f'=1}^{n} \bar{\alpha}_f \mathcal{O}_L^\alpha \kappa_{f'} V_{ff'}$$

fth quark family: $\begin{pmatrix} \alpha_f \\ \kappa_f \end{pmatrix}$, V: $n \times n$ unitary matrix, $V^\dagger V = \mathbf{1}$
Unitarity required so that appropriate linear combinations of κ_f interact with corresponding α_f like leptons and corresponding neutrinos do

$$\kappa'_f = \sum_{f'=1}^{n} V_{ff'} \kappa_{f'} \qquad (\text{recall } d' = \cos\theta_C d + \sin\theta_C s)$$

How many physically relevant parameters are contained in V?

CKM matrix (contd.)

General complex $n \times n$ matrix has $2n^2$ real parameters. Unitarity implies

$$\sum_k V_{ik} V_{jk}^* = \delta_{ij}$$

- $\Rightarrow$ n real relations $\sum_k |V_{ik}|^2 = 1$ for $i = j$
- $\Rightarrow$ $\frac{n(n-1)}{2}$ complex rel. $\sum_k V_{ik} V_{jk}^* = 0$ for $i \neq j \Rightarrow n(n-1)$ real rel.
- $\Rightarrow$ $n + n(n-1) = n^2$ real rel., reduce n. of real parameters in V to $2n^2 - n^2 = n^2$

Not all n^2 parameters are physically meaningful: can redefine $\alpha_f \to e^{i\phi_f}\alpha_f$ and $\kappa_f \to e^{i\psi_f}\kappa_f$ independently without any observable physical effect

Does not affect QCD Lagrangian which has $U(1)^{2n}$ flavour symmetry

Since $V_{jk} \to e^{-i(\phi_j - \psi_k)} V_{jk}$ one can set to zero the phase of a certain number of matrix elements (unphysical phases)

CKM matrix (contd.)

N. of independent phase factors $e^{-i(\phi_j-\psi_k)}$: write

$$\phi_j - \psi_k = (\phi_1 - \psi_1) + (\phi_j - \phi_1) - (\psi_k - \psi_1)$$

- can choose the first term on the right-hand side arbitrarily
- $n-1$ independent differences $\phi_j - \phi_1$, $n-1$ independent differences $\psi_k - \psi_1 \Rightarrow 1 + 2(n-1) = 2n-1$ independent phase factors
- alternatively: can change all phases ϕ_j, ψ_k by same amount w/out changing differences, n. of independent phase diff. reduced to $2n-1$

Number of physically meaningful real parameters in V:

$$n^2 - 2n + 1 = (n-1)^2$$

- $n \times n$ unitary matrices $\supset$ $n \times n$ real orthogonal matrices as subspace
- general $n \times n$ orthogonal matrix depends on $\frac{n(n-1)}{2}$ angles $\Rightarrow \frac{n(n-1)}{2}$ angles among n^2 real parameters of unitary matrix
- remaining $n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$ parameters are phases, but not all physical: only $\frac{n(n+1)}{2} - 2n + 1 = \frac{(n-1)(n-2)}{2}$ are physical

CKM matrix (contd.)

Summarising: $\frac{n(n-1)}{2}$ *quark mixing angles*, $\frac{(n-1)(n-2)}{2}$ physical phases

$n = 1$ family:	0 angles, 0 phases (nothing to mix)
$n = 2$ families:	1 angles, 0 phases (V is real, CP conserved)
$n = 3$ families:	3 angles, 1 phase (V complex, CP violation possible)

Matrix V: Kobayashi-Maskawa, or *Cabibbo-Kobayashi-Maskawa* matrix

Possible parameterisation: treat (d, s, b) as coordinates (z, y, x)

Most general 3d rotation (Euler angles):

1. θ_3 around z 2. θ_1 around (new) x 3. θ_2 around (new) z

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

$c_i = \cos\theta_i$, $s_i = \sin\theta_i$

CKM matrix (contd.)

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$c_i = \cos\theta_i, \; s_i = \sin\theta_i$

CKM matrix (contd.)

To make remaining phase physical include it in $(3,3)$ element of 2nd factor

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

Cannot be removed by a phase redefinition

δ: CP-violating phase

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - e^{i\delta} s_2 s_3 & c_1 c_2 s_3 + e^{i\delta} s_2 c_3 \\ s_1 s_2 c_2 & -c_1 s_2 c_3 - e^{i\delta} c_2 s_3 & -c_1 s_2 s_3 + e^{i\delta} c_2 c_3 \end{pmatrix}$$

Experimental fact: d, s have small mixing with $b \Rightarrow$ expect $\theta_{2,3}$ small

$$V \simeq \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

$\Rightarrow$ identify $\theta_1 \simeq \theta_C$

CKM matrix (contd.)

Retaining leading contributions

$$V \simeq \begin{pmatrix} 1 & s_1 & s_1 s_3 \\ -s_1 & 1 & s_3 + e^{i\delta} s_2 \\ s_1 s_2 & -s_2 - e^{i\delta} s_3 & e^{i\delta} \end{pmatrix}$$

Main processes involve transitions $d \leftrightarrow u$, $s \leftrightarrow c$ and $b \leftrightarrow t$
$\Rightarrow$ concept of families physically meaningful, dominant decays involve transitions within families

Experimental results for first row of CKM matrix:

$$|V_{ud}| = 0.97420(21) \qquad |V_{us}| = 0.2243(5) \qquad |V_{ub}| = 3.94(36) \cdot 10^{-3}$$

- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$, good agreement with unitarity
- $|V_{ub}|/|V_{ud}| = |s_3| \simeq 2 \cdot 10^{-2} \Rightarrow \theta_3$ small
- $|V_{td}|$ also small $\Rightarrow$ small $|s_2| \Rightarrow$ small θ_2

Limitations of the four-fermion theory

Main theoretical problem of four-fermion theory is lack of renormalisability

- $[G] = [m]^{d_G} = [m]^{-2} \Rightarrow$ new divergences as perturbative order increases: with F fields @vertex $\Rightarrow 4 = d_G + \frac{3}{2}F \Rightarrow F = \frac{8}{3} - \frac{2}{3}d_G$

$$D_{\rm div} = \underbrace{4(I - V + 1)}_{\substack{\text{momentum integrals} \\ \text{- conservation @vertices} \\ \text{removing overall } \delta}} - \underbrace{I}_{\text{propagators}} = 3I - 4V + 4$$

  Using topological relation $2I + E = FV$

$$D_{\rm div} = \frac{3}{2}(FV - E) - 4V + 4 = (\tfrac{3}{2}F - 4)V - \frac{3}{2}E + 4$$

$$\Longrightarrow D_{\rm div} > 0 \;\text{ if }\; E < (F - \tfrac{8}{3})V + \tfrac{8}{3} = -d_G V + \tfrac{8}{3} \underset{F=4}{=} \tfrac{4}{3}(V + 2)$$

- requires infinitely many counterterms resulting in lack of predictivity

Can treat 4-fermion theory as an effective theory: renormalised up to n counterterms is predictive up to energy where effects of $(n+1)$th type of divergence show up; theoretically unsatisfactory, however