Weak Interactions

Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

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Neutral K mass difference and GIM mechanism

$K^0, \bar{K}^0$ oscillation allowed because charge distinguishing them (S) not exactly conserved

Different from $n, \bar{n}$ differing in baryon number

$S(K^0) = 1$, $S(\bar{K}^0) = -1 \Rightarrow$ oscillation is $\Delta S = 2$ process requiring second-order weak interaction

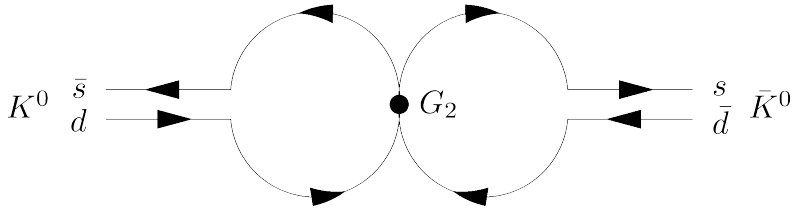
Oscillation frequency equals mass difference Δm between CP eigenstates, can be expressed in terms of $K^0, \bar{K}^0$ mixing matrix element

$$
\begin{aligned}
m_j &= \mathrm{Re}\langle K_j^0 | H_{\mathrm{eff}} | K_j^0 \rangle \\
m_1 &= \tfrac{1}{2}\mathrm{Re}\left(\langle K^0| - \langle \bar{K}^0|\right) H_{\mathrm{eff}} \left(|K^0\rangle - |\bar{K}^0\rangle\right) \\
m_2 &= \tfrac{1}{2}\mathrm{Re}\left(\langle K^0| + \langle \bar{K}^0|\right) H_{\mathrm{eff}} \left(|K^0\rangle + |\bar{K}^0\rangle\right) \\
\Delta m = m_2 - m_1 &= \mathrm{Re}\left(\langle \bar{K}^0 | H_{\mathrm{eff}} | K^0\rangle + \langle K^0 | H_{\mathrm{eff}} | \bar{K}^0\rangle\right)
\end{aligned}
$$

Neutral K mass difference and GIM mechanism (contd.)

If there was a $\Delta S = 2$ vertex with some coupling G_2

$$G_2(\bar{d}\mathcal{O}_L^\alpha s \bar{d}\mathcal{O}_{L\alpha} s + \bar{s}\mathcal{O}_L^\alpha d \bar{s}\mathcal{O}_{L\alpha} d)$$

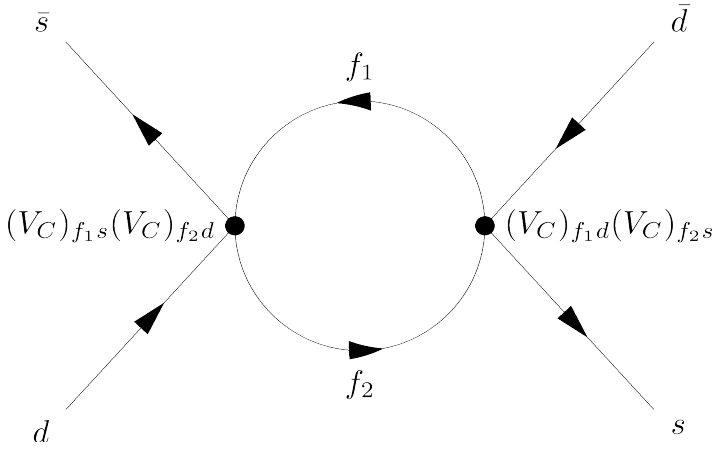


$$\Delta m = 2G_2 \,\mathrm{Re}\langle \bar{K}^0|\bar{s}\mathcal{O}_L^\alpha d \bar{s}\mathcal{O}_{L\alpha} d|K^0\rangle = 2G_2 \,\mathrm{Re} \sum_n \langle \bar{K}^0|\bar{s}\mathcal{O}_L^\alpha d|n\rangle\langle n|\bar{s}\mathcal{O}_{L\alpha} d|K^0\rangle$$
$$\sim G_2 \langle \bar{K}^0|\bar{s}\mathcal{O}_L^\alpha d|0\rangle\langle 0|\bar{s}\mathcal{O}_{L\alpha} d|K^0\rangle \propto G_2 f_K^2 m_K$$

- sum over states approximated with vacuum contribution
- f_K: kaon decay constant $\langle 0|\bar{s}\mathcal{O}_{L\alpha} d|K^0\rangle = \frac{1}{\sqrt{2}}\langle 0|\bar{s}\mathcal{O}_{L\alpha} u|K^+\rangle = -ip_\alpha f_K$
- compare with Δm from kaon oscillations $\Rightarrow$ requires $G_2 \sim 10^{-7} G_F$
- $\Delta S = 2$ vertex not present in $V - A$ theory, obtained as effective vertex in second order perturbation theory
- effective vertex involves quadratically-divergent loop integral, impose cut-off $\Lambda = \mathcal{O}(m_W)$ (four-fermion interaction picture breaks down)
- effective $G_2 = G_F^2 \Lambda^2 = G_F^2 m_W^2 \simeq 10^{-1} G$, too big to explain small Δm

Neutral K mass difference and GIM mechanism (contd.)

Way out: assume existence of charm quark c forming second family with s



- Charged weak hadronic current $\bar{u}\mathcal{O}_L^\alpha d' + \bar{c}\mathcal{O}_L^\alpha s'$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = V_C \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} (V_C)_{ud} & (V_C)_{us} \\ (V_C)_{cd} & (V_C)_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- Not one but four loop diagrams: upper/lower line same (uu, cc) or different (uc, cu), couplings $\sin^2\theta_C \cos^2\theta_C$ or $-\sin^2\theta_C \cos^2\theta_C$
- If $m_u = m_c \Rightarrow$ exact cancellation, in any case cancellation of very large loop momenta $\Rightarrow$ finite result and suppression mechanism – *GIM mechanism* after Glashow, Iliopoulos and Maiani

Neutral K mass difference and GIM mechanism (contd.)

Detailed calculation: effective coupling

$$G_2 = \frac{G^2}{(4\pi)^2} \sin^2\theta_C \cos^2\theta_C (m_c - m_u)^2$$

Treating neutral kaons as pure $q\bar{q}$ states $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$

$$\begin{aligned}
\Delta m &= 2G_2 \,\mathrm{Re}\langle \bar{K}^0 | \bar{s}\mathcal{O}_L^\alpha d \bar{s}\mathcal{O}_{L\alpha} d | K^0 \rangle \\
&\simeq 2G_2 \,\mathrm{Re}\tfrac{8}{3} \langle \bar{K}^0 | \bar{s}\mathcal{O}_L^\alpha d | 0 \rangle \langle 0 | \bar{s}\mathcal{O}_{L\alpha} d | K^0 \rangle \\
&= \tfrac{16}{3} \tfrac{G^2}{(4\pi)^2} \sin^2\theta_C \cos^2\theta_C (m_c - m_u)^2 f_K^2 m_K^2 \tfrac{1}{2m_K} \\
&\simeq \tfrac{1}{3} \tfrac{G^2}{2\pi} \sin^2\theta_C \cos^2\theta_C m_c^2 f_K^2 m_K
\end{aligned}$$

$\frac{8}{3}$ colour factor, $2m_k$ compensates relativistic normalisation of states

Comparing with $K^+ \to \mu^+ \nu_\mu$ decay width

$$\Delta m = \frac{4\cos^2\theta_C m_c^2}{3\pi m_\mu^2} \Gamma(K^+ \to \mu^+ \nu_\mu)$$

τ decays

- τ lepton discovered in 1975 @SLAC in $e^+ e^-$ collisions
- ν_τ immediately theorised, observed @Fermilab only in 2000 by DONUT experiment

$$J_\tau = \tfrac{1}{2} \qquad m_\tau = 1.78\,\mathrm{GeV} \qquad \tau_\tau = 3.4 \cdot 10^{-13}\, s$$

- Assuming leptonic universality, add current $\bar{\nu}_\tau \mathcal{O}_L^\alpha \tau$ to charged weak leptonic current with the same coupling as e, μ currents

$$J_l^\alpha = \bar{\nu}_e \mathcal{O}_L^\alpha e + \bar{\nu}_\mu \mathcal{O}_L^\alpha \mu + \bar{\nu}_\tau \mathcal{O}_L^\alpha \tau$$

- Decay rate for **leptonic decays** as in muon decay $\mu \to e\, \bar{\nu}_e \nu_\mu$; since $m_e \ll m_\mu \ll m_\tau$ can treat the final lepton as massless, just replace $m_\mu \to m_\tau$ in muon decay rate

$$\Gamma(\tau \to \ell\, \bar{\nu}_\ell\, \nu_\tau) = \frac{G^2 m_\tau^5}{192\pi^3} = \left(\frac{m_\tau}{m_\mu}\right)^5 \Gamma(\mu \to e\, \bar{\nu}_e\, \nu_\mu) \simeq 6.2 \cdot 10^{11}\, s^{-1}$$

Semi-hadronic decays of τ

- New feature: besides leptonic decays $\tau \to \ell\, \bar{\nu}_\ell\, \nu_\tau$, $\ell = e, \mu$, heavy mass of τ allows also **semi-hadronic decays** $\tau \to \nu_\tau + \mathrm{hadrons}$
- τ lighter than lightest charmed particle $m_\tau < m_{D^0} = 1.864\,\mathrm{GeV} \Rightarrow$ only decays involving u, d, s quarks allowed
- Total width $\Gamma(\tau \to \nu_\tau + \mathrm{hadrons})$ can be estimated using PT exploiting asymptotic freedom of QCD (like other high-energy inclusive decay/scattering processes into hadrons)

$$R \equiv \frac{\sigma(e^+ e^- \to \mathrm{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)} = 3 \sum_f q_f^2 \left(1 + \frac{\alpha_s(s)}{\pi} + \ldots\right) \underset{\mathrm{LO}, s<4m_c^2}{\simeq} 3\left(\tfrac{4}{9}+\tfrac{1}{9}+\tfrac{1}{9}\right) = 2$$

$\alpha_s(s)$: running coupling constant, q_f: electric charge of quark f
sum over flavours f s.t. $4m_f^2 \leq s$, $N_c = 3$ factor due to colour

Idea: total hadron production rate = total quark production rate:
$e^+ e^- \to q\bar{q}$ and hadronises
$\Rightarrow$ sum over all quark states = sum over all hadronic final states

$$\sigma(e^+e^- \to \mathrm{hadrons}) \underset{\text{large } s}{=} \sum_f \sigma(e^+e^- \to \bar{f} f) \underset{\mathrm{LO\,PT}}{\simeq} 3 \sum_f q_f^2 \sigma(e^+e^- \to \mu^+\mu^-)$$

Semi-hadronic decays of τ (contd.)

Using the same idea, estimate $\Gamma(\tau \to \nu_\tau + \text{hadrons})$ as

$$\frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu)} = \frac{\Gamma(\tau \to \nu_\tau\, u\, \bar{d}) + \Gamma(\tau \to \nu_\tau\, u\, \bar{s})}{\Gamma(\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu)}$$
$$= (\cos^2\theta_C + \sin^2\theta_C) N_c = 3$$

Ignoring $\mathcal{O}(\alpha_s(m_\tau^2))$ corrections to tree-level contributions

- Processes $\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu$, $\tau \to \nu_\tau\, u\, \bar{d}$ and $\tau \to \nu_\tau\, u\, \bar{s}$ described by the same diagram except for coupling constants, in ratio $1 : \cos\theta_C : \sin\theta_C$
- $u\bar{d}$, $u\bar{s}$ pairs can be produced in $N_c = 3$ different colours, summed over for total decay rate

Total width of the τ lepton:

$$\Gamma(\tau) = \Gamma(\tau \to \nu_\tau + \text{hadrons}) + \Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e) + \Gamma(\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu)$$
$$\simeq 5\Gamma(\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu)$$

$$\tau_\tau \simeq \frac{1}{5\Gamma(\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu)} = 3.2 \cdot 10^{-13}\, s \qquad (\text{exp.: } 3.4 \cdot 10^{-13}\, s)$$

Semi-hadronic decay modes: $\tau^- \to \pi^- \nu_\tau$

Decay amplitude for $\tau^- \to \pi^- \nu_\tau$

$$\mathcal{M}_{\rm fi} = -\frac{G\cos\theta_C}{\sqrt{2}}\bar{u}_\nu(p_\nu)\gamma^\alpha(1-\gamma^5)u_\tau(p_\tau)\langle\pi^-|(\bar{d}\gamma_\alpha(1-\gamma^5)u)(0)|0\rangle$$
$$= i\frac{G\cos\theta_C}{\sqrt{2}}f_\pi\sqrt{2}\bar{u}_\nu(p_\nu)\not{p}_\pi(1-\gamma^5)u_\tau(p_\tau)$$

Known matrix element

$$\langle 0|(\bar{u}\gamma_\alpha(1-\gamma^5)d)(0)|\pi^+\rangle = -\langle 0|(\bar{u}\gamma_\alpha\gamma^5 d)(0)|\pi^+\rangle = -i\sqrt{2}f_\pi(p_\pi)_\alpha$$

Same pion decay constant f_π as in $\pi^- \to \ell^-\bar{\nu}_\ell$

Using momentum conservation $p_\pi = p_\tau - p_\nu$

$$\bar{u}_\nu(p_\nu)\not{p}_\pi(1-\gamma^5)u_\tau(p_\tau) = \bar{u}_\nu(p_\nu)\not{p}_\tau(1-\gamma^5)u_\tau(p_\tau) = m_\tau\bar{u}_\nu(p_\nu)(1+\gamma^5)u_\tau(p_\tau)$$

Total width for unpolarised τ

$$\Gamma = \frac{1}{2}\frac{1}{2m_\tau}\Phi^{(2)}G^2\cos^2\theta_C m_\tau^2 f_\pi^2\sum_{s_\tau}\bar{u}_\nu(p_\nu)(1+\gamma^5)u_\tau(p_\tau)\bar{u}_\tau(p_\tau)(1-\gamma^5)u_\nu(p_\nu)$$
$$= \frac{G^2\cos^2\theta_C}{4}m_\tau f_\pi^2\Phi^{(2)}\mathrm{tr}\,(\not{p}_\tau+m_\tau)(1-\gamma^5)\not{p}_\nu(1+\gamma^5)$$
$$= 2G^2\cos^2\theta_C m_\tau f_\pi^2\Phi^{(2)}p_\tau\cdot p_\nu$$

Can integrate over $d\Phi^{(2)}$ before spin sum since final energies fixed by momentum conservation

Semi-hadronic decay modes: $\tau^- \to \pi^- \nu_\tau$ (contd.)

Two-body phase space

$$\Phi^{(2)} = \frac{p_{\rm CM}}{4\pi E_{\rm CM}} = \frac{E_\nu}{4\pi m_\tau}$$

Using $p_\tau \cdot p_\nu = m_\tau E_\nu$ with

$$E_\nu = \frac{m_\tau^2 - m_\pi^2}{2m_\tau} = \frac{m_\tau}{2}\left(1 - \frac{m_\pi^2}{m_\tau^2}\right)$$

Total width for $\tau^- \to \pi^- \nu_\tau$, unpolarised τ

$$\Gamma = \frac{G^2 \cos^2\theta_C f_\pi^2}{2\pi} m_\tau E_\nu^2 = \frac{G^2 \cos^2\theta_C f_\pi^2}{8\pi} m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2$$

$$= \Gamma(\tau \to \mu\, \bar{\nu}_\mu\, \nu_\tau) \frac{24\pi^2}{m_\tau^2} \cos^2\theta_C f_\pi^2 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \simeq 0.6\, \Gamma(\tau \to \mu\, \bar{\nu}_\mu\, \nu_\tau)$$

Decay width for $\tau^- \to K^- \nu_\tau$ obtained by replacing $m_\pi \to m_K$, $f_\pi \to f_K$ and $\cos\theta_C \to \sin\theta_C$

Semi-hadronic decay modes: $\tau^- \to \rho^- \nu_\tau$

ρ mesons: isotriplet ($I = 1$) of vector particles ($J^P = 1^-$), $m_\rho = 770\,\mathrm{MeV}$

Quark content same as pions ($\rho^+ = -u\bar{d}$, $\rho^0 = \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$, $\rho^- = d\bar{u}$), but the quark spin state has $s = 1$ instead of $s = 0$

Different parameterisation of hadronic matrix element:

$$H_\alpha = \langle \rho^- | \bar{d}\gamma_\alpha(1-\gamma^5)u|0\rangle = \langle \rho^- | \bar{d}\gamma_\alpha u|0\rangle = g_\rho \varepsilon_\alpha + f_\rho p_\alpha$$

- only vector current contributes
- two possible vectors: four-momentum p, polarisation vector ε_α (dimensionless) of ρ

Three independent polarisation vectors $\varepsilon^{(s)}$ corresponding to $J_z = 0, \pm 1$, satisfying $p \cdot \varepsilon = 0$

Polarisation vector has only spatial components in the rest frame of the ρ

$$\sum_s \varepsilon_\alpha^{(s)} \varepsilon_\beta^{(s)*} = -\eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_\rho^2}$$

Allow ε to be complex in order to describe circular polarisations

Semi-hadronic decay modes: $\tau^- \to \rho^- \nu_\tau$ (contd.)

Conservation of vector current (in isospin limit) $0 = p \cdot V = f_\rho m_\rho^2 \Rightarrow f_\rho = 0$ $\Longrightarrow H_\alpha = V_\alpha = g_\rho \varepsilon_\alpha$, constant g_ρ of mass-dimension 2

Decay amplitude,

$$\mathcal{M}_{\rm fi} = -\frac{G}{\sqrt{2}} \cos\theta_C g_\rho \varepsilon_\alpha \bar{u}_\nu(p_\nu)\gamma^\alpha(1-\gamma^5)u_\tau(p_\tau)$$

Spin-summed amplitude square (use $\gamma^\alpha \not{A} \gamma_\alpha = -2\not{A}$)

$$\begin{aligned}
\langle\langle|\mathcal{M}_{\rm fi}|^2\rangle\rangle &= \frac{G^2}{2}\cos^2\theta_C g_\rho^2 \left(-\eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_\rho^2}\right) \mathrm{tr}\,\gamma^\alpha(1-\gamma^5)(\not{p}_\tau + m_\tau)\gamma^\beta(1-\gamma^5)\not{p}_\nu \\
&= G^2\cos^2\theta_C g_\rho^2 \left(-\eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_\rho^2}\right) \mathrm{tr}\,\gamma^\alpha \not{p}_\tau \gamma^\beta \not{p}_\nu \\
&= G^2\cos^2\theta_C g_\rho^2 \left(-\mathrm{tr}\,\gamma^\alpha \not{p}_\tau \gamma_\alpha \not{p}_\nu + \frac{1}{m_\rho^2}\mathrm{tr}\,\not{p}\not{p}_\tau\not{p}\not{p}_\nu\right) \\
&= G^2\cos^2\theta_C g_\rho^2 \left(2\mathrm{tr}\,\not{p}_\tau\not{p}_\nu + \frac{1}{m_\rho^2}\mathrm{tr}\,\not{p}\not{p}_\tau\not{p}\not{p}_\nu\right) \\
&= 4G^2\cos^2\theta_C g_\rho^2 \left(2p_\tau\cdot p_\nu + \frac{1}{m_\rho^2}\left(2p\cdot p_\tau p\cdot p_\nu - m_\rho^2 p_\tau\cdot p_\nu\right)\right) \\
&= 4G^2\cos^2\theta_C \frac{g_\rho^2}{m_\rho^2}\left(2p\cdot p_\tau p\cdot p_\nu + p_\tau\cdot p_\nu m_\rho^2\right)
\end{aligned}$$

Semi-hadronic decay modes: $\tau^- \to \rho^- \nu_\tau$ (contd.)

Using $p \cdot p_\nu = (p_\tau - p_\nu) \cdot p_\nu = p_\tau \cdot p_\nu$, $0 = p_\nu^2 = m_\tau^2 + m_\rho^2 - 2p \cdot p_\tau$

$$\langle\langle |\mathcal{M}_{\rm fi}|^2 \rangle\rangle = 4G^2 \cos^2\theta_C \frac{g_\rho^2}{m_\rho^2} p_\tau \cdot p_\nu \left(2p \cdot p_\tau + m_\rho^2\right)$$
$$= 4G^2 \cos^2\theta_C \frac{g_\rho^2}{m_\rho^2} p_\tau \cdot p_\nu \left(2m_\rho^2 + m_\tau^2\right)$$

Decay width

$$\Gamma = \frac{1}{2}\frac{1}{2m_\tau}\Phi^{(2)} 4G^2 \cos^2\theta_C \frac{g_\rho^2}{m_\rho^2} p_\tau \cdot p_\nu m_\tau^2 \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right)$$

Phase space

$$\Phi^{(2)} = \frac{p_{\rm CM}}{4\pi E_{\rm CM}} = \frac{E_\nu}{4\pi m_\tau}$$

Using $m_\rho^2 = m_\tau^2 - 2p_\tau \cdot p_\nu = m_\tau^2 - 2m_\tau E_\nu$

$$\Gamma = \frac{1}{4m_\tau}\frac{E_\nu}{4\pi m_\tau} 4G^2 \cos^2\theta_C \frac{g_\rho^2}{m_\rho^2} m_\tau^3 E_\nu \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right)$$
$$= \frac{G^2\cos^2\theta_C}{4\pi}\left(\frac{g_\rho}{m_\rho}\right)^2 m_\tau E_\nu^2 \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right) = \frac{G^2\cos^2\theta_C}{16\pi}\left(\frac{g_\rho}{m_\rho}\right)^2 m_\tau^3 \left(1 - \frac{m_\rho^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_\rho^2}{m_\tau^2}\right)$$

Semi-hadronic decay modes: $\tau^- \to \rho^- \nu_\tau$ (contd.)

Unknown constant g_ρ can be related to EM decay $\rho^0 \to e^+ e^-$ using isospin invariance

$$\mathcal{M}_{\rm fi} = \frac{4\pi\alpha_{\rm em}}{q^2} \bar{u}_e \gamma^\mu v_e \langle 0 | J_{{\rm em}\,\mu} | \rho^0 \rangle$$

$J_{{\rm em}\,\mu}$: electromagnetic current, $q = p_{e^+} + p_{e^-}$: ρ momentum

Using conservation of the current

$$\langle 0 | J_{{\rm em}\,\mu} | \rho^0 \rangle = \frac{m_\rho^2}{\gamma} \varepsilon_\mu$$

ε_μ: polarisation vector, γ: dimensionless constant

Using explicit form of electromagnetic current

$$\begin{aligned}
\langle 0 | J_{{\rm em}\,\mu} | \rho^0 \rangle &= \langle 0 | \tfrac{2}{3} \bar{u}\gamma_\mu u - \tfrac{1}{3} \bar{d}\gamma_\mu d | \rho^0 \rangle \\
&= \langle 0 | \underbrace{\tfrac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)}_{I=1} + \underbrace{\tfrac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)}_{I=0} \underbrace{|\rho^0\rangle}_{I=1} \\
&= \frac{1}{\sqrt{2}} \langle 0 | \frac{1}{\sqrt{2}} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) | \rho^0 \rangle \underset{\substack{\text{isospin} \\ \text{invariance}}}{=} \frac{1}{\sqrt{2}} \langle 0 | \bar{u}\gamma_\mu d | \rho^- \rangle = \frac{1}{\sqrt{2}} g_\rho \varepsilon_\mu
\end{aligned}$$

$$\boxed{\Longrightarrow \frac{g_\rho}{m_\rho} = \sqrt{2} \frac{m_\rho}{\gamma}}$$

Semi-hadronic decay modes: $\tau^- \to \rho^- \nu_\tau$ (contd.)

Amplitude squared summed over spins

$$\langle\langle|\mathcal{M}_{\rm fi}|^2\rangle\rangle = \left(\frac{4\pi\alpha_{\rm em}}{\gamma}\right)^2 {\rm tr}\left[\gamma^\mu(\not{p}_{e^+} - m_e)\gamma^\nu(\not{p}_{e^-} + m_e)\right]\left(-\eta_{\mu\nu} + \frac{q_\mu q_\nu}{m_\rho^2}\right)^2$$

$$= 4\left(\frac{4\pi\alpha_{\rm em}}{\gamma}\right)^2\left(2p_{e^+}\cdot p_{e^-} + 4m_e^2 + \frac{1}{m_\rho^2}(2q\cdot p_{e^+} q\cdot p_{e^-} - q^2 p_{e^+}\cdot p_{e^-}) - \frac{m_e^2}{m_\rho^2}q^2\right)$$

$$= 4\left(\frac{4\pi\alpha_{\rm em}}{\gamma}\right)^2\left(p_{e^+}\cdot p_{e^-} + 3m_e^2 + \frac{2}{m_\rho^2}q\cdot p_{e^+} q\cdot p_{e^-}\right)$$

$m_e^2 \ll m_\rho^2 \Rightarrow$ treat leptons as massless $\Rightarrow m_\rho^2 = 2p_{e^+}\cdot p_{e^-}$, $m_\rho^2 = 2q\cdot p_{e^\pm}$

$$\langle\langle|\mathcal{M}_{\rm fi}|^2\rangle\rangle = 4\left(\frac{4\pi\alpha_{\rm em}}{\gamma}\right)^2\left(\frac{m_\rho^2}{2} + \frac{m_\rho^2}{2}\right) = 4m_\rho^2\left(\frac{4\pi\alpha_{\rm em}}{\gamma}\right)^2$$

$$\Gamma = \frac{1}{3}\frac{1}{2m_\rho}\Phi^{(2)} 4m_\rho^2\left(\frac{4\pi\alpha_{\rm em}}{\gamma}\right)^2 = \frac{4\pi}{3}\frac{\alpha_{\rm em}^2}{\gamma^2}m_\rho = \frac{4\pi}{3}\frac{\alpha_{\rm em}^2}{m_\rho}\left(\frac{m_\rho}{\gamma}\right)^2$$

$$\Phi^{(2)} = \frac{|\vec{p}_{e^\pm}|}{4\pi m_\rho} = \frac{E_{e^\pm}}{4\pi m_\rho} = \frac{\frac{m_\rho}{2}}{4\pi m_\rho} = \frac{1}{8\pi}$$

$$\left(\frac{m_\rho}{\gamma}\right)^2 = \frac{3}{4\pi}\frac{m_\rho}{\alpha_{\rm em}^2}\Gamma(\rho^0 \to e^+ e^-)$$

Comparison with experiment yields $\frac{\gamma^2}{4\pi} = \frac{\alpha_{\rm em}^2}{3}\frac{\Gamma(\rho^0\to e^+ e^-)}{m_\rho} = 2.1 \div 2.36$

Decay of charmed particles

Charmed particles: $C \neq 0$ (number of c minus number of $\bar{c}$)

Lightest charmed particles: D and D_s, pseudoscalar mesons

$$D^+ = c\bar{d} \qquad D^0 = c\bar{u}$$
$$\bar{D}^0 = u\bar{c} \qquad D^- = d\bar{c}$$
$$D_s^+ = c\bar{s} \qquad D_s^- = s\bar{c}$$

Older notation: $D_s^\pm = F^\pm$

(D^+, D^0), $(\bar{D}^0, D^-)$ isodoublets, $D_s^\pm$ isosinglets

$$m_{D^\pm} = 1.870\,\mathrm{GeV} \qquad m_{D^0,\bar{D}^0} = 1.865\,\mathrm{GeV} \qquad m_{D_s^\pm} = 1.968\,\mathrm{GeV}$$

Relevant product of currents for charmed particles decay, $|\Delta C| = 1$

Two-family approximation

$$(\cos\theta_C \bar{s}\mathcal{O}_L^\alpha c - \sin\theta_C \bar{d}\mathcal{O}_L^\alpha c)(\underbrace{\textstyle\sum_\ell \bar{\nu}_\ell \mathcal{O}_L^\alpha \ell}_{\text{semi-leptonic decays}} + \underbrace{\cos\theta_C \bar{u}\mathcal{O}_{L\alpha} d + \sin\theta_C \bar{u}\mathcal{O}_{L\alpha} s}_{\text{non-leptonic decays}}) + \mathrm{h.c.}$$

Decay of charmed particles: semi-leptonic decays

Relevant terms

$$\cos\theta_C \bar{s}\mathcal{O}_L^\alpha c\bar{\nu}_\ell\mathcal{O}_L^\alpha \ell \qquad \Rightarrow c \to s\,\nu_\ell\,\ell^+$$
$$-\sin\theta_C \bar{d}\mathcal{O}_L^\alpha c\bar{\nu}_\ell\mathcal{O}_L^\alpha \ell \qquad \Rightarrow c \to d\,\nu_\ell\,\ell^+$$

Decay widths

$$c \to s\,\nu_\ell\,\ell^+ \qquad \Gamma \propto \cos^2\theta_c$$
$$c \to d\,\nu_\ell\,\ell^+ \qquad \Gamma \propto \sin^2\theta_c$$

First type ($|\Delta S| = 1$, selection rule $\Delta C = \Delta S$) dominate over second type ($\Delta S = 0$, suppressed by $\sin^2\theta_C/\cos^2\theta_C \simeq 0.05$)

Examples:

$$\text{first type } (\Delta C = \Delta S): \qquad D^+ \to \ell^+\,\nu_\ell\,\bar{K}^0 \qquad D_s^+ \to \ell^+\,\nu_\ell$$
$$\text{second type } (\Delta S = 0): \qquad D^+ \to \ell^+\,\nu_\ell \qquad D_s^+ \to \ell^+\,\nu_\ell\,K^0$$

Decay of charmed particles: non-leptonic decays

Four types corresponding to products of currents

$$\cos^2\theta_C\,\bar{s}\mathcal{O}_L^\alpha c\bar{u}\mathcal{O}_{L\alpha}d \qquad \cos\theta_C\sin\theta_C\,\bar{s}\mathcal{O}_L^\alpha c\bar{u}\mathcal{O}_{L\alpha}s$$

$$-\sin\theta_C\cos\theta_C\,\bar{d}\mathcal{O}_L^\alpha c\bar{u}\mathcal{O}_{L\alpha}d \qquad -\sin^2\theta_C\,\bar{d}\mathcal{O}_L^\alpha c\bar{u}\mathcal{O}_{L\alpha}s$$

$c \to s\,u\,\bar{d}$	$\Gamma \propto \cos^4\theta_C$	$\Delta C = \Delta S$	dominant
$c \to s\,u\,\bar{s},\ d\,u\,\bar{d}$	$\Gamma \propto \sin^2\theta_C\cos^2\theta_C$	$\Delta C = -1,\ \Delta S = 0$	suppressed
$c \to d\,u\,\bar{s}$	$\Gamma \propto \sin^4\theta_C$	$\Delta C = -\Delta S$	doubly suppressed

General structure: $c \to s, d + (u\bar{d}), (u\bar{s})$

Trivial colour structure of charged weak current
$\Longrightarrow$ q and $\bar{q}$ in extra pair always have same colour

$$\Gamma(c \to s\,u\,\bar{d}) \simeq N_c\,\Gamma(c \to s\,\ell^+\nu_\ell) = 3\,\Gamma(c \to s\,\ell^+\nu_\ell)$$

$\cos^2\theta_C \simeq 1,\ \sin^2\theta_C \simeq 0$

Leptons, lighter quarks $\sim$ massless $(m_c \gg m_\mu \gg m_e,\ m_c \gg m_{u,d,s})$

Decay of charmed particles: leptonic decays

Creation of charmed particles optimal with ν_μ on s-rich targets (thin in the air...), second best d-rich targets (i.e., anything)

Creation via ν-beams followed by semi-leptonic decay $\Rightarrow$ dileptonic events

$$\begin{aligned} \nu_\mu\, s \to & c\, \mu^- \\ & \hookrightarrow s\, \ell^+\, \nu_\ell \end{aligned}$$

Leptonic decays (also into τ), width analogous to pion decays:

- same structure of relevant hadronic matrix elements
- only available vector: charmed particle four-momentum
- only axial current contributes (D, D_s pseudoscalars)

Unknown constants cancel in ratios, only mass dependence matters

$$\frac{\Gamma(D^+ \to \tau^+ \nu_\tau)}{\Gamma(D^+ \to \mu^+ \nu_\mu)} = \frac{m_\tau^2 \left(m_{D^+}^2 - m_\tau^2\right)^2}{m_\mu^2 \left(m_{D^+}^2 - m_\mu^2\right)^2} \simeq 2.5 \qquad \frac{\Gamma(D_s^+ \to \tau^+ \nu_\tau)}{\Gamma(D_s^+ \to \mu^+ \nu_\mu)} = \frac{m_\tau^2 \left(m_{D_s^+}^2 - m_\tau^2\right)^2}{m_\mu^2 \left(m_{D_s^+}^2 - m_\mu^2\right)^2} \simeq 17$$

D^+ decay Cabibbo-suppressed, factor $(\frac{m_\tau}{m_\mu})^2$ from chiral coupling makes it more likely for charmed mesons to decay into τs than lighter leptons

References

- R. L. Garwin, L. M. Lederman, M. Weinrich, Phys.Rev. 105 (1957) 1415
- S. Fubini, G. Furlan, Physics Physique Fizika 1 (1965) 229