Weak Interactions

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Non-leptonic decays of kaons: neutral kaons (contd.)

Using CP conservation one has predominantly $K_1^0 \to 2\pi$ and $K_2^0 \to 3\pi$

- first process violates parity, while the second one conserves it
- P-preserving and P-violating interactions have same strength at Lagrangian level
- difference between corresponding decay widths comes from the difference in phase space: less kinetic energy available for 3π, smaller available phase space in K_2^0 decays $\Rightarrow$ smaller width, longer lifetime

In the approximation of conserved CP, K_1^0 and K_2^0 correspond exactly to "short" and "long" kaons from diagonalisation of $H_{\rm eff}$ in kaon subspace

$\Gamma(K_1^0 \to 2\pi)$ about three orders of magnitude larger than $\Gamma(K_2^0 \to 3\pi)$

$$\tau_1 = 1/\Gamma_1 \simeq 10^{-10}\, s \qquad \tau_2 = 1/\Gamma_2 \simeq 5 \cdot 10^{-8}\, s$$

Differ slightly in mass: $\Delta m = m_2 - m_1 = 3.5 \cdot 10^{-12}\,\mathrm{MeV}$

Isospin wave functions in two-pion decays

Generic two-pion state

$$|\pi\,\pi\rangle = A^a B^b\,|\pi_a\,\pi_b\rangle \qquad a,b=1,2,3$$

$$\pi^0 = \pi_3 \quad \pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \pm i\pi_2) \qquad \vec{A},\vec{B}\in\mathbb{C}^3$$

Electric charge superselection rule forbids *physical* realisation of superpositions of states with different charge, but OK in the Hilbert space

$A^a B^b : \mathbf{1}\otimes\mathbf{1} = \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{2}$

$$A^a B^b = \underbrace{\tfrac{1}{3}\vec{A}\cdot\vec{B}\delta^{ab}}_{I=0} + \underbrace{\tfrac{1}{2}\varepsilon^{abc}(\vec{A}\wedge\vec{B})^c}_{I=1} + \underbrace{\tfrac{1}{2}\left(A^aB^b + A^bB^a - \tfrac{2}{3}\vec{A}\cdot\vec{B}\delta^{ab}\right)}_{I=2}$$

- Bose-Einstein symmetry $\Rightarrow$ symmetric pion total wave function, for $\ell=0$ state flavour w.f. must be symmetric $\Rightarrow$ no $I=1$ component
- $K_1^0 \to \pi^+\pi^-, \pi^0\pi^0$: $I_3=0$ in final state, both $I=0,2$ can be present
- $K^+ \to \pi^+\pi^0$: one has $I_3=1$ in final state, only $I=2$ is present

Isospin wave functions in two-pion decays (contd.)

If $\Delta I = \frac{1}{2}$ rule were exact, since Ks have $I = \frac{1}{2}$

- could only get $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$
- $K^+ \to \pi^+\pi^0$ would be forbidden
- only $I = 0$ component would be present in $K_1^0 \to \pi^+\pi^-, \pi^0\pi^0$

Amplitude $\mathcal{M} \propto \vec{A} \cdot \vec{B}$, width $\Gamma(K_1^0 \to 2\pi) \propto \frac{|\vec{A}\cdot\vec{B}|^2}{|\vec{A}|^2|\vec{B}|^2+|\vec{A}^*\cdot\vec{B}|^2}$

Normalisation of the state taken into account
Same proportionality factor in the isospin limit in the two cases

$$
\begin{aligned}
& K_1^0 \to \pi^+\pi^- : && \vec{A} = (1, i, 0) && \vec{B} = (1, -i, 0) \\
\Longrightarrow & \vec{A} \cdot \vec{B} = 2 && |\vec{A}|^2 = |\vec{B}|^2 = 2 && \vec{A}^* \cdot \vec{B} = 0 \\
& K_1^0 \to \pi^0\pi^0 : && \vec{A} = (0, 0, 1) && \vec{B} = (0, 0, 1) \\
\Longrightarrow & \vec{A} \cdot \vec{B} = 1 && |\vec{A}|^2 = |\vec{B}|^2 = 1 && \vec{A}^* \cdot \vec{B} = 1
\end{aligned}
$$

$$
\frac{\Gamma(K_1^0 \to \pi^+\pi^-)}{\Gamma(K_1^0 \to \pi^0\pi^0)} = \frac{4/4}{1/2} = 2 \qquad \text{(exp.: 2.2)}
$$

Isospin wave functions in two-pion decays (contd.)

$K^+ \to \pi^+\pi^0$ not forbidden, mediated by I_6 (contains $\Delta I = \frac{3}{2}$, allows $I = 2$)

Split full $\mathcal{L}_{\rm eff}$ in four pieces $O_{I\,I_3}$, $I = \frac{3}{2}, I_3 = \pm\frac{1}{2}$ (related by h.c./CP), $I = \frac{1}{2}, I_3 = \pm\frac{1}{2}$ (related by h.c./CP) $\Rightarrow \mathcal{L}_{\rm eff} = \sum_{I=\frac{1}{2},\frac{3}{2};I_3=\pm\frac{1}{2}} O_{I\,I_3}$

Decompose $O_{I\,I_3}|K^+, K^0\rangle = O_{I\,I_3}|\frac{1}{2}\ \pm\frac{1}{2}\rangle$ into total isospin eigenstates

$$O_{\frac{3}{2}\,\frac{1}{2}}|K^0\rangle = O_{\frac{3}{2}\,\frac{1}{2}}|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}\left(|2\,0\rangle_1 - |1\,0\rangle_1\right)$$

$$O_{\frac{1}{2}\,\frac{1}{2}}|K^0\rangle = O_{\frac{1}{2}\,\frac{1}{2}}|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}\left(|1\,0\rangle_1 - |0\,0\rangle_1\right)$$

$$O_{\frac{3}{2}\,\frac{1}{2}}|K^+\rangle = O_{\frac{3}{2}\,\frac{1}{2}}|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = \tfrac{\sqrt{3}}{2}\left(|2\,1\rangle_1 + |1\,1\rangle_1\right)$$

Decompose two-pion states:

$$|\pi^+\,\pi^-\rangle = \tfrac{1}{\sqrt{3}}|2\,0\rangle_2 + \sqrt{\tfrac{2}{3}}|0\,0\rangle_2$$

$$|\pi^0\,\pi^0\rangle = \sqrt{\tfrac{2}{3}}|2\,0\rangle_2 - \tfrac{1}{\sqrt{3}}|0\,0\rangle_2$$

$$|\pi^+\,\pi^0\rangle = |2\,1\rangle_2$$

Use Clebsch-Gordan coefficients

$|I\,I_3\rangle_{1,2}$ *different* states

Isospin wave functions in two-pion decays (contd.)

In K_1^0 decays using $I_3(2\pi) = 0,\ CP(2\pi) = 1$

$$\begin{aligned}\langle \pi\,\pi | O_{I\,\frac{1}{2}} + O_{I\,-\frac{1}{2}} | K_1^0 \rangle &= \frac{1}{\sqrt{2}} \left[\langle \pi\,\pi | O_{I\,\frac{1}{2}} | K^0 \rangle - \langle \pi\,\pi | O_{I\,-\frac{1}{2}} | \bar{K}^0 \rangle \right] \\ &= \frac{1}{\sqrt{2}} \left[\langle \pi\,\pi | O_{I\,\frac{1}{2}} | K^0 \rangle - \langle \pi\,\pi | (CP)^\dagger O_{I\,\frac{1}{2}} (CP) | \bar{K}^0 \rangle \right] \\ &= \sqrt{2} \langle \pi\,\pi | O_{I\,\frac{1}{2}} | K^0 \rangle\end{aligned}$$

Three relevant amplitudes in terms of two definite-isospin amplitudes

$$\begin{aligned}\langle \pi^+\,\pi^- | \mathcal{L}_{\rm eff} | K_1^0 \rangle &= \frac{1}{\sqrt{3}}\, {}_2\langle 2\,0 | 2\,0 \rangle_1 - \sqrt{\frac{2}{3}}\, {}_2\langle 0\,0 | 0\,0 \rangle_1 = \frac{1}{\sqrt{3}} \mathcal{A}_2 - \sqrt{\frac{2}{3}} \mathcal{A}_0 \\ \langle \pi^0\,\pi^0 | \mathcal{L}_{\rm eff} | K_1^0 \rangle &= \sqrt{\frac{2}{3}}\, {}_2\langle 2\,0 | 2\,0 \rangle_1 + \frac{1}{\sqrt{3}}\, {}_2\langle 0\,0 | 0\,0 \rangle_1 = \sqrt{\frac{2}{3}} \mathcal{A}_2 + \frac{1}{\sqrt{3}} \mathcal{A}_0 \\ \langle \pi^+\,\pi^0 | \mathcal{L}_{\rm eff} | K^+ \rangle &= \frac{\sqrt{3}}{2}\, {}_2\langle 2\,1 | 2\,1 \rangle_1 = \frac{\sqrt{3}}{2}\, {}_2\langle 2\,0 | 2\,0 \rangle_1 = \frac{\sqrt{3}}{2} \mathcal{A}_2\end{aligned}$$

Expect $|\mathcal{A}_2| \ll 1$ due to $\Delta I = \frac{3}{2}$ suppression

Isospin wave functions in two-pion decays (contd.)

Ratio of widths

$$\frac{\Gamma(K_1^0 \to \pi^+\pi^-)}{\Gamma(K_1^0 \to \pi^0\pi^0)} = \frac{p_\pm}{p_0} \frac{\left|\frac{1}{\sqrt{3}}\mathcal{A}_2 - \sqrt{\frac{2}{3}}\mathcal{A}_0\right|^2}{\left|\sqrt{\frac{2}{3}}\mathcal{A}_2 + \frac{1}{\sqrt{3}}\mathcal{A}_0\right|^2} = \frac{p_\pm}{p_0} \frac{2|\mathcal{A}_0|^2 + |\mathcal{A}_2|^2 - 2\sqrt{2}\mathrm{Re}\mathcal{A}_2^*\mathcal{A}_0}{|\mathcal{A}_0|^2 + 2|\mathcal{A}_2|^2 + 2\sqrt{2}\mathrm{Re}\mathcal{A}_2^*\mathcal{A}_0}$$

$$\simeq \left(2 - 3\sqrt{2}\,\frac{\mathrm{Re}\mathcal{A}_2^*\mathcal{A}_0}{|\mathcal{A}_0|^2}\right)\frac{p_\pm}{p_0}$$

$$\frac{\Gamma(K^+ \to \pi^+\pi^0)}{\Gamma(K_1^0 \to \pi^+\pi^-) + \Gamma(K_1^0 \to \pi^0\pi^0)} \simeq \frac{\frac{3}{4}|\mathcal{A}_2|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_2|^2} \simeq \frac{3}{4}\frac{|\mathcal{A}_2|^2}{|\mathcal{A}_0|^2}$$

$\frac{p_\pm}{p_0} \simeq 0.98$: ratio of rest-frame final-state momenta of charged/neutral pion pairs, equals phase-space ratio

Experiments show suppression factor $\simeq 700 \Rightarrow \frac{|\mathcal{A}_2|}{|\mathcal{A}_0|} \simeq 0.045 \div 0.046$

Neutral kaon oscillations

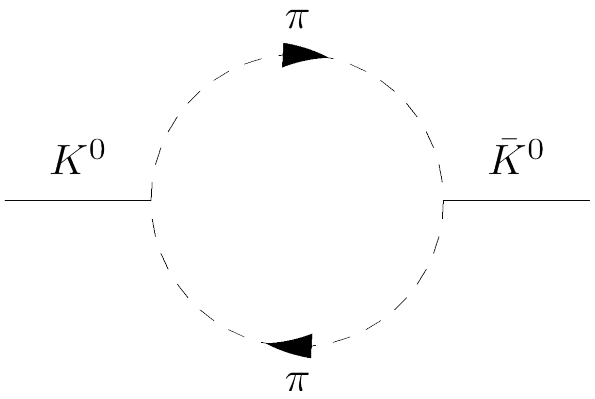
$K^0, \bar{K}^0$ produced by strong interactions as eigenstates of S

E.g., via $\pi^- p \to K^0 \Lambda$

Decays governed by weak interactions (almost) CP-conserving but not S-conserving, proceed through CP-even and CP-odd components

$K^0, \bar{K}^0 \to 2\pi, 3\pi$

K^0 and $\bar{K}^0$ can *oscillate* into each other: both can decay into, e.g., 2π, can also oscillate into each other through π loop



Projection $|K^0(t)\rangle$ on K subspace (projecting out decay states)

$$|K^0(t)\rangle = c_1(t)|K^0\rangle + c_2(t)|\bar{K}^0\rangle$$

Non-unitary temporal evolution governed by non-Hermitian H_{eff}

Kaons can decay, projecting full unitary evolution to K subspace makes it non-unitary

Neutral kaon oscillations (contd.)

Effective Schrödinger equation

$$i\frac{\partial}{\partial t}|K^0(t)\rangle = H_{\rm eff}|K^0(t)\rangle$$

Diagonalise $H_{\rm eff} \Rightarrow$ "short" and "long" neutral kaons $K^0_{S,L}$

$$H_{\rm eff}|K^0_{S,L}\rangle = \left(m_{S,L} - \tfrac{i}{2}\Gamma_{S,L}\right)|K^0_{S,L}\rangle \qquad \Gamma_S > \Gamma_L$$

Im(eigenvalue) governs exponential decay in time $\Rightarrow$ decay width

Exact CP limit $\Rightarrow [CP, H_{\rm eff}] = 0$, common eigenvectors
K^0_1 short lived (large decay rate because of 2π decays)

$$\Rightarrow K^0_1 = K^0_S,\ K^0_2 = K^0_L$$

$m_{S,L} = m_{1,2}$, $\Gamma_{S,L} = \Gamma_{1,2}$

Create K^0 at $t = 0$

$$|K^0(0)\rangle = |K^0\rangle = \frac{|K^0_1\rangle + |K^0_2\rangle}{\sqrt{2}}$$

State at time t

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\left(m_1 - i\frac{\Gamma_1}{2}\right)t}|K^0_1\rangle + e^{-i\left(m_2 - i\frac{\Gamma_2}{2}\right)t}|K^0_2\rangle\right)$$

Neutral kaon oscillations (contd.)

K^0 and $\bar{K}^0$ amplitudes at time t

$$\langle K^0|K^0(t)\rangle = \frac{1}{2}\left(e^{-i\left(m_1 - i\frac{\Gamma_1}{2}\right)t} + e^{-i\left(m_2 - i\frac{\Gamma_2}{2}\right)t}\right)$$

$$\langle \bar{K}^0|K^0(t)\rangle = \frac{1}{2}\left(e^{-i\left(m_1 - i\frac{\Gamma_1}{2}\right)t} - e^{-i\left(m_2 - i\frac{\Gamma_2}{2}\right)t}\right)$$

Number of K^0 and $\bar{K}^0$ observed in the neutral kaon beam at time t

$$N_{K^0,\bar{K}^0}(t) \propto \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} \pm 2\cos(m_2 - m_1)te^{-\frac{\Gamma_1+\Gamma_2}{2}t}\right)$$

How to measure $N_{K^0,\bar{K}^0}(t)$? Processes seeing S-content of K-beam

- measure flux of $e^\pm$ from semileptonic decays: $\Delta Q = \Delta S$ implies

$$K^0 \to e^+\,\nu_e\,\pi^- \qquad \bar{K}^0 \to e^-\,\bar{\nu}_e\,\pi^+$$

- hyperon production from scattering on ordinary matter: $\bar{K}^0$ can be absorbed ($\bar{K}^0\,p \to \Lambda\,\pi^+$), S conservation forbids hyperons from K^0

Non-leptonic decays into pions see CP content of K-beam (K_1^0 and K_2^0 amplitudes): different, non-compatible aspects of kaon state

References

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