Weak Interactions

Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

ELTE
09/09/2020

Matteo Giordano

(I am the one in the back)

How to find me:

- Elmeleti Fizika Tanszék – room 1.119
- email: `giordano@bodri.elte.hu`

Lecture notes and other info: `http://bodri.elte.hu/~giordano/`

Miscellaneous info

Lectures on Wednesday 10–12 and Thursday 14–15

Exam:

- solve one of the exercises listed on the webpage (written part)
- discuss one subject from the syllabus (oral part)
- sacrifice a chicken to Yog-Sothoth

Introduction

Weak interactions: one of the four fundamental interactions in Nature

- Responsible for
  - β-decays of nuclei and other hadronic decays (pions, kaons, hyperons)
  - decays of elementary particles (muons and taus)
  - reactions of astrophysical relevance involving neutrinos
  - parity-violating effects, including in atomic spectra
  - ...
- All elementary particles (quarks and leptons) interact weakly, in an essentially universal manner
- Least symmetric of interactions, violate P, C, CP, T, and most flavour symmetries
- Only symmetries fully respected: Poincaré, CPT, baryon and lepton number

  Lepton family number also a good symmetry if neutrinos were massless

  Tiny nonperturbative effects leave only $B - L$ as a symmetry

Introduction (contd.)

Modern perspective: weak interactions unified with electromagnetism in terms of a spontaneously broken gauge theory of the group SU(2)×U(1)

- EM part = exchange of massless, electrically neutral photons
- weak part = exchange of massive *intermediate vector bosons*
  - charged $W^{\pm}$ bosons ($m_W \simeq 80\,\mathrm{GeV}$), mediate *charged weak interactions*
  - neutral Z^0 boson ($m_Z \simeq 90\,\mathrm{GeV}$), mediates *neutral weak interactions*

Low energy approximation (sufficient for most phenomenology at low energy): four-fermion interaction

Brief history

1896, H. Becquerel: discovery of radioactivity

1899, E. Rutherford: classification of α, β and γ rays as radiation of increasing penetrating power

1900, H. Becquerel: measures mass and charge-to-mass ratios of the β rays, shows that they are electrons (discovered in 1897 by J. J. Thomson)

α rays= helium nuclei ${}^4_2\mathrm{He}$; γ rays= highly energetic photons

1913, N. Bohr: suggests β-rays originate from atomic nucleus (discovered by E. Rutherford in 1911)

1914, J. Chadwick: continuous energy spectrum of β-rays

1927, C. D. Ellis and W. A. Wooster: missing energy in ${}^{210}_{83}\mathrm{Bi} \to {}^{210}_{84}\mathrm{Po}$

energy release of the reaction in a calorimeter $\neq$ max. energy of β-rays, only $=$ average energy

1930, W. Pauli: neutrino hypothesis – new spin-$\frac{1}{2}$ particle emitted in β-decay, which goes undetected and carries away the missing energy

1932, J. Chadwick: discovery of neutron

1933-34, E. Fermi: first theory of β-decay based on reaction $n \to p\, e^- \bar{\nu}_e$

Fermi theory of β-decay

Inspiration: QED + Heisenberg's nucleon (p and $n \sim$ same particle)

In QED the interaction couples two vector: EM current and photon field

Replace the proton electric current with a neutron-proton current, and gauge field with neutrino-electron current

$$H^{\rm int}_{\rm Fermi} = G \int d^3x \left(\bar{p}(x)\gamma^\mu\, n(x)\right)\left(\bar{e}(x)\gamma_\mu\, \nu(x)\right) + \text{h.c.}$$

Fermi concluded neutrino massless or very light, and $G \simeq 0.3 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$ (Fermi constant, modern value: $G \simeq 1.1 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$)

Neutrinos worked phenomenologically, but detected only in 1956 (Reines and Cowan, studying the inverse reaction $\bar{\nu}_e + p \to n + e^+$)

Fermi theory of β-decay (contd.)

1936, Gamow: generalisation of Fermi's Hamiltonian
(most general four-fermion non derivative couplings)

$$H_\beta^{\rm int} = -\int d^3x\, \mathcal{L}_\beta^{\rm int}(x)$$

$$\mathcal{L}_\beta^{\rm int}(x) = -\sum_{j=1}^{5} g_j \big(\bar{p}(x) M_j\, n(x)\big)\big(\bar{e}(x) M^j\, \nu(x)\big)$$

$$+\, g_j' \big(\bar{p}(x) M_j\, n(x)\big)\big(\bar{e}(x) M_j \gamma^5\, \nu(x)\big) + \text{h.c.}$$

$M^j = \mathbf{1}\,, \gamma^5\,, \gamma^\mu\,, \gamma^\mu\gamma^5\,, \sigma^{\mu\nu}$ and g_j, g_j' (generally complex) couplings

Terms differing by a permutation of the fields can be reduced to these by Fierz transformations

T invariance $\Rightarrow g_j, g_j' \in \mathbb{R}$, P invariance $\Rightarrow g_j' = 0$

These seemed perfectly reasonable requirements at the time

Muon and universality of the weak interactions

1936, Anderson and Neddermayer: discovery of muon in cosmic rays (mistaken for Yukawa's meson)

1947, Powell, Occhialini and Lattes: discovery of pion in cosmic rays, was actually Yukawa's meson, and decays into muon

muon does not interact strongly, decayed weakly $\mu^- \to e^- \bar{\nu}_e \, \nu_\mu$

1962, Lederman *et al.*: two types of neutrino

1947, Pontecorvo: suggests that weak interactions couple muons and electrons to hadrons in the same way (μ-e universality)

1948, Puppi: approximate equality of couplings in muon decay and in β-decays

$\Rightarrow$ universality of weak interactions, i.e., they affect equally leptons and nuclei

Parity violations

θ-τ puzzle:

$$\theta^+ \to \pi^+ \pi^+ \pi^- \qquad \tau^+ \to \pi^+ \pi^0$$

- scalar particles
- decays suggest different parity $P_\theta = -1$ and $P_\tau = +1\ldots$
- ...but same mass and lifetime!

1956, T. D. Lee and C. N. Yang: they are the same particle, $\theta = \tau = K$, but weak interactions **do not conserve parity**
1957, Wu *et al.*; Garwin *et al.*: experimental confirmation of parity violations in weak processes

Parity violations seen but not recognised in Cox (1928) and Chase (1930)

$V - A$ structure of the interaction

1956-57: accepting P breaking led to understand $V - A$ structure

- P breaking $\Rightarrow g_j' \neq 0$ allowed (10 couplings)
- two-component neutrino hypothesis (Salam; Landau; Lee & Yang): neutrinos have definite helicity $\Rightarrow$ 5 couplings, specific definite-handedness part of the ν field in $\mathcal{L}$
- extended to all fields (Feynman & Gell-Mann; Sudarshan & Marshak):

$$\mathcal{L}_\beta^{\rm int} = -\frac{G_\beta}{\sqrt{2}}\left(\bar{p}(x)\gamma^\alpha(1 - \tfrac{g_V}{g_A}\gamma^5)\, n(x)\right)\left(\bar{e}(x)\gamma_\alpha(1-\gamma^5)\,\nu_e(x)\right) + \text{h.c.}$$
$$\mathcal{L}_\mu^{\rm int} = -\frac{G_\mu}{\sqrt{2}}\left(\bar{\mu}(x)\underbrace{\gamma^\alpha(1-\gamma^5)}_{V-A}\,\nu_\mu(x)\right)\left(\bar{e}(x)\gamma_\alpha(1-\gamma^5)\,\nu_e(x)\right) + \text{h.c.}$$

  G_β, G_μ: dimensions M^{-2}, $G_\beta/G_\mu \simeq 0.98$; g_V/g_A real dimensionless
- same coupling for μ (pointlike) and n (extended) $\sim$ electric charge (same for e^+ and p) $\Rightarrow$ conserved vector current (CVC) hypothesis (conservation of hadronic current: 1956, Gershtein; 1958, Feynman)

Hadronic currents and the quark model

1950s-1960s: hadron "zoo" – many new hadrons, often decaying weakly, w/ leptons in the final state (*semileptonic*) or w/out (*nonleptonic decay*) Should a new hadronic current be added to $\mathcal{L}$ for each new hadron?

1958, Feynman: only a few hadronic currents with the appropriate quantum numbers suffice; have to be assumed since no fundamental description available for hadrons

1964, M. Gell-Mann; G. Zweig: quark hypothesis – hadrons are bound states of quarks, and quark currents appear in the weak Lagrangian

Nuclear β-decay and charged-pion decay $\sim$ same decay process of the d quark, $d \to u\, e^- \,\bar{\nu}_e$, with quark current

$$\bar{u}\gamma^\alpha(1-\gamma^5)\, d$$

Hadronic currents and the quark model (contd.)

Incomplete since strangeness-changing processes not allowed

$K^+ \to \mu^+ \nu_\mu$ (semileptonic)
$K^+ \to \pi^+\pi^+\pi^-, K^+ \to \pi^+\pi^0$ (nonleptonic)

1963, Cabibbo: "rotate" the d quark

$$\bar{u}\gamma^\alpha(1-\gamma^5)\, d \longrightarrow \bar{u}\gamma^\alpha(1-\gamma^5)\, d' \qquad d' = \cos\theta_C d + \sin\theta_C s$$

θ_C: *Cabibbo angle*
$\cos\theta_C = G_\beta / G_\mu \simeq 0.98$ from β and muon decays
$\sin\theta_C \simeq 0.21$ from K^+ semileptonic decays

$\cos^2\theta_C + \sin^2\theta_C \simeq 1$, consistent

- explains strangeness-changing processes
- explains the difference between G_μ and G_β retaining universality of the charged current (with $d \to d'$)

Neutral currents and the charm quark

1973, Hasert *et al.* (Gargamelle experiment): observation of weak neutral currents in antineutrino-electron scattering $\bar{\nu}_e \, e^- \to \bar{\nu}_e \, e^-$ and elastic (anti)neutrino scattering on nuclei

Neutral currents expected in unified electroweak theory (see below)

1974, B. Richter/S. Ting: J/ψ resonance and *charm* quark c (*November revolution*)

Proposed by Glashow, Iliopoulos and Maiani (1970, *GIM mechanism*) to explain the suppression of certain weak processes

- support for unified electroweak theory
- support for Quantum Chromodynamics (QCD), microscopic theory of strong interactions emerged from quark model

$\boxed{\Longrightarrow \text{Standard Model of Particle Physics}}$

Electroweak theory

1935, Yukawa: weak interactions mediated by exchange of intermediate boson instead of four-fermion coupling

Yukawa's idea: same boson for weak and strong interactions

- EM interaction: massless photon, long range interaction
- weak interaction: very massive boson, very short range interaction

$$V_{\mathrm{Coulomb}}(\vec{r}\,) = \frac{e^2}{4\pi r} \qquad\qquad V_{\mathrm{Yukawa}}(\vec{r}\,) = \frac{g^2}{4\pi r}\, e^{-m_W r}$$

g coupling, m_W mass of intermediate boson

If $m_W \to 0$, $V_{\mathrm{Yukawa}} \to V_{\mathrm{Coulomb}}$; if $m_W \to \infty$

$$V_{\mathrm{Yukawa}}(r) \underset{m_W \to \infty}{\to} \frac{g^2}{m_W^2}\, \delta^{(3)}(\vec{r}\,)$$

Point-like interaction with coupling $G = \frac{g^2}{m_W^2}$

Electroweak theory (contd.)

Equivalently: exchange of a massive boson in relativistic QFT $\Rightarrow$ factor $\frac{g^2}{m_W^2 - p^2}$ in scattering amplitude $\to \frac{g^2}{m_W^2}$ if $\frac{p^2}{m_W^2} \ll 1$

Assuming equal strength $g^2 \simeq e^2$

$$m_W^2 = \frac{g^2}{G} \simeq \frac{e^2}{G} = \frac{4\pi\alpha}{G} \simeq (90\,\mathrm{GeV})^2$$

modern measurements: $m_W \simeq 80\,\mathrm{GeV}$

- Four-fermion theory badly behaved at high energy (cf. the mass dimension of G), partly cured by massive intermediate boson
- Further trick needed: boson masses generated via spontaneous symmetry breaking – Higgs mechanism
(1964, Higgs; Brout & Englert; Guralnik, Hagen & Kibble)
- Intermediate vector boson + Higgs mechanism $\Rightarrow$ unified electroweak theory (1961, Glashow; 1967, Weinberg; 1968, Salam)

$\Longrightarrow$ well-behaved theory, phenomenologically very successful

Overview of weak interactions at low energy

Low-energy limit of electroweak theory $\rightarrow$ W, Z-boson exchanges replaced by four-fermion local interaction

- excellent approximation in many cases of interests
- avoids technicalities of the full theory

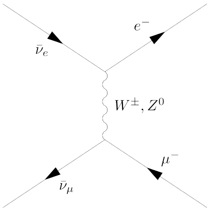
In the low-energy limit $\mathcal{L}_W^{\text{int}} = \mathcal{L}_{W,\,\text{charged}}^{\text{int}} + \mathcal{L}_{W,\,\text{neutral}}^{\text{int}}$

$$\mathcal{L}_{W,\,\text{charged}}^{\text{int}} = -\frac{G}{\sqrt{2}}\, J^{\alpha\dagger} J_\alpha \qquad \mathcal{L}_{W,\,\text{neutral}}^{\text{int}} = -\frac{G}{\sqrt{2}}\, J_0^\alpha J_{0\,\alpha}$$

Charged current $J^\alpha = J_l^\alpha + J_h^\alpha$
(leptonic + hadronic)

$$J_l^\alpha = \sum_{\ell=e,\mu,\tau} \bar{\ell}\, \mathcal{O}_L^\alpha\, \nu_\ell$$

$$J_h^\alpha = \bar{d}'\, \mathcal{O}_L^\alpha\, u + \bar{s}'\, \mathcal{O}_L^\alpha\, c + \bar{b}'\, \mathcal{O}_L^\alpha\, t$$



$(\bar{\nu}_\mu \mathcal{O}_L^\alpha \mu)(\bar{e}\mathcal{O}_{L\alpha}\nu_e)$

Overview of weak interactions at low energy

Low-energy limit of electroweak theory $\rightarrow$ W, Z-boson exchanges replaced by four-fermion local interaction

- excellent approximation in many cases of interests
- avoids technicalities of the full theory

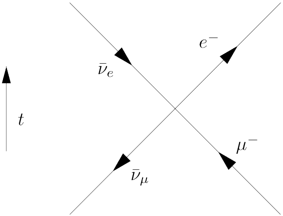
In the low-energy limit $\mathcal{L}_W^{\rm int} = \mathcal{L}_{W,\,\rm charged}^{\rm int} + \mathcal{L}_{W,\,\rm neutral}^{\rm int}$

$$\mathcal{L}_{W,\,\rm charged}^{\rm int} = -\frac{G}{\sqrt{2}}\, J^{\alpha\dagger} J_\alpha \qquad \mathcal{L}_{W,\,\rm neutral}^{\rm int} = -\frac{G}{\sqrt{2}}\, J_0^\alpha J_{0\,\alpha}$$

Charged current $J^\alpha = J_l^\alpha + J_h^\alpha$ (leptonic + hadronic)

$$J_l^\alpha = \sum_{\ell=e,\mu,\tau} \bar{\ell}\, \mathcal{O}_L^\alpha\, \nu_\ell$$

$$J_h^\alpha = \bar{d}'\, \mathcal{O}_L^\alpha\, u + \bar{s}'\, \mathcal{O}_L^\alpha\, c + \bar{b}'\, \mathcal{O}_L^\alpha\, t$$



$(\bar{\nu}_\mu \mathcal{O}_L^\alpha \mu)(\bar{e}\mathcal{O}_{L\alpha}\nu_e)$

Charged current

Charged current = sum of left-handed currents $\bar{\psi}_2 \mathcal{O}_L^\alpha \psi_1$

$V - A$ current, left *chirality* only $\boxed{\mathcal{O}_L^\alpha = \gamma^\alpha (1 - \gamma^5)}$

$\gamma^{\mu,5}$: Dirac matrices, $\bar{\psi} = \psi^\dagger \gamma^0$: Dirac adjoint

Lorentz/colour (for quarks) indices are suppressed; $\mathcal{O}_L^\alpha$ is trivial in colour space

"Rotated" negative-charge quark fields

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \underset{\text{2-gen. approx.}}{\to} \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$V_{\rm CKM}$: unitary *Cabibbo-Kobayashi-Maskawa* matrix

d', s', b' (= eigenstates of weak-interaction flavour) are linear superpositions of mass eigenstates of quarks d, s, b (= eigenstates of strong-interaction flavour) $\to$ not definite-mass fields

Neutral current

Neutral current: both left- and right-handed currents

$$J_0^\alpha = \sum_f g_f^L \bar{f}\, \mathcal{O}_L^\alpha\, f + g_f^R \bar{f}\, \mathcal{O}_R^\alpha\, f \qquad \mathcal{O}_{L,R}^\alpha = \gamma^\alpha (1 \mp \gamma^5)$$

$$g_f^L = \begin{cases} \frac{1}{2}\,, & f = \nu_{e,\mu,\tau} \\ -\frac{1}{2} + \xi\,, & f = e, \mu, \tau \\ \frac{1}{2} - \frac{2}{3}\xi\,, & f = u, c, t \\ -\frac{1}{2} + \frac{1}{3}\xi\,, & f = d, s, b \end{cases} \qquad g_f^R = \begin{cases} 0\,, & f = \nu_{e,\mu,\tau} \\ \xi\,, & f = e, \mu, \tau \\ -\frac{2}{3}\xi\,, & f = u, c, t \\ \frac{1}{3}\xi\,, & f = d, s, b \end{cases}$$

You don't have to remember them by heart – for now...

$\xi = \sin^2\theta_W$, θ_W weak or Weinberg angle

...introduced by Glashow: cf. Arnol'd principle

No flavour-changing neutral currents

Flavour-changing currents also change electric charge

Free fermion field

Free field operator for spin-$\frac{1}{2}$ fermion of mass m $\quad (p^0 = \sqrt{\vec{p}^{\,2} + m^2})$

$$\psi(x) = \int d\Omega_p \sum_{s=\pm\frac{1}{2}} \left\{ b_s(\vec{p}\,) u_s(\vec{p}\,) e^{-ip\cdot x} + d_s(\vec{p}\,)^\dagger v_s(\vec{p}\,) e^{ip\cdot x} \right\}$$

$b_s(\vec{p}\,)$, $d_s(\vec{p}\,)$: fermion/antifermion annihilation operators
$b_s(\vec{p}\,)^\dagger$, $d_s(\vec{p}\,)^\dagger$: fermion/antifermion creation operators

$$\{b_s(\vec{p}\,), b_{s'}(\vec{p}\,'\,)^\dagger\} = \delta_{ss'}(2\pi)^3 2p^0 \delta^{(3)}(\vec{p} - \vec{p}\,') \qquad \{b_s(\vec{p}\,), b_{s'}(\vec{p}\,'\,)\} = 0$$

$$\ldots \qquad\qquad \ldots$$

$u_s(\vec{p}\,)$, $v_s(\vec{p}\,)$ (bispinors): positive/negative-energy sol.s of Dirac equation

$$(\not{p} - m) u_s(\vec{p}\,) = 0 \qquad (\not{p} + m) v_s(\vec{p}\,) = 0$$

$$\bar{u}_{s'}(\vec{p}\,) u_s(\vec{p}\,) = 2m\delta_{s's} \qquad \bar{v}_{s'}(\vec{p}\,) v_s(\vec{p}\,) = -2m\delta_{s's}$$

$\bar{u} = u^\dagger \gamma^0$, $\not{A} = A_\mu \gamma^\mu$

Phase-space measure $d\Omega_p = \dfrac{d^3 p}{(2\pi)^3 2p^0}$

Dirac matrices

Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$

$\{A, B\} = AB + BA$, Minkowski metric tensor $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$

$$\gamma^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix} \; i = 1, 2, 3 \qquad \sigma^i\text{: Pauli matrices}$$

$$\gamma^5 = -\frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = -i\varepsilon_{0123}\gamma^0\gamma^1\gamma^2\gamma^3 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$$

$\varepsilon_{\mu\nu\rho\sigma}$ is the totally-antisymmetric tensor with $\varepsilon_{0123} = -1$

Anticommutation property: $\{\gamma^5, \gamma^\mu\} = 0$

Generators of $s = \frac{1}{2}$ irrep of Lorentz transformations: $\sigma^{\mu\nu} = \frac{1}{2i}[\gamma^\mu, \gamma^\nu]$

$$U(\Lambda)^\dagger\psi(x)U(\Lambda) = S(\Lambda)\psi(\Lambda^{-1}x) \qquad S(e^{\frac{i}{2}\omega_{\mu\nu}J^{(\mu\nu)}}) = e^{\frac{i}{4}\omega_{\mu\nu}\sigma^{\mu\nu}}$$

$\{\mathbf{1}, \gamma^\mu, \sigma^{\mu\nu}, i\gamma^5\gamma^\mu, \gamma^5\}$: basis of the vector space of complex 4×4 matrices

Chirality

Weak interactions are *chiral*: different chirality are treated differently

For any bispinor $\psi = \psi_+ + \psi_-$, $\psi_\pm$: chiral components

$\psi_\pm$ eigenvectors of γ^5, $\gamma^5\psi_\xi = \xi\psi_\xi$, with *chirality* ξ

Use chiral projectors $P_\pm$, $\gamma^5 P_\pm = \pm P_\pm$, so $\psi_\pm = P_\pm\psi$:

$$P_\pm = \frac{1 \pm \gamma^5}{2}\,, \quad P_\pm = P_\pm^\dagger = P_\pm^2\,, \quad P_+P_- = 0\,, \quad P_+ + P_- = \mathbf{1}$$

Notice $\gamma^\alpha P_\pm = P_\mp\gamma^\alpha$

- $\mathcal{O}_L^\alpha$ contains only P_-, charged current only involves fields with negative chirality, $f_- = P_- f$ (notice $\overline{f_-} = \bar{f} P_+$)
- neutral current has different couplings for the terms involving f_- and $f_+ = P_+ f$

Chirality conflated with *helicity*= spin projection in direction of motion

- coincide only for massless fermions
- nonetheless customary to denote L, R the negative/positive chirality components (left/right refers to "handedness", i.e., helicity)

Neutrinos

For massless fermions chirality and helicity coincide,
Neutrinos will be massless almost until the end of the course $\Rightarrow$ drop one of the two helicity components (appears nowhere in $\mathcal{L}_W^{\mathrm{int}}$)
$\{\partial\!\!\!/, \gamma^5\} = 0 \Rightarrow$ can choose definite-chirality solutions of massless Dirac eq.

$$i\partial\!\!\!/\psi_\pm = 0 \qquad \gamma^5\psi_\pm = \pm\psi_\pm$$

Positive-energy solutions $\psi = ue^{-ip\cdot x} \Rightarrow p\!\!\!/u = 0$

$$u(\vec{p}\,) = \sqrt{|\vec{p}\,|}\begin{pmatrix} \xi \\ \hat{p}\cdot\vec{\sigma}\xi \end{pmatrix} \qquad \xi^\dagger\xi = 1 \qquad \bar{u}\gamma^0 u = 2p^0 = 2|\vec{p}\,|$$

Definite helicity solutions $\hat{p}\cdot\vec{\sigma}\xi_{R,L} = \pm\xi_{R,L}$

$$u_{R,L}(\vec{p}\,) = \sqrt{|\vec{p}\,|}\begin{pmatrix} \xi_{R,L} \\ \pm\xi_{R,L} \end{pmatrix} \qquad \gamma^5 u_{R,L}(\vec{p}\,) = \pm u_{R,L}(\vec{p}\,)$$

Particle w/ positive helicity = *right-handed*, and *positive chirality*
Particle w/ negative helicity = *left-handed*, and *negative chirality*

Neutrinos (contd.)

Negative-energy solutions $\psi = v e^{ip\cdot x} \Rightarrow \not{p} v = 0$

$$v(\vec{p}\,) = \sqrt{|\vec{p}\,|}\begin{pmatrix} \tilde{\xi} \\ \hat{p}\cdot\vec{\sigma}\tilde{\xi} \end{pmatrix} \qquad \tilde{\xi}^\dagger\tilde{\xi} = 1 \qquad \bar{v}\gamma^0 v = 2p^0 = 2|\vec{p}\,|$$

Lorentz transformation properties of fermion field impose that if $u(\xi)$=particle, spin s along $\vec{n}$, $v(\tilde{\xi})$=antiparticle, same spin s along same $\vec{n}$ then we must choose $\tilde{\xi} = -i\sigma_2\xi^* \Rightarrow$ for definite helicity $\tilde{\xi}_{R,L} = -i\sigma_2\xi^*_{R,L}$

$$\hat{p}\cdot\vec{\sigma}\tilde{\xi}_{R,L} = \hat{p}\cdot\vec{\sigma}(-i\sigma_2)\xi^*_{R,L} = i\sigma_2(\hat{p}\cdot\vec{\sigma}\xi_{R,L})^* = \mp(-i\sigma_2)\xi^*_{R,L} = \mp\tilde{\xi}_{R,L}$$

$$v_{R,L}(\vec{p}\,) = \sqrt{|\vec{p}\,|}\begin{pmatrix} \tilde{\xi}_{R,L} \\ \mp\tilde{\xi}_{R,L} \end{pmatrix} \qquad \gamma^5 v_{R,L}(\vec{p}\,) = \mp v_{R,L}(\vec{p}\,)$$

Antiparticle w/ positive helicity = *right-handed*, and *negative chirality*
Antiparticle w/ negative helicity = *left-handed*, and *positive chirality*

Helicity Lorentz-invariant only for $m = 0$ (for $m \neq 0$ can always find ref. frame where particle flips $\vec{p}$ and so h), better quantum number as energy increases (particle closer to behaving as massless)

References