Weak interactions - homework

31/05/2023

1 Semileptonic hyperon decays

Consider the semileptonic hyperon decays

$$\Sigma^+ \to \Lambda\, e^+\, \nu_e\,, \qquad \Sigma^- \to \Lambda\, e^-\, \bar{\nu}_e\,. \tag{$*$}$$

Assuming SU(3) invariance and working in the static approximation, the matrix elements of the hadronic currents are of the general form

$$\mathcal{M}_b(c \to a) = \langle a | j_b^\mu(0) | c \rangle = F^\mu f_{abc} + D^\mu d_{abc}\,, \tag{$**$}$$

where

$$\begin{aligned} F^\mu &= \bar{u}\gamma^\mu(1 - F_A\gamma^5)u\,, \\ D^\mu &= -D_A\bar{u}\gamma^\mu\gamma^5 u\,, \end{aligned} \tag{$***$}$$

with u the usual Dirac bispinor for a particle of mass equal to the common octet mass.

1. After identifying the relevant components of $j_b^\mu(0)$ and the relevant combinations of basis vectors $|a\rangle$ and $|c\rangle$ corresponding to the initial and final hadrons, show that for the processes of interest the F^μ term does not contribute. Using the explicit expressions of the SU(3) generators in terms of Gell-Mann matrices and the definition of d_{abc}, compute the coefficient of the D^μ term.

2. Write down the decay amplitudes for the processes in $(*)$. Compute the differential decay width for unpolarised initial particles and unobserved spin of the final particles. Show that the recoil of the Λ is negligible, and determine explicitly the three-body phase space element in the static approximation. Find the angular correlation between the charged lepton and the (anti)neutrino. Show that replacing $u \to u_\Sigma$ and $\bar{u} \to \bar{u}_\Lambda$ in $(**)$ and $m \to m_\Lambda$ in the phase space element and in the prefactor of $|\mathcal{M}_b(c \to a)|^2$ appearing in the decay width formula does not lead to any correction.

3. Compute the total decay width for unpolarised initial particles and unobserved spin of the final particles. Show that the electron mass can be neglected, and determine all the numerical factors exactly.

4. Using the experimental values for the lifetimes of $\Sigma^\pm$ and the branching ratios for the processes $(*)$ (as well as those for the various coupling constants appearing in the decay widths), provide estimates of D_A.

2 Charged pion decay

Consider the leptonic decay

$$\pi^- \to \mu^- \, \bar{\nu}_\mu \, .$$

Compute the decay amplitude and the differential decay width $\Gamma(\vec{p}, \vec{\eta})$ for final muons with momentum $\vec{p}$ and spin $+\frac{1}{2}$ in direction $\vec{\eta}$, and the polarisation

$$P(\vec{p}, \vec{\eta}) = \frac{\Gamma(\vec{p}, \vec{\eta}) - \Gamma(\vec{p}, -\vec{\eta})}{\Gamma(\vec{p}, \vec{\eta}) + \Gamma(\vec{p}, -\vec{\eta})} \, .$$

Explain the result in terms of symmetries and of the chiral nature of weak interactions.