Particle physics

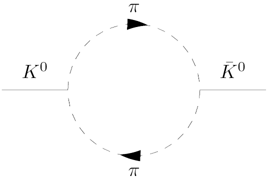
Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

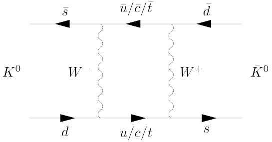
ELTE
08/09/2020

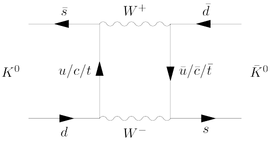
K^0–$\bar{K}^0$ system

K^0, $\bar{K}^0$: same quantum numbers except strangeness, S is not a conserved quantity $\Rightarrow$ K^0, $\bar{K}^0$ can mix



$K^0, \bar{K}^0 \to 2\pi \Rightarrow$ oscillation through virtual pion loop, $K^0 \to 2\pi \to \bar{K}^0$





In modern terms: second-order process via W-exchange

K^0–$\bar{K}^0$ system (contd.)

K^0, $\bar{K}^0$: flavour eigenstates of the *strong interactions*, produced in the lab by strong processes, but decays governed by *weak interactions*

Oscillations in K^0–$\bar{K}^0$ system $\Leftrightarrow \langle \bar{K}^0 | H_{\rm weak} | K^0 \rangle \neq 0$

If kaons could mix but were stable, unitary temporal evolution of kaon state $|\psi(t)\rangle$ limited to kaon subspace $\Rightarrow$ unitary within this subspace

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = c_1(t)|K^0\rangle + c_2(t)|\bar{K}^0\rangle$$

Kaons do decay, temporal evolution leads state outside of kaon subspace

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = c_1(t)|K^0\rangle + c_2(t)|\bar{K}^0\rangle + |R(t)\rangle$$

$|R(t)\rangle$ orthogonal to the kaon states

Projection of state at time t back on kaon subspace compared to state at time 0 $\Rightarrow$ evolution not unitary

K^0–$\bar{K}^0$ system (contd.)

$$|\psi_K(t)\rangle \equiv \Pi_K|\psi(t)\rangle = \Pi_K U(t)|\psi(0)\rangle = \Pi_K U(t)\Pi_K|\psi(0)\rangle$$
$$= c_1(t)|K^0\rangle + c_2(t)|\bar{K}^0\rangle$$

Π_K: projector on kaon subspace

$\Pi_K U(t)\Pi_K = \Pi_K e^{-iHt}\Pi_K$ not unitary, $H = H_{\mathrm{strong}} + H_{\mathrm{weak}}$

Weisskopf-Wigner approximation:

$$\Pi_K e^{-iHt}\Pi_K \simeq e^{-iH_{\mathrm{eff}}t}$$

$H_{\mathrm{eff}} \neq H_{\mathrm{eff}}^\dagger$: effective non-Hermitian Hamiltonian

non-Hermiticity reflects the fact that kaons can decay

Temporal evolution:

$$i\frac{\partial}{\partial t}|\psi_K(t)\rangle = H_{\mathrm{eff}}|\psi_K(t)\rangle$$

Solved by finding eigenvalues/eigenvectors of H_{eff}:

- eigenvalues are generally complex
- eigenvectors are generally non-orthogonal

K^0–$\bar{K}^0$ system (contd.)

$$H_{\rm eff}|K_{S,L}\rangle = \lambda_{S,L}|K_{S,L}\rangle = (m_{S,L} - i\tfrac{\Gamma_{S,L}}{2})|K_{S,L}\rangle$$

$\langle K_S|K_S\rangle = \langle K_L|K_L\rangle = 1$, but $\langle K_S|K_L\rangle \neq 0$ in general
$m_{S,L}, \Gamma_{S,L} \in \mathbb{R}$, $\Gamma_S \geq \Gamma_L$ by convention

Let $|\psi(0)\rangle = |\psi_K(0)\rangle = c_S|K_S\rangle + c_L|K_L\rangle$

$$|\psi_K(t)\rangle = c_S e^{-i\left(m_S - i\frac{\Gamma_S}{2}\right)t}|K_S\rangle + c_L e^{-i\left(m_L - i\frac{\Gamma_L}{2}\right)t}|K_L\rangle$$

$m_{S,L}$ = mass, $\Gamma_{S,L}$ = decay width

$$|\langle\psi_K(t)|\psi_K(t)\rangle|^2 = |c_S|^2 e^{-\Gamma_S t} + |c_L|^2 e^{-\Gamma_L t} + 2{\rm Re}\left\{c_S^* c_L e^{i(m_S - m_L)t} e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\langle K_S|K_L\rangle\right\}$$

- $|K_S\rangle$: "K-short", $|K_L\rangle$: "K-long": definite masses $m_{S,L}$ and decay times $\Gamma_{S,L}^{-1}$ ($|K_S\rangle$ shorter-lived than $|K_L\rangle$) $\Rightarrow$ definite decay properties
- K_S, $K_L \neq K^0$, $\bar{K}^0$ $\Rightarrow$ a K^0 beam develops a $\bar{K}^0$ component in time

K^0–$\bar{K}^0$ system and CP symmetry

If CP is a symmetry of weak interactions $\Rightarrow [CP, H] = [CP, \Pi_K] = 0$

CP is a symmetry of strong interactions, kaon subspace is a CP eigenspace $\Rightarrow [CP, \Pi_K] = 0$

$$0 = [CP, \Pi_K e^{-iHt} \Pi_K] = [CP, e^{-iH_{\rm eff} t}] \Rightarrow [CP, H_{\rm eff}] = 0$$

Common basis exists, $H_{\rm eff}$ non degenerate

$$\Rightarrow CP|K_{S,L}\rangle = \eta_{S,L}|K_{S,L}\rangle \qquad |\eta_{S,L}| = 1$$

K^0, $\bar{K}^0$ pseudoscalars, choosing phases $C|K^0\rangle = |\bar{K}^0\rangle \Rightarrow CP$ eigenstates

QFT requires $C^2 = 1$

$$|K_1\rangle = \tfrac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad |K_2\rangle = \tfrac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$CP|K_1\rangle = |K_1\rangle \qquad CP|K_2\rangle = -|K_2\rangle$$

$K_{S,L}$ can be identified with $K_{1,2} \Rightarrow \eta_{S,L} = \pm 1$, $\langle K_S|K_L\rangle = 0$

K^0–$\bar{K}^0$ system and CP symmetry (contd.)

Before 1956: $K^0 \to 2\pi$ in $\tau_S \simeq 0.89 \cdot 10^{-10}s$

- final state has $P = 1$: since $\ell = 0 \Rightarrow (-1)^\ell \eta_\pi^2 = 1$

Notice $P_i \neq P_f$

- if $2\pi = \pi^0\pi^0$: automatically $C = 1$ state ($\xi_{\pi^0} = 1$) $\Rightarrow$ $CP = 1$
- if $2\pi = \pi^+\pi^-$: $C|\pi^+\pi^-\rangle = |\pi^-\pi^+\rangle = (-1)^\ell|\pi^+\pi^-\rangle \Rightarrow CP = 1$
- $CP = -1$ state cannot decay into 2π, but can decay into 3π
  ℓ, L: relative angular momentum of first pair, and of third particle
  wrt CM of the pair; $J = 0 \Rightarrow \ell = L$
  Under P: 3π state gets phase $\eta_P = (-1)^{3+\ell+L} = -1$;
  under C: $3\pi^0$ gets $\eta_C = 1^3 = 1$; $\pi^+\pi^-\pi^0$ gets $(-1)^\ell \cdot 1$
  $CP = (-1)^{\ell+1}$ for $\pi^+\pi^-\pi^0$, $CP = -1$ for $\pi^0\pi^0\pi^0$

Identification:

- $CP = 1$ state K_1 identified with K_S
- $CP = -1$ state K_2 identified with K_L

1956: Lederman and collaborators observed $K^0 \to 3\pi$ further down the beam = "long" component of K^0, $\tau_L = 5.2 \cdot 10^{-8}s$

K^0–$\bar{K}^0$ system and CP symmetry (contd.)

Summary (so far):

- neutral K produced via strong interactions as strangeness eigenstates, K^0, $\bar{K}^0$
- in terms of CP eigenstates, $K_{1,2}$

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle + |K_2\rangle\right) \qquad |\bar{K}^0\rangle = -\frac{1}{\sqrt{2}}\left(|K_1\rangle - |K_2\rangle\right)$$

- K decay process governed by weak interactions, S not conserved
- if CP conserved K_1, K_2 are states with definite lifetime, and a beam of kaons will see
  - $K_1 = K_S$ component ($CP = 1$) decay first mostly into 2π

    $K_1 \to \pi^+\pi^-\pi^0$ is possible if final angular momentum chosen properly
  - $K_2 = K_L$ component ($CP = -1$) decay later mostly into 3π, *never* into 2π
- in passing: tiny mass difference $m_L - m_S \simeq 3.5 \cdot 10^{-6}\ \mathrm{eV}$

K^0–$\bar{K}^0$ system and CP symmetry (contd.)

But what if CP is not a symmetry of weak interactions?

- no reason for the physical state K_L to be a pure $CP = -1$ state

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left(|K_2\rangle + \varepsilon|K_1\rangle\right)$$

- down the beam where K_S has already decayed, some CP-forbidden decays in 2π should be observed
- 1964: Cronin, Fitch and collaborators, using a very long kaon beam, observed 2π decays of $K_L \Rightarrow CP$ violated by weak interactions
- rather small violation: exp. $|\varepsilon| = 2.2 \cdot 10^{-3}$
- *indirect* CP violation, due to physical states not being CP eigenstates
- possible to have *direct* CP violation: CP is not anymore a symmetry $\Rightarrow$ $CP = -1$ eigenstate (e.g., K_2) can decay in 2π (observed expt.lly)
- for explicit CP breaking in the SM least 3×3 CKM matrix:
  - need at least three families of quarks
  - CP-violating phase remains in CKM matrix after all irrelevant phases are reabsorbed in a redefinition of fermion states
  - suggested by Kobayashi and Maskawa before 2nd family complete

Scattering theory

Scattering experiment:

- throw bunch of particles against fixed target/other bunch of particles
- study what comes out of the collision

Empirical fact #1: particles far enough from each other do not interact appreciably, travelling essentially undisturbed on straight-line trajectories

A posteriori explanation: interactions are typically short-ranged (even long-ranged EM interactions are effectively short-ranged in most cases due to screening effects)

Experimenters can prepare states of spatially separated, non-interacting, freely evolving particles, used as *initial states* of scattering experiments

Empirical fact #2: after a sufficiently long time has elapsed after the collision, the state of the system looks again like a state of freely-evolving, spatially well-separated particles

"Sufficiently long" depends on the type of interaction, but anyway a very short time on human scales: upper bound estimated as $10^{-10}s$

Experimenters, by making measurements, can characterise these states – *final states* of scattering experiments

Scattering theory (contd.)

For all practical purposes we can treat the system as follows:

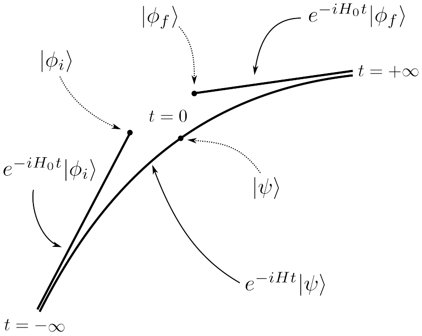
- system prepared in initial state in distant past (formally: $t = -\infty$), with particles far away (formally: infinitely far away) from each other
- observations on the system made in distant future (formally: $t = +\infty$), with particles again far away (formally: infinitely far away) from each other, after interaction is over
- measuring energy, momentum, electric charge, etc., of the final particles projects the state of the system on a particle state with definite particle content and particle momenta

In summary: in the far past and far future the state of the system looks like a freely-evolving particle state

Free particles = localised objects travelling on straight lines

Formal theory of scattering

Mathematically: as $t \to -\infty$, $t \to +\infty$, exact temporal evolution $e^{-iHt}|\psi\rangle$ of the system state (H: full Hamiltonian) practically indistinguishable from freely evolving states $e^{-iH_0 t}|\phi_i\rangle$, $e^{-iH_0 t}|\phi_f\rangle$ (H_0: free Hamiltonian)



Formally: given $|\psi\rangle$, $\exists|\phi_{i,f}\rangle$

$$\lim_{t\to-\infty} \left\| e^{-iHt}|\psi\rangle - e^{-iH_0 t}|\phi_i\rangle \right\| = 0 \qquad \lim_{t\to+\infty} \left\| e^{-iHt}|\psi\rangle - e^{-iH_0 t}|\phi_f\rangle \right\| = 0$$

Formal theory of scattering (contd.)

Turn the argument around:

if system prepared in the distant past looking like $e^{-iH_0 t}|\phi_i\rangle$, then state vector describing exact temporal evolution of the system with the full Hamiltonian is

$$|\psi_+\rangle = \lim_{t\to-\infty} e^{iHt} e^{-iH_0 t}|\phi_i\rangle$$

$|\phi_i\rangle$ encodes particle type, momenta, spin,. . . , of the experimental setup

if the state observed in the distant future looks like $e^{-iH_0 t}|\phi_f\rangle$, then state vector describing exact temporal evolution is

$$|\psi_-\rangle = \lim_{t\to+\infty} e^{iHt} e^{-iH_0 t}|\phi_f\rangle$$

$|\phi_f\rangle$ encodes particle type, momenta, spin,. . . , seen by the detectors

States $|\psi_+\rangle$ and $|\psi_-\rangle$ are the *in* and *out* states corresponding to the *asymptotic states* $|\phi_i\rangle$ (initial) and $|\phi_f\rangle$ (final)

Formal theory of scattering (contd.)

$|\psi_\pm\rangle = \lim_{t\to\mp\infty} e^{iHt}e^{-iH_0t}|\phi_{i,f}\rangle$ define *scattering* (or *Møller*) operators

$$\Omega_\pm \equiv \lim_{t\to\mp\infty} e^{iHt}e^{-iH_0t}$$

- $\Omega_\pm$ limit of unitary operators, conserve the norm $\|\Omega_\pm|\phi\rangle\| = \||\phi\rangle\|$
  Initial/final state arbitrary (prepare/detect what we want)
  $\Rightarrow |\phi_{i,f}\rangle$ range over a complete set of states
  $\Rightarrow \Omega_\pm^\dagger\Omega_\pm = \mathbf{1}$
- Assume all states of the system accessible in a scattering experiment
  $\Leftrightarrow$ all states look like freely-evolving states as $t\to\mp\infty$

  Not always true, e.g., bound states in non-relativistic QM

  $\Rightarrow \forall|\psi\rangle\,\exists|\phi_\pm\rangle$ s.t. $|\psi\rangle = \Omega_\pm|\phi_\pm\rangle \Rightarrow \exists\lim_{t\to\mp\infty} e^{iH_0t}e^{-iHt} = \Omega_\pm^\dagger$

  $$|\phi_\pm\rangle = \Omega_\pm^\dagger\Omega_\pm|\phi_\pm\rangle = \Omega_\pm^\dagger|\psi\rangle \Rightarrow \Omega_\pm\Omega_\pm^\dagger|\psi\rangle = \Omega_\pm|\phi_\pm\rangle = |\psi\rangle$$

  $\Rightarrow \Omega_\pm\Omega_\pm^\dagger = \mathbf{1}$

$\Omega_\pm$ are unitary operators

The S-matrix

Exact temporal evolution of the system is experimentally inaccessible

What is measured are transition probabilities for a system in the given initial state to be observed in some prescribed final state

Initial state: $|\psi_+(t)\rangle = e^{-iHt}|\psi_+\rangle \underset{t\to-\infty}{\to} e^{-iH_0 t}|\phi_i\rangle$

Final state: $|\psi_-(t)\rangle = e^{-iHt}|\psi_-\rangle \underset{t\to+\infty}{\to} e^{-iH_0 t}|\phi_f\rangle$

Project on (= detect) final state at $t = T_f$ is t-indep.):

$$\begin{aligned}
\langle\psi_-(T_f)|\psi_+(T_f)\rangle &= \langle\psi_-|e^{iHT_f}e^{-iHT_f}|\psi_+\rangle = \langle\psi_-|\psi_+\rangle \quad (t\text{-indepedent}) \\
\langle\psi_-|\psi_+\rangle &= \lim_{T_f\to+\infty}\langle\psi_-|e^{iHT_f}e^{-iHT_f}|\psi_+\rangle \\
&= \lim_{T_f\to+\infty, T_i\to-\infty}\langle\psi_-|e^{iHT_f}e^{-iHT_f}e^{iHT_i}e^{-iHT_i}|\psi_+\rangle \\
&= \lim_{T_f\to+\infty, T_i\to-\infty}\langle\phi_f|e^{iH_0T_f}e^{-iHT_f}e^{iHT_i}e^{-iH_0T_i}|\phi_i\rangle \\
&= \langle\phi_f|\Omega_-^\dagger\Omega_+|\phi_i\rangle \equiv \langle\phi_f|S|\phi_i\rangle = S_{fi}
\end{aligned}$$

$S = \Omega_-^\dagger\Omega_+$: *S-operator*, matrix elements S_{fi}: *S-matrix*

The S-matrix (contd.)

S-matrix encodes all relevant information about scattering processes: from transition amplitudes S_{fi} one gets transition *probabilities* $P_{fi} = |S_{fi}|^2$ measured (up to factors) in experiments

1. S is unitary: $S^\dagger S = SS^\dagger = \mathbf{1}$

Also when $\Omega_\pm \Omega_\pm^\dagger \neq \mathbf{1}$

Expresses conservation of probabilities: $\sum_f P_{fi} = \sum_f |S_{fi}|^2 = 1$

$P(i \to \text{anything}) = \sum_f P(i \to f) = 1$

$$\sum_f |S_{fi}|^2 = \sum_{\phi_f} \langle \phi_i | S^\dagger | \phi_f \rangle \langle \phi_f | S | \phi_i \rangle = \langle \phi_i | S^\dagger S | \phi_i \rangle = \langle \phi_i | \phi_i \rangle = 1$$

$P(f \leftarrow \text{something}) = \sum_i P(i \to f) = 1$

$$\sum_i |S_{fi}|^2 = \sum_{\phi_i} \langle \phi_f | S | \phi_i \rangle \langle \phi_i | S^\dagger | \phi_f \rangle = \langle \phi_f | SS^\dagger | \phi_f \rangle = \langle \phi_f | \phi_f \rangle = 1$$

The S-matrix (contd.)

2. Since $\forall s$

$$e^{iHs}\Omega_\pm e^{-iH_0 s} = \lim_{t\to\mp\infty} e^{iHs}e^{iHt}e^{-iH_0 t}e^{-iH_0 s} = \lim_{t\to\mp\infty} e^{iH(t+s)}e^{-iH_0(t+s)}$$
$$= \lim_{t\to\mp\infty} e^{iHt}e^{-iH_0 t} = \Omega_\pm$$

Taking $\partial/\partial s|_{s=0} \Rightarrow$ intertwining relations

$$H\Omega_\pm = \Omega_\pm H_0 \qquad H_0\Omega_\pm^\dagger = \Omega_\pm^\dagger H$$

Energy conservation in a scattering process:

$$H_0 S = H_0\Omega_-^\dagger\Omega_+ = \Omega_-^\dagger H\Omega_+ = \Omega_-^\dagger\Omega_+ H_0 = SH_0 \Rightarrow [H_0, S] = 0$$

3. If symmetry generator G commutes with both free and full Hamiltonians

$$[G, H_0] = [G, H] = 0 \Rightarrow [G, \Omega_\pm] = 0 \Rightarrow [G, S] = 0$$

Interactions $V = H - H_0$ usually translationally and rotationally invariant
$\Rightarrow$ generators $\vec{P}$ (momentum) and $\vec{J}$ (angular momentum)
Conservation of momentum and angular momentum in scattering processes

$$[\vec{P}, S] = [\vec{J}, S] = 0$$