Particle physics

Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

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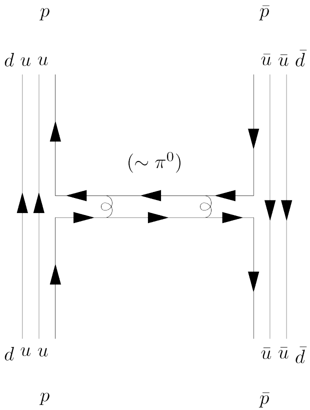
Strong processes

- Interaction vertices involve quarks and gluons which carry colour, but physical particles are colour neutral due to confinement
- Physical processes must involve exchange of gluons and quarks in colour-neutral combinations
- Physical processes can be effectively described as exchange of hadrons $\Longrightarrow$ lightest mediator is not the gluon (massless), but the pion (massive)



$p\,\bar{p} \to p\,\bar{p}$

Range of interactions

Range of interaction $\sim$ inverse of the mass of the lightest mediator
Back-of-an-envelope calculation: use $\Delta E \Delta t \geq \frac{\hbar}{2}$:

- exchange of particle of mass M requires violation of energy conservation over time $\Delta t \sim \hbar/\Delta E \sim \hbar/(Mc^2)$
- echanged particles travels no more than $\Delta x = c\Delta t$ in Δt
- $\Delta x \sim \hbar/(Mc) = \lambda_{\mathrm{Compton}}$

More detailed:

- Schrödinger equation from NR energy-momentum relation $E = \frac{\vec{p}^{\,2}}{2m}$ by $E \to i\partial_t$ and $\vec{p} \to -i\vec{\nabla}$

$$E = \frac{\vec{p}^{\,2}}{2m} \Rightarrow i\partial_t \psi(t,\vec{x}) = -\frac{\vec{\nabla}^{\,2}}{2m}\psi(t,\vec{x})$$

- relativistic case $E^2 - \vec{p}^{\,2} = M^2 \to$ *Klein-Gordon equation*

$$(\Box + M^2)\psi(t,\vec{x}) = (\partial_t^2 - \vec{\nabla}^2 + M^2)\psi(t,\vec{x}) = 0$$

Range of interactions (contd.)

Correct description of relativistic particles requires

- relativistic equation
- quantisation of the wave function $\psi \to \hat{\Psi}$ into a field operator that creates/annihilates particles

Field obeying Klein-Gordon eq. describes propagation of free particles of mass M

- $M = 0$, $\Box \vec{A} = 0$ is classical EM field in vacuum (Coulomb gauge) $\to$ massless quanta (photons) upon quantisation
- $M \neq 0 \to$ massive quanta upon quantisation

Consider low-energy limit (non-relativistic) and classical limit (exchange of a large number of particles)

- Maxwell equation $\to$ non-relativistic Coulomb potential
- Klein-Gordon equation $\to$ non-relativistic potential corresponding to exchange of massive particle

Range of interactions (contd.)

To find the potential, solve the equation in the static (time-independent) case adding fixed source with "charge" g

$$(-\vec{\nabla}^2 + M^2)u(\vec{x}) = g\delta(\vec{x}) \Longrightarrow (\vec{p}^{\,2} + M^2)\tilde{u}(\vec{p}\,) = g$$

Fourier transform $u(\vec{x}) = \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{x}}\tilde{u}(\vec{p}\,)$

Solution in momentum space

$$\tilde{u}(\vec{p}\,) = \frac{g}{\vec{p}^{\,2} + M^2}$$

Coordinate space solution ($r = |\vec{x}|$)

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&= \frac{g}{(2\pi)^2 ir} (2\pi i) e^{-Mr} \frac{iM}{2iM} = \frac{g}{4\pi r}\, e^{-Mr}
\end{aligned}$$

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Yukawa potential

Range of interactions (contd.)

Yukawa potential $V_{\text{Yukawa}}(r) = \frac{g}{4\pi r}\, e^{-Mr}$, range $\sim 1/M$

$$\text{range} = \frac{\hbar}{Mc} = \frac{\hbar c}{Mc^2} = \frac{197\ \text{MeV}\cdot\text{fm}}{M[\text{MeV}/c^2]\text{MeV}} = \frac{197}{M[\text{MeV}/c^2]}\,\text{fm}\,.$$

- weak interactions: lightest mediator $= W$, $M_W = 80\,\text{GeV}/c^2$

$$\text{range}_{\text{weak}} = \frac{197}{8\cdot 10^4}\,\text{fm} = 2.5\cdot 10^{-3}\,\text{fm}$$

- strong interactions: lightest mediator $= \pi^0$, $M_{\pi^0} = 135\,\text{MeV}/c^2$ (recall: confinement)

$$\text{range}_{\text{strong}} = \frac{197}{135}\,\text{fm} = 1.5\,\text{fm} \qquad \text{(typical scale of nuclei)}$$

- electromagnetic interactions: mediator $= \gamma$, massless $\to$ infinite range ($V_{\text{Yukawa}} \to_{m\to 0} V_{\text{Coulomb}} \propto 1/r$)

Conversely: mediator mass $= \text{range}^{-1} \sim$ typical energy scale

strong: $\mathcal{O}(100\,\text{MeV})$, weak: $\mathcal{O}(100\,\text{GeV})$

Strength of the interactions

Relative strength of interactions can be estimated from lifetimes of particle decays or from cross sections mediated by them

- "weight" of Feynman diagram $\propto \prod_{v\in\text{vertices}} \text{coupling}_v$

  "weight" since it is generally a complex quantity
- process *amplitude* = sum of weights of all corresponding diagrams
- probability per unit time of a process (decay width/scattering rate) $\propto$ $|\text{amplitude}|^2$

Consider decay with width $\Gamma = \tau^{-1}$ governed by interaction w/ coupling g

- simplest diagrams for a process: two vertices $\to \Gamma \propto (g^2)^2 = g^4$

  stronger interaction $\Rightarrow$ shorter lifetime
- relative strength of interactions 1 and 2

$$\frac{g_1^4}{g_2^4} \sim \frac{\tau_2}{\tau_1} \Longrightarrow \frac{g_1^2}{g_2^2} \sim \sqrt{\frac{\tau_2}{\tau_1}}$$

Strength of the interactions (contd.)

Typical decay times:

$$\tau_{\rm strong} \sim 10^{-23} \div 10^{-20} s \qquad \tau_{\rm weak} \sim 10^{-13} \div 10^{3} s \qquad \tau_{\rm em} \sim 10^{-16} s$$

- strong vs. weak:
  - Δ^0 resonance, $\tau_\Delta = 5.6 \cdot 10^{-24} s$ (strong, $\Delta^0 \to p\pi^-,\ n\pi^0$)
  - neutron, $\tau_n = 880\, s$ (weak, $n \to pe^-\bar{\nu}_e$)

$$\frac{g_w^2}{g_s^2} \sim \sqrt{\frac{\tau_\Delta}{\tau_n}} \sim 10^{-13}$$

- EM vs. weak:
  - neutral pion π^0, $\tau_{\pi^0} = 8.4 \cdot 10^{-17} s$ (EM, $\pi^0 \to \gamma\gamma$)
  - charged pion $\pi^\pm$, $\tau_{\pi^+} = 2.6 \cdot 10^{-8} s$ (weak, $\pi^+ \to \mu^+\nu_\mu$)

$$\frac{g_w^2}{g_{em}^2} \sim \sqrt{\frac{\tau_{\pi 0}}{\tau_{\pi+}}} \sim 10^{-4} \div 10^{-3}$$

strong force > electromagnetic force > weak force

different estimates from different processes but qualitatively unchanged

Similar estimate from scattering processes: $\sigma \propto |\text{amplitude}|^2 \propto g^4$

stronger interaction ⇒ larger cross section

Strength of weak interactions and the weak coupling

Weak coupling used above is not the true one but an effective one

In GWS theory, (true) weak coupling g_w related to e and $e^2 \sim 0.2 g_w^2$

Reason for weakness of weak interaction is not the smallness of the coupling, but the large mass of the mediator:

- exchange of boson brings factor $(p^2 - m^2)^{-1}$ in Feynman diagram, (p: four-momentum, m: mass of the virtual boson)
- M_W large, at low energies the effective coupling is the *Fermi constant* $G_F = g_w^2/M_W^2 \sim 1.1 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$
- $1/\sqrt{G_F} \gg$ typical m and $p \to$ weak interactions are indeed weak
- applies as long as $p^2 \ll M_W^2$, then weak interaction become stronger

Fermi constant appeared first in Fermi's theory of β decay (1933): p, n, e, ν interact via a 4-fermion vertex with coupling G_F

- good approximation at low energy ($p^2 \ll M_W^2$)
- $[G_F] = m^{-2}$ = trouble, "more fundamental" theory had to exist
- assuming unification $e \sim g$ and using $G_F \to M_W \sim \sqrt{4\pi\alpha/G_F} \sim 90\,\mathrm{GeV}$

How to tell the nature of a decay process

How do we tell what interaction governs a given process?

- signature particles: $\gamma \to$ EM, $\nu \to$ weak, $\pi \to$ strong (but also weak)
- lifetime (related to strength)
- conservation laws

Dynamical conservation laws originate in the symmetries of the interactions

- some conservation laws apply to all interactions (energy-momentum, angular momentum, electric charge, baryon and lepton number)
- some are interaction specific (e.g., flavour-type symmetries)

If a conservation law is violated by a process, an interaction that respects it cannot be the one responsible for the process

Applies to scattering as well

Symmetries of fundamental interactions

Degree of symmetry varies: strong interactions "most symmetrical", weak interactions "least symmetrical"

Quark flavour: n. of quarks minus antiquarks of each type

- strangeness $S = n_{\bar{s}} - n_s$, charm $C = n_c - n_{\bar{c}}$ (sign of S is a historical accident)
- $U = n_u - n_{\bar{u}}$, $D = n_{\bar{d}} - n_d$ $\leftrightarrow$ baryon number $\mathcal{B}$ and **electric charge** Q, conserved by all interactions

Strong: yes	EM: yes	Weak: no

e.g.: neutral kaon $\to$ pions: $\not{S}$ (+ long lifetime), cannot be via strong (it is weak)

Lepton flavour: n. of leptons minus antileptons of each type

Strong: not relevant	EM: yes	Weak: no

Lepton family: n. of leptons minus antileptons of each family

Strong: not relevant	EM: yes	Weak: yes*

Baryon/lepton number: n. of baryons/leptons minus antibaryons/leptons

Strong: yes/n.r.	EM: yes	Weak: yes**

Symmetries of fundamental interactions (contd.)

Isospin $\vec{I}$: hadrons organised in multiplets of $2I+1$ particles with nearly identical masses, same $\vec{I}^2 = I(I+1)$, distinguished by I_3 (non-hadrons: $I=0$)

- $\vec{I} \to$ SU(2) symmetry

  like spin but physically unrelated

- we will see that $I_3 = \frac{1}{2}(U+D)$
- for light quarks (made of u, d, s)

  $I_3 = Q - \frac{1}{2}(\mathcal{B}+S)$

  including c: $I_3 = Q - \frac{1}{2}(\mathcal{B}+S+C)$

pions	triplet	$I=1$	$I_3 = Q$
p, n	doublet	$I=\frac{1}{2}$	$I_3 = Q-\frac{1}{2}$
K^+, K^0	doublet	$I=\frac{1}{2}$	$I_3 = Q-1$
Δ's	quartet	$I=\frac{3}{2}$	$I_3 = Q-\frac{1}{2}$

Strong: yes	EM: no	Weak: no

EM: $q_u \neq q_d$, weak: $d' \neq d$

$$\pi^0(I=1) \underset{\text{EM}}{\to} \gamma\gamma(I=0) \qquad \pi^+(I=1) \underset{\text{weak}}{\to} \mu^+\nu_\mu(I=0)$$

Symmetries of fundamental interactions (contd.)

Discrete symmetries:

- **parity** P (spatial inversion $\vec{x} \to -\vec{x}$)
  $=$ cannot tell if process takes place here or in the mirror
- **charge conjugation** C (exchange of particles with antiparticles)
  $=$ sign of all conserved charges is conventional
- **time reversal** T ($t \to -t$)
  $=$ cannot tell if process takes place in real time or in reversed film

Strong: all	EM: all	Weak: none

CPT: conserved in any local QFT (so $CP = T$)

- physics unchanged if $\vec{x} \to -\vec{x}$, $t \to -t$, particles $\leftrightarrow$ antiparticles
- CPT conservation is a theorem in QFT, violations $\Rightarrow$ QFT inadequate

Parity non-conservation in weak interactions

Physical phenomena that we see in the mirror are not always possible physical phenomena in the real world

mirror image = parity and 180° rotation around an axis orthogonal to the mirror

- neutrinos left-handed ($h = -1$), antineutrinos right-handed ($h = +1$)

  helicity $h = \frac{\vec{s}\cdot\vec{p}}{|\vec{p}\,|}$, Lorentz-invariant for massless particles

- $h \xrightarrow{P} -h \Rightarrow$ mirror neutrinos/antineutrinos are right/left-handed!

Lee and Yang suggest parity non-conservation in weak interactions, immediately confirmed experimentally (1956)

- Wu *et al.*: polarised cobalt-60 decay ${}^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e$
  - e^- emitted preferentially opposite to nucleus spin
  - in mirror ($\parallel \vec{S}_{\mathrm{Co}}$) spin reversed, e^- pref. emitted in spin direction
- Garwin *et al.*: polarised muon decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$
  - e^- angular distribution $\propto 1 - \frac{1}{3}\cos\theta$, θ: angle between $\vec{p}_{e^-}$ and $\vec{s}_\mu$
  - under P, $\vec{s}_\mu$ unchanged, $\vec{p}_{e^-}$ reversed $\Rightarrow$ e^- angular distribution $\propto 1 + \frac{1}{3}\cos\theta$ in the mirror
  - this is the angular distribution of e^+ in the μ^+ decay $\Rightarrow CP$ seems still good (but it is not in the end)

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