Particle physics

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Cross section

Operative definition

$$\sigma = \frac{N_{\text{events}}}{N_t \frac{N_b}{A_b}}$$

Uniformly distributed beam, crossing the whole target in time t

$$N_b = \frac{N_b}{t} t = \frac{\Delta N_b}{\Delta t} \Delta t = \rho_b A_b \frac{\Delta x}{\Delta t} \Delta t = \rho_b A_b v \Delta t = \Phi A_b \Delta t$$

ρ_b: beam density (n. particle/unit volume)
$\Phi = \rho_b v$: beam *flux* (n. particle crossing unit area $\perp$ beam per unit time)

Constant event rate $N_{\text{events}} = \frac{N_{\text{events}}}{t} t = \frac{\Delta N_{\text{events}}}{\Delta t} \Delta t$

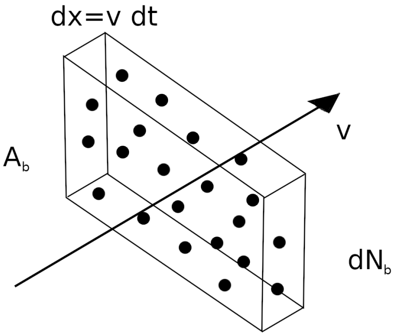
$$\sigma = \frac{\frac{\Delta N_{\text{events}}}{\Delta t}}{N_t \frac{1}{A_b} \frac{\Delta N_b}{\Delta t}} = \frac{\Delta N_{\text{events}}}{\Delta t N_t \Phi}$$

Cross section = n. scattering events per unit time, unit target, unit flux

Cross section

Operative definition

Uni



ρ_b:
Φ =
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Co

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Cross section = n. scattering events per unit time, unit target, unit flux

Scattering processes

Elastic scattering

$$a\,b \longrightarrow a\,b$$

- same type (and number) of particles in and out
- momenta and spin component can change

Inelastic scattering

$$a\,b \longrightarrow X_1\,X_2\,\ldots\,X_n$$

- different particles in and out
- kinematical and dynamical constraints restrict the allowed inelastic processeses

Resonances

Distinctive sign of unstable particle being created as intermediate state: peak in the cross section as a function of energy (*resonance*)

- Position of the peak $\rightarrow$ mass m
- Width of the peak $\rightarrow$ decay width $\Gamma = 1/\text{lifetime}$

"Hand-waving" argument: wave-function of unstable system of $E \approx m$ decaying exponentially in time with lifetime $1/\Gamma$

$$\psi(t) = \psi(0) e^{-imt} e^{-\frac{\Gamma}{2}t} \longrightarrow |\psi(t)|^2 = |\psi(0)| e^{-\Gamma t}$$

$$\tilde{\psi}(E) = \int dt e^{iEt} \psi(t) = \frac{i\psi(0)}{E - m + i\frac{\Gamma}{2}}$$

If the unstable system is formed in a scattering experiment at energy E, $\sigma(E)$ near $m \propto$ probability of observing the unstable system with energy E

$$\sigma(E) \propto |\tilde{\psi}(E)|^2 = \frac{|\psi(0)|^2}{(E-m)^+ \left(\frac{\Gamma}{2}\right)^2} = \sigma_{\max} \frac{\left(\frac{\Gamma}{2}\right)^2}{(E-m)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

Breit-Wigner distribution describes accurately many resonances

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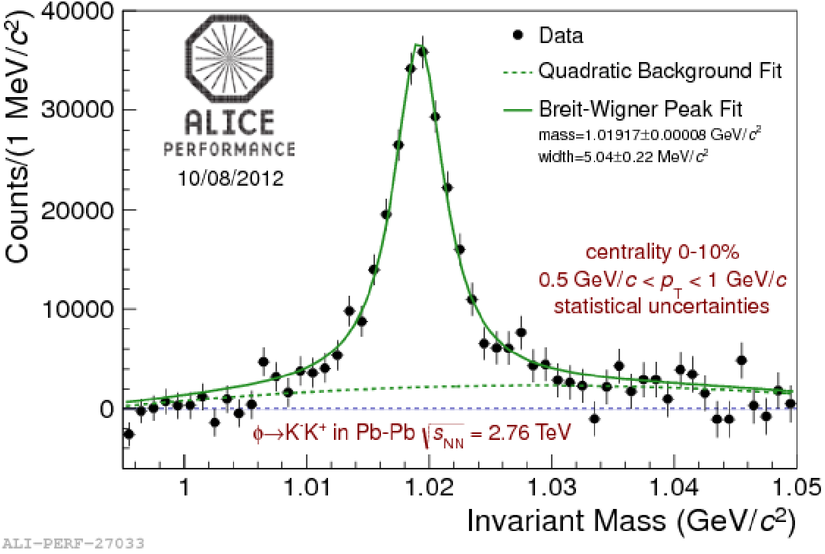


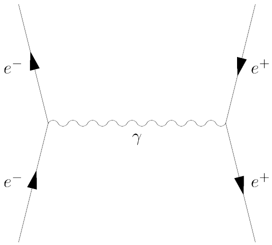
ϕ meson ($s\bar{s}$) in lead-lead collisions

$m_\phi = 1019.461 \pm 0.016\,\mathrm{MeV}$, $\Gamma_\phi = 4.249 \pm 0.013\,\mathrm{MeV}$ [4]

Describing interactions: Feynman diagrams

Interactions can be described as exchange of particles

E.g.: e^-e^+ scattering mediated by EM = exchange of (one or more) γ



- particle 1 emits/absorbs mediator absorbed/emitted by particle 2...
- ...or the other way around?
- both, and none of the two: not well defined who does what on such short time-scales

What matters is the exchange, not who emits/absorbs

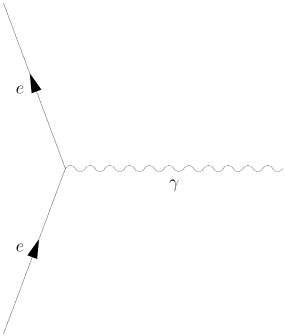
(Also: do not take this picture too literally)

Fundamental processes: emission/absorption of an interaction particle from matter particle or from another interaction particle (*vertex*)

Quantities conserved at vertex → automatically conserved by interaction: energy/momentum, angular momentum, electric charge...

Electromagnetic interactions

Interaction vertex in Quantum ElectroDynamics (QED)



- electron enters, emits/absorbs photon, exits (time flows upwards)
- only vertex
- same for any other negatively charged lepton, or for quarks
- same for antiparticles, except arrow is drawn reversed (time still flows upwards)

Diagrams like these are known as *Feynman diagrams*: more than pictorial representation of a process (in due time)

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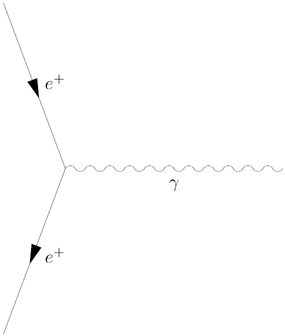

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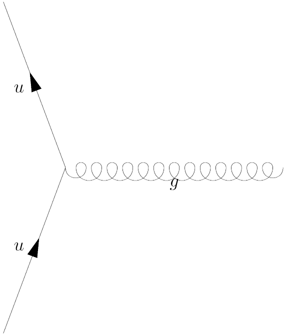
Strong interactions

Interaction vertices in Quantum ChromoDynamics (QCD)



- quark enters, exchanges gluon, exits
- similar to QED, but quarks and gluons carry also *colour*
- colour of q can change but overall conserved at vertex ("difference" carried by g)
- 3 quark colours, 8 gluon types
  ($3 \times 3 = 9$ combinations, but the one leaving all colours unchanged is absent)
- also vertices involving only 3 or 4 gluons (gluons self-interact)

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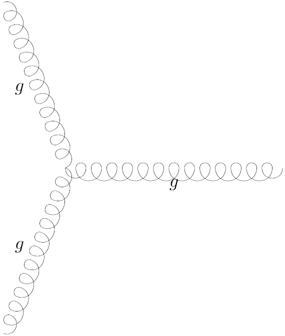

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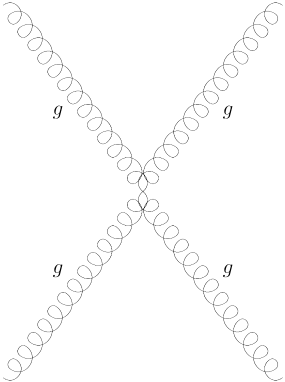

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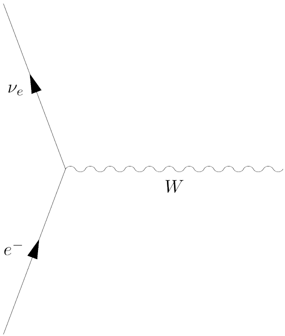
Weak interactions

Interaction vertices for weak interactions



- charged current: negatively charged lepton enters, emits W^-/absorbs W^+ and turns into neutrino...
- ...or neutrino enters, emits W^+/absorbs W^- and turns into neg. charged lepton
- similarly with antiparticles
- leptons from same family are involved: (l^-, ν_l), $(l^+, \bar{\nu}_l)$
- neutral current: lepton enters, exchanges Z^0, exits
- also vertices involving only 3 or 4 IVB (self-interactions) and/or γ ($W^\pm$ are charged)

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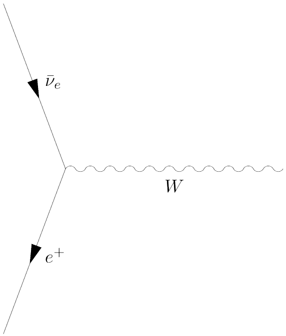

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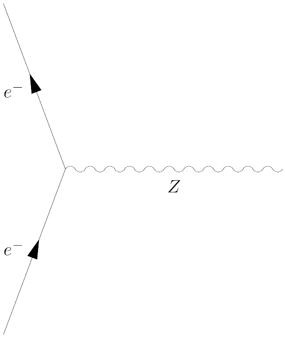

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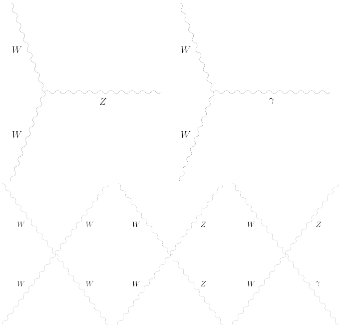

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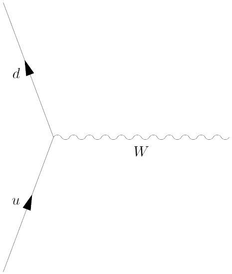
Weak interactions of quarks

N. current same as with leptons, ch. current analogue $(e^-, \nu_e) \to (u, d)$
Instead $(e^-, \nu_e) \to (u, d')$ with d' a superposition of d and s quarks

$$|d'\rangle = \cos\theta_C |d\rangle + \sin\theta_C |s\rangle$$



Needed to explain $K \to \mathrm{hadrons}$, where $s/\bar{s}$ turns into $d/\bar{d}$

Better: $(\ell^-, \nu_\ell) \to (u, d'), (c, s'), (t, b')$ with d', s', b' lin. sup. of d, s, b

Unitary matrix of mixing coefficients is the Cabibbo-Kobayashi-Maskawa (CKM) matrix

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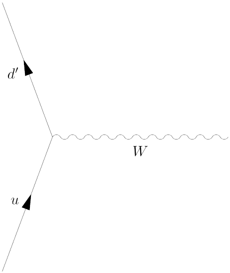
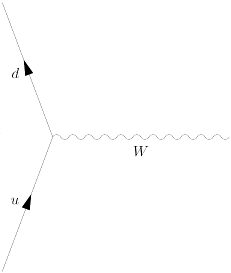
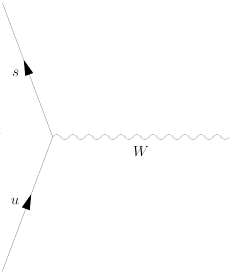


$= \cos\theta_C$



$+ \sin\theta_C$



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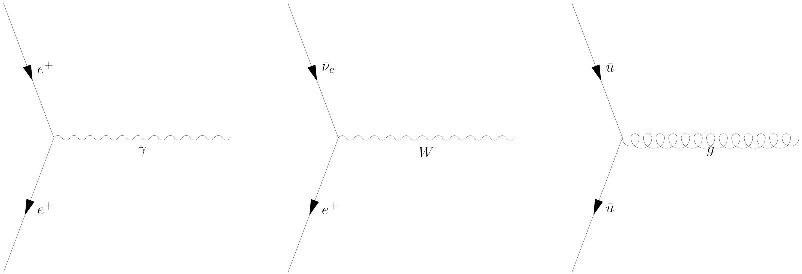
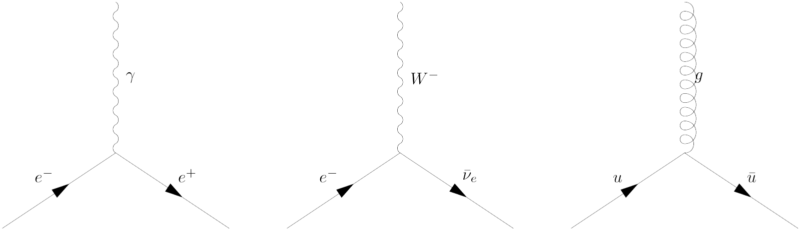
"Reflecting" a fermion line replaces particle with antiparticle



Vertices can be "rotated" to put fermion and antifermion on the same side of the process

Diagrams with antiparticles

"Reflecting" a fermion line replaces particle with antiparticle



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Conservation laws

From interaction vertices one can read off conservation laws

	EM	strong	weak
electric charge	yes	yes	yes
lepton type	yes	–	no
flavour (=quark type)	yes	yes	no
lepton family	yes	–	yes (if massless)
quark family	yes	yes	no
lepton number	yes	–	yes
quark number	yes	yes	yes

Lepton type/flavour: $n_f - n_{\bar{f}}$

Lepton family number: $L_\ell = (n_{\ell^-} - n_{\ell^+}) + (n_{\nu_\ell} - n_{\bar{\nu}_\ell})$

"Quark family": $(n_u - n_{\bar{u}}) + (n_d - n_{\bar{d}})$, etc.

(never used: either more detailed cons. law exists, or not conserved due to quark mixing)

Lepton number: $L = \sum_{\ell=e,\mu,\tau} L_\ell$

Quark number: $\mathcal{Q} = \sum_q n_q - n_{\bar{q}}$

Conservation laws (contd.)

Flavour numbers:

$$U = n_u - n_{\bar{u}} \qquad C = n_c - n_{\bar{c}} \qquad T = n_t - n_{\bar{t}}$$

$$D = -n_d + n_{\bar{d}} \qquad S = -n_s + n_{\bar{s}} \qquad B = -n_b + n_{\bar{b}}$$

Quark number is equivalent to baryon number

$$\sum_q n_q = 3n_{\mathrm{baryons}} + n_{\mathrm{mesons}}$$

$$\sum_q n_{\bar{q}} = 3n_{\mathrm{antibaryons}} + n_{\mathrm{mesons}}$$

$$\Longrightarrow \mathcal{Q} = 3(n_{\mathrm{baryons}} - n_{\mathrm{antibaryons}}) = 3\mathcal{B}$$

If flavour is conserved, U and D are traded for $\mathcal{B}$ and electric charge Q

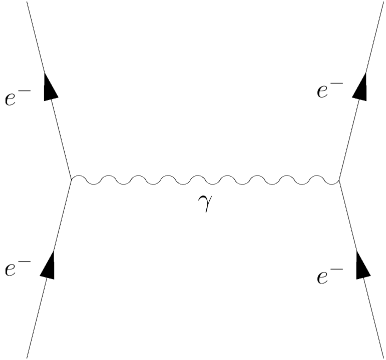
$$\mathcal{B} = \tfrac{1}{3}(U + C + T - D - S - B)$$

$$Q = \tfrac{2}{3}(U + C + T) + \tfrac{1}{3}(D + S + B)$$

If an interaction conserves a certain particle number it cannot be responsible for decays in which this number is violated
(e.g., strangeness changing processes cannot be due to strong int.)

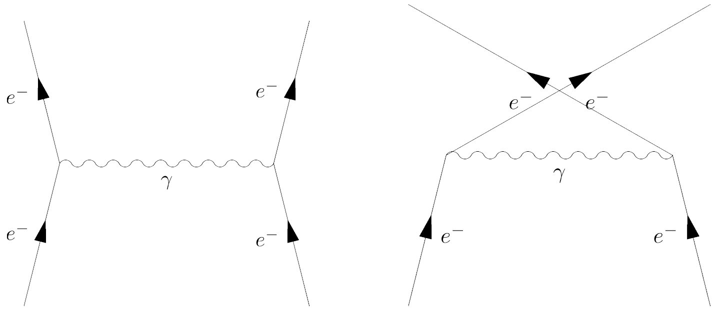
Basic processes

- Vertex diagrams describe how the interaction works at the most fundamental level but cannot represent a true physical process due to lack of energy-momentum conservation,
- Actual physical process described combining two or more vertices



- energy and momentum are conserved, but exchanged photon not on *mass shell*, $p_\gamma^2 \neq 0 = m_\gamma^2$
- internal lines represent *virtual particles*, not real particles, so not on shell
- external lines represent real particles, must be on-shell $p_i^2 = m_i^2$

Basic processes in QED

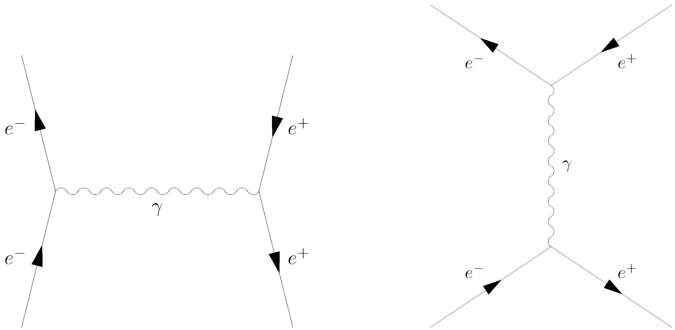


Møller scattering

$$e^- \, e^- \to e^- \, e^-$$

electrons are indistinguishable, cannot say which one is going left/right after photon exchange $\to$ must take both possibilities into account

Basic processes in QED

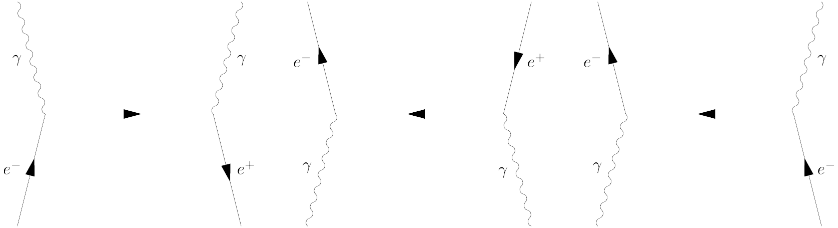


Bhabha scattering

$$e^- \, e^+ \to e^- \, e^+$$

$e^- \, e^+$ can exchange a photon or annihilate into a photon $\to$ must take both possibilities into account

Basic processes in QED



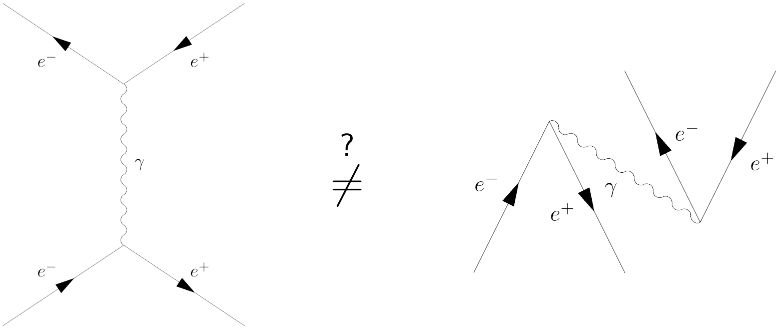
Three more basic QED processes:

- electron-positron annihilation $e^- \, e^+ \to \gamma \, \gamma$
- electron-positron pair creation $\gamma \, \gamma \to e^- \, e^+$
- Compton scattering $\gamma \, e^- \to \gamma \, e^-$

(For each of these processes there is a second diagram: can you draw it?)

What matters in a Feynman diagram

Feynman diagrams are *not* an accurate depiction of particle trajectories!



- "time" not well defined, except "before" and "after" of the process
- "when" annihiliation/creation happens are meaningless questions
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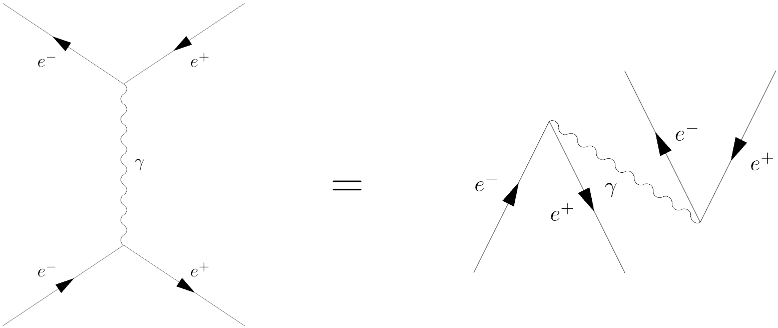

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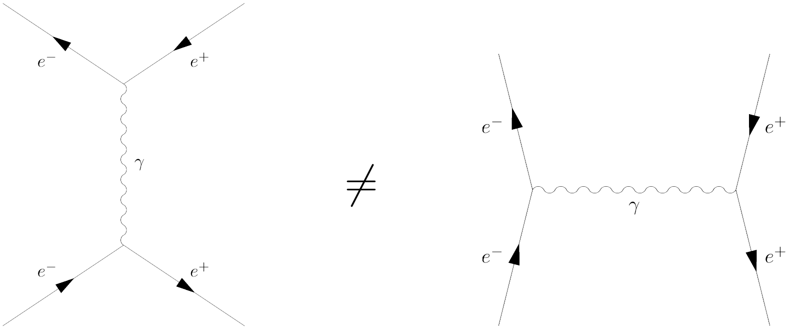

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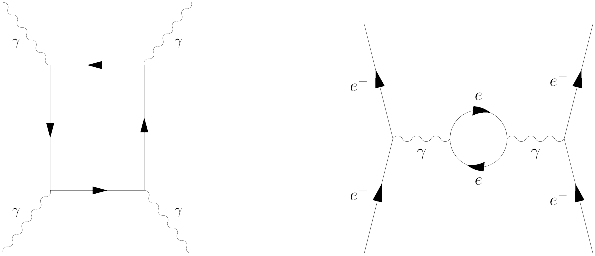
Higher order diagrams

Infinity of ways in which one can combine vertices

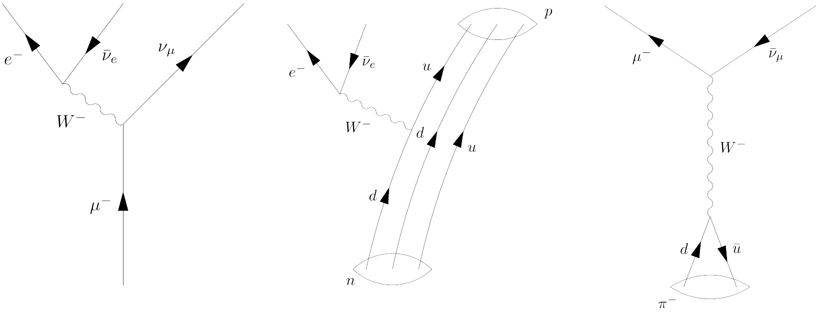


Left: light-by-light (Delbrück) scattering, Right: $e^- \, e^-$ elastic scattering

- Diagrams have different "weights", i.e., are more or less important
- Each vertex contributes a factor e to the weight of a diagram
  - two vertices $\to \propto \alpha$, $\alpha = e^2/(4\pi) \simeq 1/137$ *fine structure constant*
  - four vertices $\to \alpha^2$, relatively suppressed
- Describing process to given precision requires limited n. of diagrams
- Vertex weighting factor: *coupling constant*

  1 EM (e), 1 strong, 2 weak both $\propto e$ via *Weinberg angle* (EW unification)

Basic charged weak current processes



Muon decay $\mu^- \to e^- \, \bar{\nu}_e \, \nu_\mu$

Neutron β-decay $n \to p \, e^- \, \bar{\nu}_e$ (same diagram plus spectator quarks)

Pion decay: $\pi^- \to \mu^- \bar{\nu}_\mu$ (replace e with μ)

Strangeness-changing processes: $\Lambda^0(uds) \to p(uud) \, \pi^-(d\bar{u}) \quad (\Delta S = 1)$

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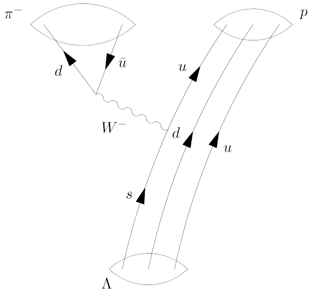

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References

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- Douglas Adams, *The Hitchhiker's Guide to the Galaxy*
- P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)