Particle physics

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Feynman diagrams (contd.)

Propagator (internal line):

$$D(x) = \langle 0 | T\big(\phi(x)\phi(y)\big) | 0 \rangle = \langle 0 | \theta(x^0 - y^0)\phi(x)\phi(y) + \theta(y^0 - x^0)\phi(y)\phi(x) | 0 \rangle$$
$$= \int d\Omega_p \int d\Omega_q \langle 0 | a(\vec{p}\,) a(\vec{q}\,)^\dagger | 0 \rangle \big[\theta(x^0 - y^0) e^{-ip\cdot x} e^{iq\cdot y} + \theta(y^0 - x^0) e^{-ip\cdot y} e^{iq\cdot x}\big]$$
$$= \int d\Omega_p \big[\theta(x^0 - y^0) e^{-ip\cdot(x-y)} + \theta(y^0 - x^0) e^{-ip\cdot(y-x)}\big]$$

$D(x)$ obeys inhomogeneous Klein-Gordon equation

$$\partial_{x0}^2 T\big(\phi(x)\phi(y)\big) = \partial_{x0}\big\{ T\big(\partial_{x0}\phi(x)\phi(y)\big) + \delta(x^0 - y^0)[\phi(x), \phi(y)]_{\mathrm{ET}} \big\}$$
$$= \partial_{x0} T\big(\partial_{x0}\phi(x)\phi(y)\big) = T\big(\partial_{x0}^2\phi(x)\phi(y)\big) + \delta(x^0 - y^0)[\partial_{x0}\phi(x), \phi(y)]_{\mathrm{ET}}$$
$$= T\big(\partial_{x0}^2\phi(x)\phi(y)\big) + \delta(x^0 - y^0)[\pi(x), \phi(y)]_{\mathrm{ET}} = T\big(\partial_{x0}^2\phi(x)\phi(y)\big) - i\delta^{(4)}(x-y)$$
$$(\Box_x + m^2) D(x-y) = \langle 0 | T\big((\Box_x + m^2)\phi(x)\phi(y)\big) - i\delta^{(4)}(x-y) | 0 \rangle = -i\delta^{(4)}(x-y)$$

In momentum space $\Rightarrow \tilde{D}(q) = \dfrac{i}{q^2 - m^2}$ except near $q^2 = m^2$

Singularity requires prescription $\Rightarrow \tilde{D}(p) = \dfrac{i}{p^2 - m^2 + i\epsilon}$, reproduces $D(x)$

Feynman diagrams (contd.)

Using residue theorem (poles at $p^0 = \pm\sqrt{\vec{p}^{\,2} + m^2} \mp i\epsilon$)

$$\begin{aligned}
\int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot x}\tilde{D}(p) &= \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot x}\frac{i}{p^2 - m^2 + i\epsilon} \\
&= i\int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot x}\frac{1}{p^0 - \sqrt{\vec{p}^{\,2}+m^2} + i\epsilon}\,\frac{1}{p^0 + \sqrt{\vec{p}^{\,2}+m^2} - i\epsilon} \\
&= i\int \frac{d^3p}{(2\pi)^4}\left\{\theta(x^0)(-2\pi i)e^{-ip\cdot x}\frac{1}{2p^0} + \theta(-x^0)(2\pi i)e^{ip\cdot x}\frac{1}{-2p^0}\right\} \\
&= i\int \frac{d^3p}{(2\pi)^3 2p^0}\left\{\theta(x^0)e^{-ip\cdot x} + \theta(-x^0)e^{ip\cdot x}\right\} = D(x)
\end{aligned}$$

$2 \to 2$ elastic scattering amplitude in ϕ^3 theory to lowest order

$$\begin{aligned}
\mathcal{M}_{fi} &= i\lambda^2\left\{\frac{i}{(p_1+p_2)^2 - m^2} + \frac{i}{(p_1' - p_1)^2 - m^2} + \frac{i}{(p_2' - p_1)^2 - m^2}\right\} \\
&= -\lambda^2\left\{\frac{1}{s - m^2} + \frac{1}{t - m^2} + \frac{1}{u - m^2}\right\}
\end{aligned}$$

Displays crossing symmetry manifestly

Fermions

Matter particles (quarks, leptons) are spin-$\frac{1}{2}$ fermions
Basis of one-particle states: diagonalise momenta and one spin component

Usually z component

$$|\vec{p}\,, s_z\rangle \qquad s_z = \pm\tfrac{1}{2}$$

Multiparticle states must obey Fermi-Dirac statistics

$$|\vec{p}_1\,, s_{1\,z}; \vec{p}_2\,, s_{2\,z}; \ldots \vec{p}_n\,, s_{n\,z}\rangle = (-1)^{\sigma_{\mathrm{P}}}|\vec{p}_{\mathrm{P}(1)}\,, s_{\mathrm{P}(1)\,z}; \vec{p}_{\mathrm{P}(2)}\,, s_{\mathrm{P}(2)\,z}; \ldots \vec{p}_{\mathrm{P}(n)}\,, s_{\mathrm{P}(n)\,z}\rangle$$

P: permutation, $\sigma_{\mathrm{P}} = 0, 1$ signature of the permutation

Annihilation/creation operators $b_s(\vec{p}\,)$, $b_s(\vec{p}\,)^\dagger$, $s = \pm\frac{1}{2}$

- extra index for spin
- obey *anticommutation relations* due to Fermi-Dirac statistics

$$\{b_s(\vec{p}\,), b_t(\vec{q}\,)^\dagger\} \equiv b_s(\vec{p}\,)b_t(\vec{q}\,)^\dagger + b_t(\vec{q}\,)^\dagger b_s(\vec{p}\,) = \delta_{st}(2\pi)^3 2p^0\delta^{(3)}(\vec{p}-\vec{q}\,)$$
$$\{b_s(\vec{p}\,), b_t(\vec{q}\,)\} = \{b_s(\vec{p}\,)^\dagger, b_t(\vec{q}\,)^\dagger\} = 0$$

- creation/annihilation operators of different fermion species anticommute with each other (commute with bosonic operators)