Particle Physics Homework

5. Quark model/Neutral kaon decays

2023/10/16
(deadline: 2023/10/24)

Problem 1

Diquark states $q_1 q_2$ can be organised in symmetric and antisymmetric combinations, that transform respectively in the $\mathbf{6}$ and $\bar{\mathbf{3}}$ irreducible representations of SU(3).

1. [**4 points**] Draw the $\mathbf{6}$ and $\bar{\mathbf{3}}$ representations in the (I_3, Y) plane, labelling each state with their quantum numbers I, I_3, Y, and with their quark flavour content.

2. [**4 points**] Find the combinations of two spin-$\frac{1}{2}$ particles spin states that have definite total spin, and determine their symmetry properties under particle exchange.

3. [**5 points**] Write the possible diquark wave functions in spin and flavour space, and determine the exchange symmetry properties of the total space/spin/flavour wave function (assume that the orbital angular momentum is $\ell = 0$).

4. [**5 points**] Show that a $q_1 q_2$ state cannot be a colour singlet (so it cannot be an acceptable physical state).

5. [**7 points**] After determining the coefficients of the linear relation $I = \alpha I + \beta$ between the total isospin and the hypercharge of the diquark states in the $\mathbf{6}$ and $\bar{\mathbf{3}}$ representations, write down the Gell-Mann–Okubo mass formula for these representations, and extract as many independent relations between diquark masses as possible.

Problem 2

A beam of K^0 is produced in the lab with energy E.

1. [5 points] Write the possible two-pion decays of K^0 compatible with electric charge conservation, and show that they do not conserve parity and total isospin.

2. [10 points] Knowing that the short component of neutral kaons has lifetime $\tau_S \simeq 0.9 \cdot 10^{-10}\, s$, determine how far from the beam source one has to place the detectors in order to detect only[1] decays of the long component of neutral kaons.

3. [10 points] Determine the magnitudes of the K^0 and $\bar{K}^0$ components of the beam,

$$|\langle K^0|\Pi_{K^0}U(t)|K^0\rangle|^2\,, \qquad |\langle \bar{K}^0|\Pi_{K^0}U(t)|K^0\rangle|^2\,,$$

where $\Pi_{K^0} = |K^0\rangle\langle K^0| + |\bar{K}^0\rangle\langle\bar{K}^0|$ is the projector on the neutral kaon subspace and $U(t)$ the temporal evolution operator, as a function of the distance from the beam source. Use the Wigner-Weisskopf approximation $\Pi_{K^0}U(t) \simeq e^{-iH_{\text{eff}}t}$ with non-Hermitean H_{eff}, and assume CP invariance.

Hints

Exercise 1:

(1.) you can find the quark content of a product representation graphically by centering the weight diagram of the second one on the states of the first one;

(4.) write the decomposition of the product of the colour representations of the quarks in the diquark in terms of irreducible representations;

(5.) you can start from the general form of the Gell-Mann–Okubo formula derived in class, valid for general representations, and specialise it to the cases at hand.

[1] Take this to mean that the number of short kaons in the beam has been reduced to a reasonably small of the initial amount.

Exercise 2:

(1.) write the possible total isospin states of two pions;

(2.) use the exponential decay law to estimate the number of short kaons reaching a certain distance form the source.

Grade

Your grade is given by the sum of points scored for each exercise, divided by 10, and rounded to the closest integer.