Particle Physics Homework

4. Symmetries

2023/10/09
(deadline: 2023/10/16)

1. [5 points] Using only rotation invariance, show that for a two-body scattering process $a\,b \to c\,d$ involving only spinless particles one has in the centre-of-mass frame

$$\langle c(\vec{p}\,')\, d(-\vec{p}\,')|S|a(\vec{p}\,)\, b(-\vec{p}\,)\rangle = \langle c(-\vec{p}\,')\, d(\vec{p}\,')|S|a(-\vec{p}\,)\, b(\vec{p}\,)\rangle\,,$$

where the argument in brackets is the momentum of the corresponding particle. Using invariance under parity, show that the process is allowed only if the intrinsic parities of the particles satisfy $\eta_a\eta_b = \eta_c\eta_d$.

2. [4 points] Show that if the S-operator is isometric, i.e., it satisfies $S^\dagger S = \mathbf{1}$, and if the theory is invariant under time reversal, then one has automatically $SS^\dagger = \mathbf{1}$ (and so S is unitary).

3. [5 points] Denote with S_a, $a = 1, 2, 3$, the generators of SU(2), and with $|s, m\rangle$ the eigenstates of $\vec{S}^{\,2}$ and S_3, i.e., $\vec{S}^{\,2}|s, m\rangle = s(s + 1)|s, m\rangle$ and $S_3|s, m\rangle = m|s, m\rangle$. Use the commutation relations $[S_a, S_b] = i\varepsilon_{abc}S_c$ to show that

$$\langle s, m|S_1|s, m\rangle = \langle s, m|S_2|s, m\rangle = 0\,.$$

Compute the commutator $[S_3, S_1S_2]$, and use the result to show that

$$\langle s, m|S_1^2|s, m\rangle = \langle s, m|S_2^2|s, m\rangle\,.$$

Exploiting this identity, compute these expectation values explicitly.

4. [6 points] The J/ψ meson can decay into an electron-positron pair e^-e^+. Knowing that the spin and intrinsic parity of the J/ψ are $s_{J/\psi} = 1$ and $\eta_P = -1$, determine the possible values ℓ of the orbital angular momentum $\vec{L}$ (where $\vec{L}^2 = \ell(\ell + 1)$) and s of the total spin $\vec{S} = \vec{S}_{e^-} + \vec{S}_{e^+}$ (where $\vec{S}^2 = s(s + 1)$) in the final state compatible with rotation and parity invariance (this is an electromagnetic decay, so parity is conserved).

Hints

Exercise 1: find the rotation that sends $\vec{p} \to -\vec{p}$ and $\vec{p}\,' \to -\vec{p}\,'$ simultaneously.

Exercise 2: use the transformation properties of S.

Exercise 3: recall the identity

$$[A, BC] = [A, B]C + B[A, C]\,.$$

Exercise 4: use states $|\ell, \ell_z; s, s_z\rangle$ with definite orbital angular momentum and total spin in the final state; determine first the possible eigenvalues of the total spin $\vec{S}^2$, then those of the total angular momentum $\vec{J} = \vec{L} + \vec{S}$; recall the relation between the parity and the orbital angular momentum ℓ for a state $|\ell, \ell_z\rangle$, and that between the intrinsic parities of a particle and its antiparticle.

Grade

Your grade is given by the sum of points scored for each exercise, divided by 4, and rounded to the closest integer.