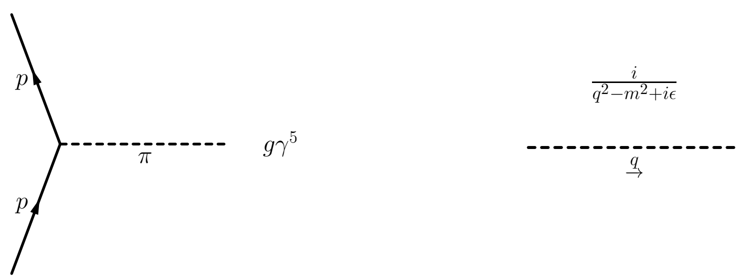
Particle Physics Homework

3. Feynman diagrams and scattering amplitudes

2023/10/02
(deadline: 2023/10/09)

The elastic scattering of nucleons can be effectively described in terms of pion exchange. The proton-proton-pion vertex and the pion propagator and the corresponding Feynman rules are given in the figure below.



Recall that $\{\gamma^5, \gamma^\mu\} = 0$ and $(\gamma^5)^2 = \mathbf{1}$.

1. [10 points] Draw the lowest-order Feynman diagrams for the elastic scattering process

$$p(\vec{p}_1, s_1)\ p(\vec{p}_2, s_2) \longrightarrow p(\vec{p}_1', s_1')\ p(\vec{p}_2', s_2')\,, \tag{$*$}$$

labelling the external lines with the quantum numbers of the corresponding particles and assigning a four-momentum to each (external or internal) line. Use four-momentum conservation to determine the momentum of the internal lines.

2. [10 points] Write down the contribution of each graph to the scattering amplitude M_{fi} for the process in $(*)$, including their sign.

3. [15 points] Compute the squared amplitude $|M_{fi}|^2$ averaged over the spins of the initial protons and summed over the spins of the final protons, and express it in terms of Mandelstam variables.

4. [10 points] Compute the differential cross section $\frac{d\sigma}{d\Omega_{\mathrm{CM}}}$ as a function of s and θ_{CM}, where the scattering angle θ_{CM} is the angle between $\vec{p}_1$ and $\vec{p}_1'$ in the centre-of-mass frame, and $d\Omega_{\mathrm{CM}}$ denotes the infinitesimal element of solid angle in the centre-of-mass frame, $d\Omega_{\mathrm{CM}} = d\theta_{\mathrm{CM}} d\phi_{\mathrm{CM}}$. Express the Mandelstam variables t and u in terms of s and of the scattering angle θ_{CM}, and determine the minimal value s_0 allowed for s. Determine the high-energy asymptotic behaviour of the differential cross section at fixed θ_{CM}, i.e., its dependence on s for $s \gg s_0$.

Hints

For (1.) and (2.), follow the approach discussed in detail in class, that you can also find in the lecture notes. For (3.) you will need the completeness relation for the positive-energy solutions $u_s(\vec{p})$ of the time-independent Dirac equation,

$$\sum_{s=1}^{2} u_s(\vec{p})\bar{u}_s(\vec{p}) = \not{p} + m \,,$$

and the following trace formulas for gamma matrices,

$$\begin{aligned} \mathrm{tr}\,\gamma^\mu\gamma^\nu &= 4\eta^{\mu\nu}\,, \\ \mathrm{tr}\,\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma &= 4\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)\,. \end{aligned}$$

For (4.) you will need the cross section formula and the expression for the two-body phase space element.

Grade

Your grade is given by the sum of points scored for each exercise, divided by 9, and rounded to the closest integer.