Particle physics problem sheet

- A pion with mass M is moving by velocity β in the lab frame and decays to two photons. Determine the relationship between the energies of the photons and their angles (relative to the motion of the pion)!
- A photon collides with a proton of mass M at rest in the lab frame. What is the necessary minimal energy for the photon such that in the final states we will have a neutral pion of mass M_0 in addition to the original proton?
- Under which conditions will the 4-momentum of a system of 2 photons correspond to zero mass?
- Consider the process $e^+e^- \to Z\gamma$. Find the processes related to this one by the crossing relations and also the kinematical bounds on the Mandelstam variables s, t and u!
- It is often useful to use the I_3 and step operators $I_\pm = I_1 \pm iI_2$ instead of the original I_j $j = 1, 2, 3$ isospin components. Show that the commutation relations are

$$[I_+, I_-] = 2I_3, \qquad [I_3, I_\pm] = \pm I_\pm$$

- The irreducible representation of $SU(2)$ labelled by j is spanned by the basis vectors $|j, m\rangle$ ($m \in -j, -j+1, \ldots, j$). The action of $I_\pm = I_1 \pm iI_2$ and I_3 on these is given by

$$I_\pm|j, m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}|j, m \pm 1\rangle, \quad I_3|j, m\rangle = m|j, m\rangle$$

  Construct the 3×3 representation matrices for I_1, I_2 and I_3 for the $j = 1$ representation!
- The Casimir operator of $SU(2)$ is given by $I = I_1^2 + I_2^2 + I_3^2$. Express I by I_3 and $I_\pm$ and using the expressions from the previous exercise show that

$$I|j, m\rangle = j(j + 1)|j, m\rangle$$

- Plot the pseudo-scalar meson octet consisting of a quark and anti-quark and the baryon octet consisting of 3 quarks on the I_3, Y plane and label each particle by its quark content (e.g. $\pi^+ = (u, \bar{d})$, $n = (d, d, u)$)!
- Consider the flavour $SU(3)$ decuplet in the I_3, Y plane. For each particle calculate $I(I + 1) - Y^2/4$ where I is the total isospin and show that it is a linear function of Y and find the coefficients α, β in $I(I+1)-Y^2/4 = \alpha Y+\beta$!
- Plot the states corresponding to the $\bar{3}$ representation of flavour $SU(3)$ in the I_3, Y plane and label each with its quark content! These states correspond to $\bar{q}$ anti-quarks.

- Plot the states corresponding to the 6 representation of flavour $SU(3)$ in the I_3, Y plane and label each with its quark content! The 6 representation is the 2-index-symmetric representation hence these states correspond to qq-states or diquarks.
- Determine which processes can take place and which ones can not:

$$\pi^+ \to e^+ e^- e^+ \qquad \pi^0 \to \gamma\gamma \qquad \mu^- \to e^- \gamma$$

$$K^+ \to \mu^+ \nu_\mu \qquad K^- + n \to \Sigma^- + \pi^0$$

- Let $a^\dagger(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ be the creation operators of free, 0 spin, mass m particles corresponding to a complex field. Show that the following states are eigenstates of the energy, momentum and charge operators and find the eigenvalues as well:

$$|ab\rangle = a^\dagger(\mathbf{p}) b^\dagger(-\mathbf{p})|0\rangle \qquad \text{és} \qquad |aa\rangle = a^\dagger(\mathbf{p}) a^\dagger(\mathbf{p})|0\rangle$$

- Let $b_1^\dagger$ and $b_2^\dagger$ be the creation operators of the ground state and excited state of an atom

$$|G\rangle = b_1^\dagger|0\rangle, \quad |X\rangle = b_2^\dagger|0\rangle, \quad b_1|0\rangle = 0 = b_2|0\rangle$$

  fulfilling the commutation relations $[b_i, b_j^\dagger] = \delta_{ij}$ $i, j = 1, 2$. Show that the following holds for $\Pi = b_1^\dagger b_2$ and $\Pi^\dagger = b_2^\dagger b_1$:

$$\Pi^\dagger|G\rangle = |X\rangle, \quad \Pi|X\rangle = |G\rangle, \quad \Pi^\dagger|X\rangle = 0, \quad \Pi|G\rangle = 0\ .$$

- Let ϕ be a real scalar field whose self-interaction is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + g\phi^3$$

  Determine the following
  (a) mass dimension of g,
  (b) the expression for the vertex in Feynman diagrams
  (c) lowest order Feynman diagrams for the $2 \to 2$ and the $2 \to 3$ scatterings
- Let $\phi(x)$ be a real scalar field and $\psi(x)$ a Dirac spinor field and assume that their interaction is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + \bar{\psi} i\gamma^\mu \partial_\mu \psi + M\bar{\psi}\psi + g\phi\bar{\psi}\psi$$

  (The last term is often called Yukawa interaction.) Determine the following
  (a) mass dimension of g
  (b) the expression for the vertex in Feynman diagrams
- Show that the time ordered product of spin 1/2 fields satisfies the equation

$$(i\gamma^\mu \partial_\mu^x - m)_{\alpha\beta} T(\psi_\beta(x)\bar{\psi}_\gamma(y)) = i\delta_{\alpha\gamma}\delta^{(4)}(x - y)$$