# *Introduction to flavor physics*

Biplab Dey

Eötvös Loránd University (ELTE)
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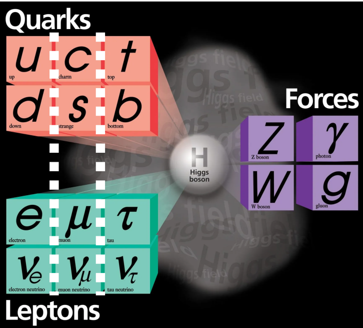
# Recap: The Standard Model

- Developed over the past $\sim 60$ years. Renormalizable QFT of gauge symmetries.



Electromagnetism (light)

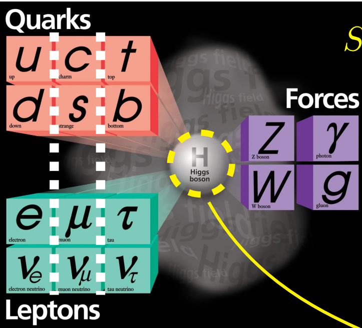
Weak (radioactivity)

Strong (solar fusion)

Three identical generations ("**flavors**") except for mass

# THE STANDARD MODEL (CONTD.)

- Higgs discovered in 2012 and have been intensely scrutinised since (couplings, width, mass...).

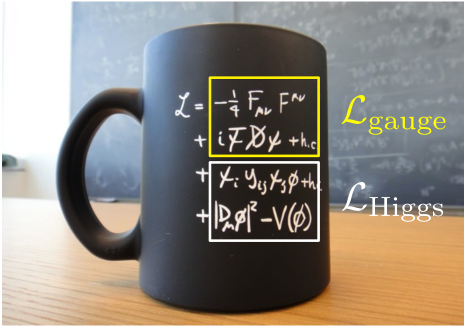


$$SU(3)_{QCD} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak broken by Higgs}}$$

Higgs couplings ⇒ particle masses and flavor mixing

Three identical generations (“**flavors**”) except for mass

# FLAVOR STRUCTURE

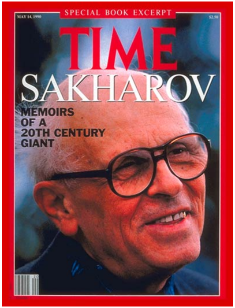


- $\mathcal{L}_{\text{gauge}} \Rightarrow$ symmetries. Beautiful!
- $\mathcal{L}_{\text{Higgs}} \Rightarrow$ "arbitrary" parameters (masses, couplings). Messy!

# MATTER-ANTIMATTER ASYMMETRY

- The Big Bang produced equal amounts of matter and antimatter.
- Outside of particle colliders, in cosmology, today's Universe is completely matter-dominated.
- Where did the antimatter go?

# THE SAKHAROV CONDITIONS



- In 1967, Andrei Sakharov gave three necessary conditions for baryon asymmetry in the universe (BAU).
- There has to be a process that includes:
  - Baryon number violation
  - Both $C$ and $CP$ violation
  - The process is out of thermal equilibrium.

- SM: EM and strong forces are $C$ and $P$ invariant.
- Only the weak interaction can provide $C$ and $CP$ violation.

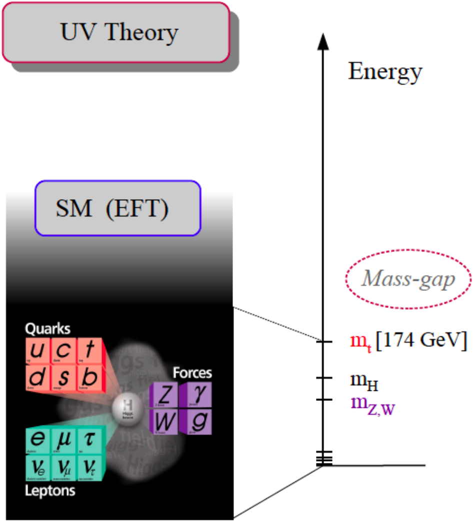
# KNOWN ISSUES WITH THE SM

- Neutrinos have zero mass in the SM. Neutrino oscillations ⇒ small but non-zero masses.
- What is the nature of Dark Matter and Dark energy?
- 19 free parameters with unexplained hierarchy.

# LESSONS FROM A DECADE OF THE LHC

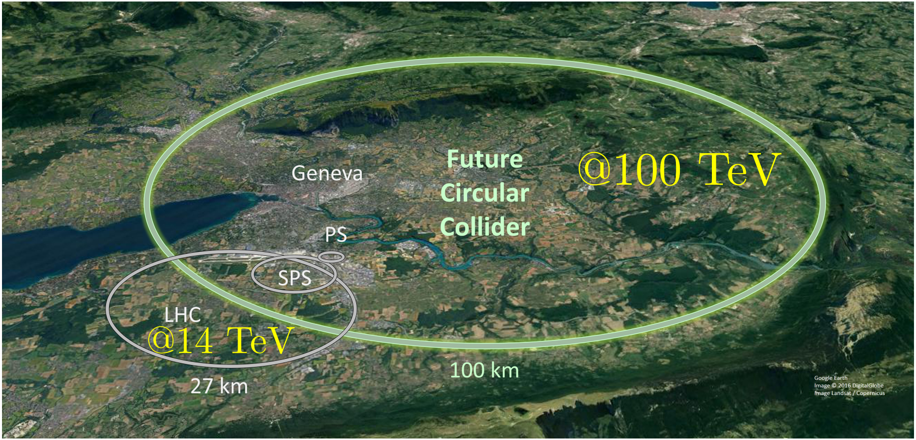
- The Higgs is light (hierarchy problem) and so-far behaves like the SM Higgs.
- Establishes EWSB (spontaneous symmetry breaking), but doesn't explain why.
- No new heavy particles yet in direct searches (SUSY, heavy $W/Z$...).
- Hints of NP via indirect searches in flavor anomalies.
- The SM works as an effectively theory (EFT) below TeV-scale and we have a fair understanding of the light degrees of freedom.

# How to access the full theory?



- Given rich physics we've discovered in the low-energy regime, it's weird that there be a "desert" till the Planck scale.
- LHC at the TeV scale ⇒ naturalness demands New Physics (NP) at the EWSB scale ($\sim 246$ GeV).
- If naturalness doesn't hold, then NP can be beyond direct searches at LHC.

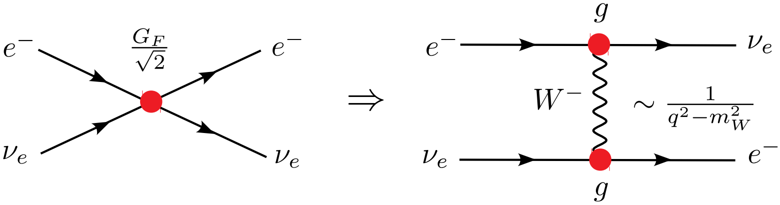
# WE'RE GONNA NEED A BIGGER BOAT



- Yes, but what else...

# INDIRECT SEARCHES

- Historically, all heavy particles were first surmised indirectly by their effects, prior to direct detection.
- Fermi effective theory of $\beta$-decay at low energies:



- $W/Z$ weak bosons are heavy: $G_F$ is small due to $1/m_W^2$ in the propagator.

# ELECTROWEAK UNIFICATION

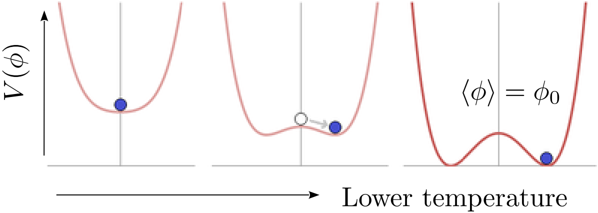
- The neutral $Z^0$ boson mixes with the photon. Whenever you exchange a photon, you can also exchange a $Z^0$ boson.
- On the other hand, $W^\pm$ shares the same charge as EM. Different from the strong interaction ⇒ gluons don't carry EM charge.
- Salam-Weinberg theory: extend $U(1)_{\text{QED}} \to SU(2)_L \times U(1)_Y$ for weak+EM. The couplings are $g$ and $g'$.
- $SU(2)_L$ is the weak-isospin and $U(1)_Y$ is the weak hypercharge.
- Isospin is kind of like spin: $I = 0$ is a singlet, $I = 1/2$ is a doublet, $I = 1$ is a triplet...

# GAUGE SECTOR AND MASS TERMS

- For any symmetry group, the gauge fields live in the adjoint representation. That is, number of fields = number of generators.
- For $SU(2)_L$, there are three generators (Pauli matrices), so three fields, $A_\mu^{1,2,3}$.
- For $U(1)_Y$, a single field, $B_\mu$.
- Gauge symmetry forbids mass terms, both for the matter and gauge boson sectors.
- If $P_{L,R} = (1 \mp \gamma^5)/2$ are projection operators, $\psi_{L,R} = P_{L,R}\psi$. Fermion mass term $m\overline{\psi}\psi = m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$. Under $SU(2)_L$, this is not invariant.
- So how do we give masses to particles?

# SPONTANEOUS SYMMETRY BREAKING

- Condensed matter: below a critical temperature, the ground state get a constant value (condensate) that breaks the $U(1)$ symmetry.



- $[e^-e^-]$ Cooper-pairs in superconductivity. Photon in the medium talks to the CPs and gets "effective" mass.

- Higgs mechanism: QFT generalisation of this idea to $SU(2)_L \times U(1)_Y$.

# THE HIGGS MECHANISM

- Introduce an elementary scalar field (what else?) that is a $\mathrm{SU}(2)_L$ doublet with hypercharge $Y = 1$.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- The ground state gets a vacuum expectation value (don't ask how/why – no one knows!), $v$.

$$\langle\phi\rangle_{\text{vev}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad \phi_{\text{physical}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

- The charge operator $Q = I_3 + Y/2 = (-\sigma^3 + I)/2$ leaves this state unchanged. All other combinations of $\{\sigma^{1,2,3}, I\}$ change the state.

# GIVING MASSES TO THE $W^{\pm}/Z^0$

- The vector boson "mass terms" with the Higgs vev come from

$$\frac{1}{2}\begin{pmatrix}0 & v\end{pmatrix}\left(g\frac{\sigma^i}{2}A^i_\mu + g'\frac{1}{2}B_\mu\right)^2\begin{pmatrix}0\\ v\end{pmatrix} = m_W^2(W^{+2}+W^{-2}) + m_Z^2 Z^2 + m_A^2 A^2$$

- $W^{\pm}_\mu = (A^1_\mu \pm iA^2_\mu)/\sqrt{s}$ with mass $m_W^2 = g^2v^2/4$
- The neutral bosons get masses $m_Z^2 = (g^2 + g'^2)v^2/4$, and $m_A = 0$, and the mixing is:

$$\begin{pmatrix}Z^0_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}A^3_\mu\\ B_\mu\end{pmatrix} \quad e = g\sin\theta_W \quad \tan\theta_W = g'/g$$

- From $\{G_F, m_W, m_Z\} \Rightarrow v \sim 246$ GeV, and is known as the electroweak scale.

# FERMION CONTENT IN $SU(2)_L \times U(1)_Y$

- Left(right)-handed fermions are doublets(singlets) under $SU(2)_L$.
- For $I = 1/2$ doublets, the generators are the Pauli matrices, $\sigma_{1,2,3}$. Eigenvectors for $\sigma_3$ are the basis states. EM charge $Q = I_3 + Y/2$.
- 5 fields × 3 generations:

$$L_e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L L_\mu = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L L_\tau = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \qquad I = \frac{1}{2}, Y = -1$$

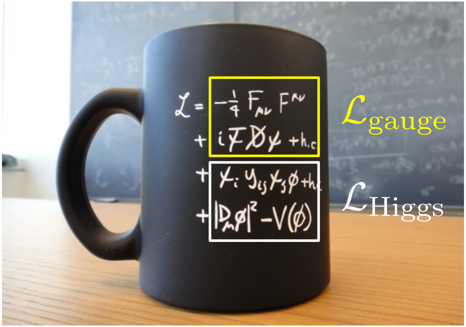
$$R_{e,\mu,\tau} = e_R, \mu_R, \tau_R \qquad I = 0, Y = -2$$

$$Q_L^1 = \begin{pmatrix} u \\ d \end{pmatrix}_L Q_L^2 = \begin{pmatrix} c \\ s \end{pmatrix}_L Q_L^3 = \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad I = \frac{1}{2}, Y = \frac{1}{3}$$

$$R_u = u_R, c_R, t_R \qquad I = 0, Y = \frac{4}{3}$$

$$R_d = d_R, s_R, b_R \qquad I = 0, Y = -\frac{2}{3}$$

# FLAVOR STRUCTURE



- $\mathcal{L}_{\text{gauge}}$ has huge flavor-degeneracy between the 3 generations.
- Global symmetry: $U(1)_L \times U(1)_B \times U(1)_Y \times U(3)^5_F$.
- The flavor-degeneracy is broken by the Higgs Yukawas.

# DIAGONALIZING THE QUARK YUKAWAS

- The quark masses and mixings all derive from the Yukawas

$$\mathcal{L}_{\text{Higgs}} \supset -\overline{Q}^i_L Y^{ik}_D d^k_R \phi - \overline{Q}^i_L Y^{ik}_U u^k_R \epsilon \phi^*$$

- $Y$ are $3 \times 3$ complex matrices, diagonalised as

$$V_D^\dagger Y_D U_D = \text{diag}(y_b, y_s, y_d) \qquad V_U^\dagger Y_U U_U = \text{diag}(y_t, y_c, y_u)$$

- The diagonal mass matrix elements are $y_i = \dfrac{m_{q_i}}{174 \text{ GeV}}$.

- Can't diagonalise $Y_{U,D}$ simultaneously. Remnant is the Cabibbo-Kobayashi-Maskawa mixing matrix $V_{\text{CKM}} = V_U V_D^\dagger$

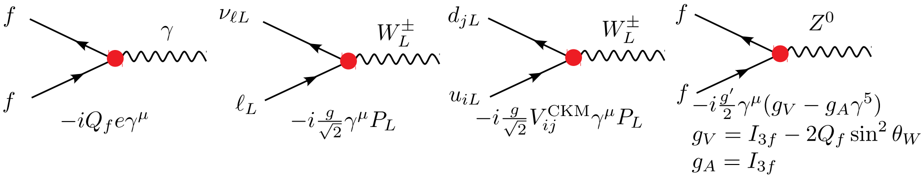
- By construction, unitarity: $V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$.

# MASS BASIS AND WEAK BASIS

- Mass or physical basis ⇒ stuff that propagates in time during flight. Mass from the Higgs.
- Weak or flavor basis ⇒ identifies a specific generation.
- The fact that the two are rotated leads to (quark) flavor mixing.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{\text{mass}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{flavor}}$$

# EW VERTICES SUMMARY



$-iQ_f e\gamma^\mu$

$-i\frac{g}{\sqrt{2}}\gamma^\mu P_L$

$-i\frac{g}{\sqrt{2}}V_{ij}^{\mathrm{CKM}}\gamma^\mu P_L$

$-i\frac{g'}{2}\gamma^\mu(g_V - g_A\gamma^5)$
$g_V = I_{3f} - 2Q_f \sin^2\theta_W$
$g_A = I_{3f}$

- $g = e/\sin\theta_W \sim 2e$. The weak *coupling* is not weak.
- No flavor changing neutral current (FCNC) at tree level.
- $b \to s$ and $b \to d$ neutral current transitions only at loop level.

# EXPLORING THE CKM MATRIX

- How many independent parameters if $N$ generations of quarks present?
- $N \times N$ complex matrix has $2N^2$ real parameters. Unitarity means $N^2$ parameters remaining.
- $2N-1$ phases are unphysical for the $2N$ quarks, sans overcounting 1 global phase.
- Remnant: $\frac{1}{2}N(N-1)$ rotation angles and $\frac{1}{2}(N-1)(N-2)$ phases.
- With the first two generations, single Cabibbo angle:

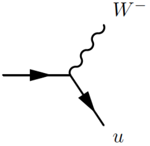
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

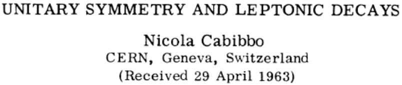
# GIM MECHANISM AND THE CHARM QUARK

- By the 60's, it was known that the $d$ and $s$ mixed.

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

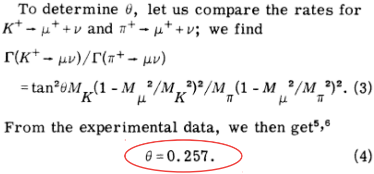
$d\cos\theta_C + s\sin\theta_C$





UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

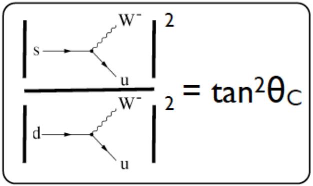


To determine $\theta$, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu) = \tan^2\theta M_K(1 - M_\mu{}^2/M_K{}^2)^2/M_\pi(1 - M_\mu{}^2/M_\pi{}^2)^2. \quad (3)$$

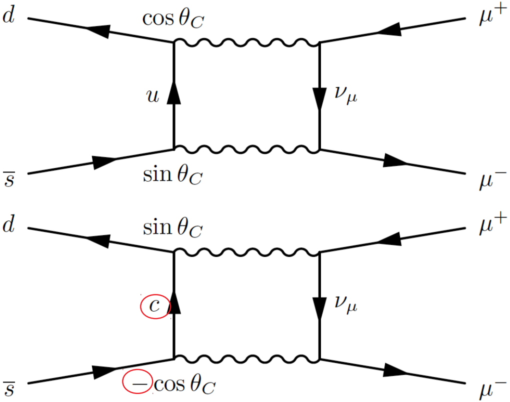
From the experimental data, we then get[5,6]

$$\theta = 0.257. \quad (4)$$



# GIM MECHANISM AND THE CHARM QUARK (CNTD.)

- However suppression of $K^0 \to \mu^+\mu^-$ could not be explained.
- GIM mechanism: add a fourth quark, charm.

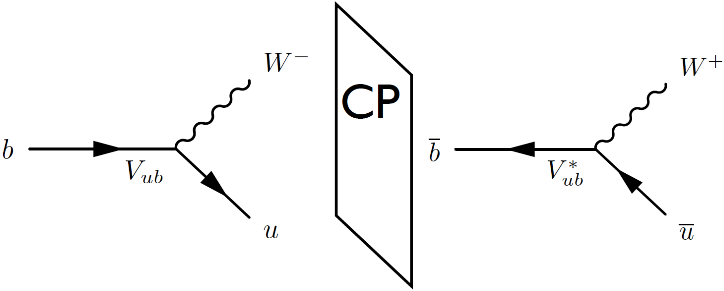


GIM cancellation due to 4th charm quark

Flavor changing neutral currents (FCNC) are suppressed

# THIRD GENERATION ⇒ CP VIOLATION

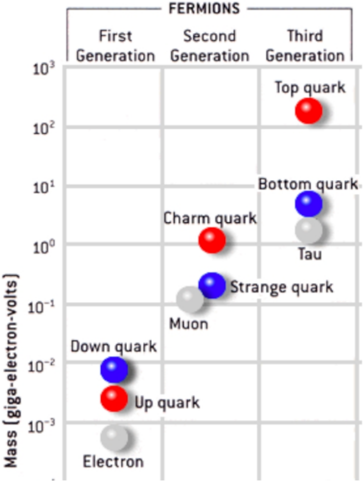
- With $N = 3$, three angles and one weak phase. Flips sign under CP-conjugation. Single source of CPV in the SM.



- To produce CPV, need two amplitudes with both a relative strong and weak phase.

$$|A_{\text{tot}}|^2 - |\overline{A}_{\text{tot}}|^2 \propto A_1 A_2 \sin \delta_W \sin \delta_S$$

# PATTERN OF HIERARCHY: MASS



- Both the mass and mixing matrices show a remarkable pattern of hierarchy.
- Higgs couplings to the different generations span five orders of magnitude!
- Unlike the gauge sector, Yukawas not fixed by any symmetry.

# Pattern of hierarchy: mixing

- Wolfenstein parameterisation of $V_{\text{CKM}}$ with $\lambda \sim 0.22$:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \mathcal{O}(\lambda^1)$$

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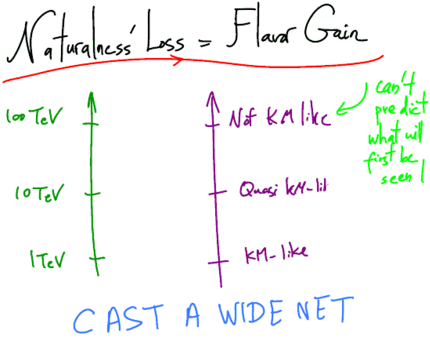
- Source of $CP$ violation in the SM.

# MINIMAL FLAVOR VIOLATION

- SM Higgs couples strongest to the 3rd (and 2nd) generation and breaks flavor-degeneracy.
- MFV ⇒ any NP should have similar structure in the couplings.
- Expect NP to show up in decays involving heavy $b$, $\tau$.



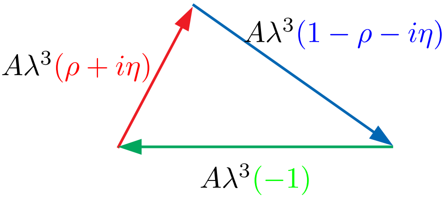
Nima Arkani Hamed
Intensity Frontier Workshop'11

# UNITARITY TRIANGLES

- That $V_{\rm CKM}$ is unitary leads to six “unitarity triangles”.
- Most are highly squashed, except for the $b \to d$ one:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



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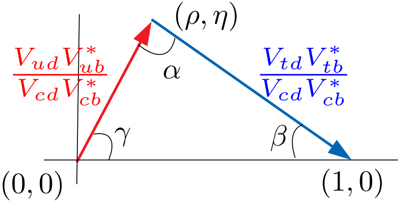
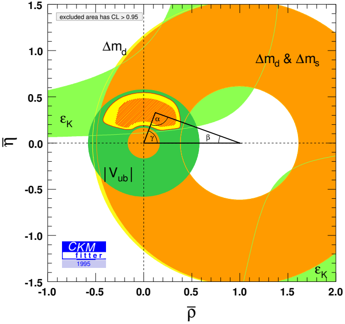


# UT CLOSURE

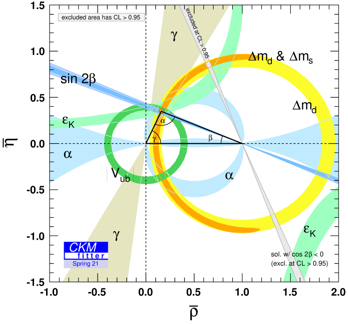
- Measure the sides and angles of the UT in all possible ways (loop-dominated, tree-level)...
- Do they all agree? Does the UT close “exactly”?

1995:



$\Longrightarrow$

2021:



- Fabulous progress, but still room for NP.

# AMOUNT OF $CPV$ IN THE SM

- Jarlskog parameter
  $J = |\mathrm{Im}(V_{us}V_{cb}V_{ub}^{*}V_{cs}^{*})| = \lambda^6 A^2 \eta (1-\lambda^2/2) \sim 10^{-5} = 2 \times \text{Area}_{\text{UT}}$
- Note: the $CPV$ phase $\eta$ enters directly.
- Also need mass splitting among the three generations:
  $P_U = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$, $P_D = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$

$$\text{BAU} \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma} = \left.\frac{P_U P_D J}{(m_W)^{12}}\right|_{\text{SM}} \approx 10^{-20}$$

- Observed $\text{BAU}_{\text{obs}} \sim 10^{-10} \Rightarrow$ huge discrepancy!

# WORD ABOUT LEPTONS

- Main difference is that that no $\nu_R$, so can diagonalise the charged leptons: $y_\ell[(\overline{L}_\ell\langle\phi\rangle)R_\ell + \overline{R}_\ell(\langle\phi\rangle^T L_\ell)]$
- Charged leptons get $m_\ell \sim y_\ell v$. Neutrinos get no mass in the SM.

- Direct measurement of $\nu_e$ mass by the KATRIN experiment puts the limit around 1 eV.
- However, neutrino oscillations mean that the masses can't be exactly zero and also mix.

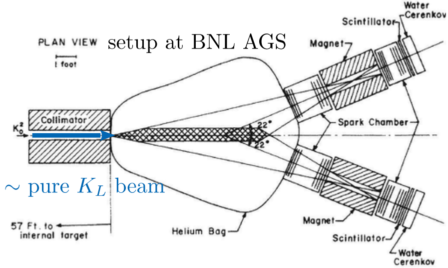
- Neutrino's in fact mix heavily (PMNS matrix), but the masses are so small, would require long distances (solar) to be detected.
- Effectively, in $B$-physics, care about only one lepton basis.

# $CP$ VIOLATION: THE DISCOVERY IN KAON SYSTEMS

- The kaon system had already shown itself to be potent source of discoveries.
- Existence of both $K^+ \to \pi^+\pi^0$ $(P = +1)$ and $K^+ \to \pi^+\pi^-\pi^+$ $(P = -1)$ showed parity not conserved (Lee and Yang).
- $CP$ violation was discovered by Cronin and Fitch in 1964 in neutral $K^0$ mesons at Brookhaven. Came as a great surprise!
- Two neutral $K^0$ mesons with seeming same mass, spin, parity, except one is long-lived ($K_L$, $c\tau \sim 2.7$ cm) and the other is short-lived ($K_S$, $c\tau \sim 15$ m).
- $K_S \to \pi^+\pi^-$ $(CP = +1)$ and $K_L \to \pi^+\pi^-\pi^0$ $(CP = -1)$. The $K_L$ decay has very little phase-space that explains the long lifetime.

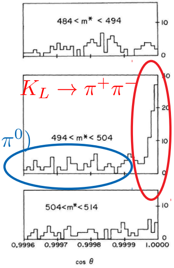
# $CP$ VIOLATION: DISCOVERY (CNTD.)

- Quark assignment: $|K^0\rangle = |d\bar{s}\rangle$. $CP|K^0\rangle = |\overline{K^0}\rangle = |\bar{d}s\rangle$
- $CP$ eigenstates are $|K_\pm\rangle = \frac{|K^0\rangle \pm |\overline{K^0}\rangle}{\sqrt{2}}$. W/o CPV, $K_{S,L} \equiv K_{+,-}$.
- Experimentally, $K_{S,L}$ separated by letting the $K^0$ beam propagate for $\sim 17$m $\Rightarrow$ almost pure $K_L$ beam.
- C&F found that out of $\sim 22700$ $K_L$ events, there were 45 $K_L \to \pi^+\pi^-$ candidates. CP violation!

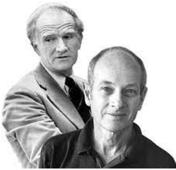

setup at BNL AGS

$\sim$ pure $K_L$ beam

$K_L \to \pi^+\pi^-(\pi^0)$ continuum

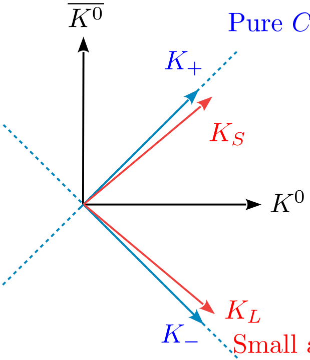

$K_L \to \pi^+\pi^-$


1980



# SM PICTURE OF THE KAON SYSTEM

- Physical (mass eigenstates) $K_{S,L}$ are not $CP$ eigenstates



Pure $CP$ eigenstates

$$|K_L\rangle = \frac{|K_+\rangle + \varepsilon|K_-\rangle}{\sqrt{1+|\varepsilon|^2}}$$

$$|K_S\rangle = \frac{|K_-\rangle + \varepsilon|K_+\rangle}{\sqrt{1+|\varepsilon|^2}}$$

$$|\varepsilon| \ll 1$$

$$\langle K_S|K_L\rangle = \frac{2\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}$$

Small admixture $\varepsilon$ induces $CPV$

- NB: amount of $CPV$ here is quite small ($|\varepsilon| \sim 10^{-3}$) unlike parity, which is maximally violated.

# NEUTRAL MESON OSCILLATIONS

- Let's recast the notation as follows

$$\begin{pmatrix} K_S \\ K_L \end{pmatrix} = Q \begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}, Q = \begin{pmatrix} p & q \\ p & -q \end{pmatrix} \qquad \begin{aligned} p &= (1+\varepsilon)/(\sqrt{2}\sqrt{1+\varepsilon|^2}) \\ q &= (1-\varepsilon)/(\sqrt{2}\sqrt{1+\varepsilon|^2}) \end{aligned}$$

- Time-evolution of the mass eigenstates is given by a Schrodinger's equation using an effective Hamiltonian in the flavor basis:

$$i\frac{\partial}{\partial t}\psi(t) = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \psi(t),$$

- Solutions of the form $\psi_\pm(t) = e^{i\omega_\pm t}\begin{pmatrix} p \\ \pm q \end{pmatrix}$ with $\omega_\pm = m_\pm - \frac{i}{2}\Gamma_\pm$
  - $m_\pm = M \pm \frac{1}{2}\Delta m, \Gamma_\pm = \Gamma \pm \frac{1}{2}\Delta\Gamma$
  - $\{\Delta m, \Delta\Gamma\} = 2\{\mathrm{Re}, \mathrm{Im}\}\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}$
  - $(q/p)^2 = \sqrt{(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)/M_{12} - \frac{i}{2}\Gamma_{12})}$

# NEUTRAL MESON OSCILLATIONS

- Time-evolution of the mass eigenstates:

$$\begin{pmatrix} K_S(t) \\ K_L(t) \end{pmatrix} = \begin{pmatrix} e^{-i\omega_S t} & 0 \\ 0 & e^{-i\omega_L t} \end{pmatrix} \begin{pmatrix} K_S(0) \\ K_L(0) \end{pmatrix},\ \omega_i = m_i - i\Gamma_i/2$$

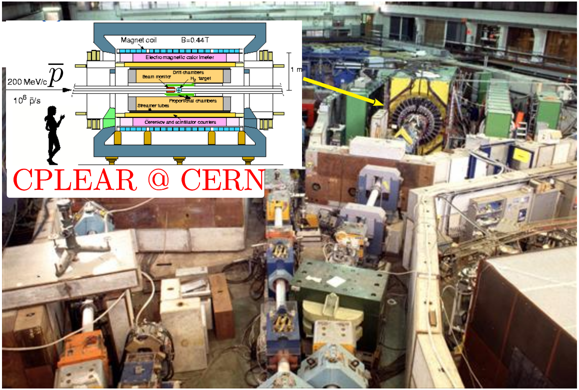
- Translate this to time evolution of the flavor eigenstates:

$$\begin{aligned} \begin{pmatrix} K^0(t) \\ \overline{K^0}(t) \end{pmatrix} &= Q^{-1} \begin{pmatrix} e^{-i\omega_S t} & 0 \\ 0 & e^{-i\omega_L t} \end{pmatrix} Q \begin{pmatrix} K^0(0) \\ \overline{K^0}(0) \end{pmatrix} \\ &= \begin{pmatrix} g_+(t) & \frac{q}{p} g_-(t) \\ \frac{p}{q} g_-(t) & g_+(t) \end{pmatrix} \begin{pmatrix} K^0(0) \\ \overline{K^0}(0) \end{pmatrix},\ g_\pm(t) = \frac{e^{-i\omega_S t} \pm e^{-i\omega_L t}}{2} \end{aligned}$$

- The $g_-(t)$ terms lead to matter-antimatter oscillations!

# $K^0$-$\overline{K^0}$ OSCILLATIONS: EXPERIMENTAL DISCOVERY

- Flavor tagging: need to know the flavor at $t = 0$.

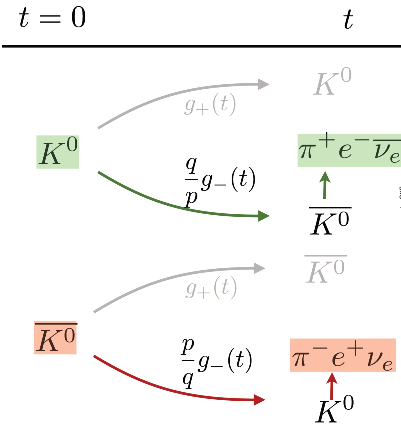


$$p\overline{p} \to \begin{cases} \pi^- K^+ \overline{K^0} \\ \pi^+ K^- K^0 \end{cases}$$

- The $K^\pm$ tags the $K^0$ flavor at production.

# $K^0$-$\overline{K^0}$ OSCILLATIONS: EXPERIMENTAL DISCOVERY

- Electron charge tags the flavor at decay.



$t = 0$

$t$

$K^0$

$g_+(t)$

$K^0$

$\frac{q}{p} g_-(t)$

$\pi^+ e^- \overline{\nu_e}$

$\overline{K^0}$

$\overline{K^0}$

$g_+(t)$

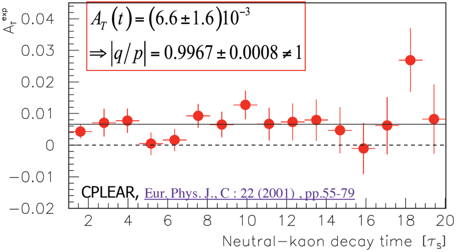
$\overline{K^0}$

$\frac{p}{q} g_-(t)$

$\pi^- e^+ \nu_e$

$K^0$

$$A_T(t) = \frac{\overline{I}_{\pi^- e^+ \nu}(t) - I_{\pi^+ e^- \overline{\nu}}(t)}{\overline{I}_{\pi^- e^+ \nu}(t) + I_{\pi^+ e^- \overline{\nu}}(t)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} = 4\mathcal{R}\epsilon$$



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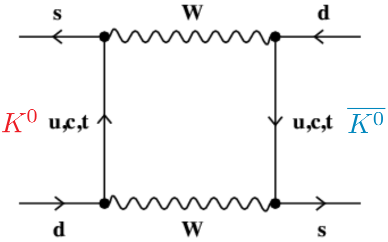
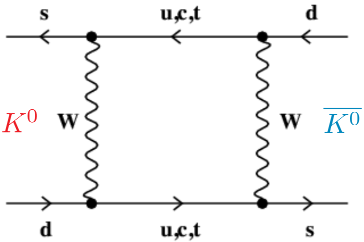
- $CP$-violation (also $T$-violation) in mixing ⇒ indirect $CPV$.

# THE “BOX DIAGRAMS”

- Microscopic picture of the mixing process:

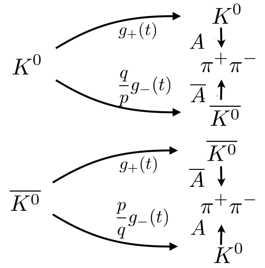




- Occurs only at the loop level. New heavy particles can be exchanged in the loop.
- Therefore these processes are very sensitive to New Physics (NP) outside the SM.

# THREE TYPES OF $CPV$ FOR NEUTRAL MESONS

| $t=0$ | $t$ | Amplitude |
|---|---|---|



$$A\left[g_+(t)+\lambda g_-(t)\right]$$

$$\overline{A}\left[g_+(t)+\tfrac{1}{\lambda}g_-(t)\right]$$

$$A \equiv \langle \pi^+\pi^-|K^0\rangle,$$
$$\overline{A} \equiv \langle \pi^+\pi^-|\overline{K^0}\rangle,$$
$$\lambda \equiv \tfrac{q}{p}\tfrac{\overline{A}}{A}$$

$$\Gamma(K^0\to\pi^+\pi^-) \propto |A|^2\left[|g_+(t)|^2+|\lambda|^2|g_-(t)|^2+2\mathrm{Re}(\lambda g_+^*(t)g_-(t))\right]$$

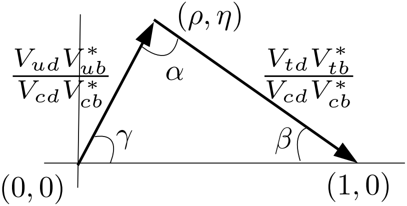
$$\Gamma(\overline{K^0}\to\pi^+\pi^-) \propto |\overline{A}|^2\left[|g_+(t)|^2+|\frac{1}{\lambda}|^2|g_-(t)|^2+\frac{2}{|\lambda|^2}\mathrm{Re}(\lambda^* g_+^*(t)g_-(t))\right]$$

- $CPV$ in decay (or direct $CPV$): $\frac{A}{\overline{A}} \neq 1$
- $CPV$ in mixing (or indirect $CPV$): $\frac{q}{p} \neq 1$
- $CPV$ in interference between mixing and decay: $\mathrm{Im}\lambda \neq 0$

# $B^0_{(s)}$ OSCILLATIONS, AKA TIME-DEPENDENT CPV

- $B^0 = |\bar{b}d\rangle$, $B^0_s = |\bar{b}s\rangle$ oscillations in decays to self-conjugate final states, called $f_{\rm CP}$, sensitive to $CPV$ weak phases $\lambda_f = e^{-i\phi_{\rm weak}}$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix}$$



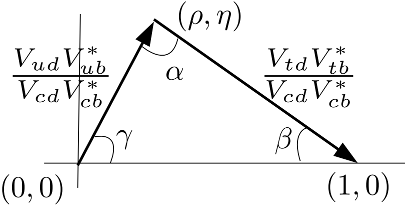
$$B^0 \to J/\psi\, K^0_S : 2\beta, \quad B^0 \to \pi^+\pi^- : 2(\beta+\gamma)$$

$$B^0_s \to J/\psi\, \phi : 2\beta_s, \quad B^0_s \to K^+K^- : 2(\beta_s+\gamma)$$

# $B^0_{(s)}$ OSCILLATIONS, AKA TIME-DEPENDENT CPV

- $B^0 = |\bar{b}d\rangle$, $B_s^0 = |\bar{b}s\rangle$ oscillations in decays to self-conjugate final states, called $f_{\mathrm{CP}}$, sensitive to $CPV$ weak phases $\lambda_f = e^{-i\phi_{\mathrm{weak}}}$

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix}$$



$B^0 \to J/\psi\, K_S^0 : 2\beta$, $\quad B^0 \to \pi^+\pi^- : 2(\beta+\gamma)$

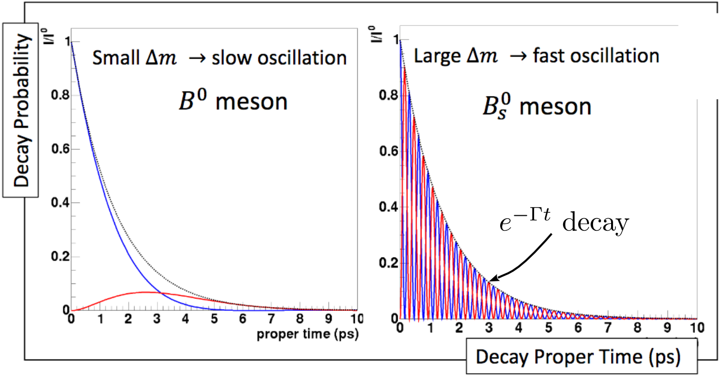
$B_s^0 \to J/\psi\, \phi : 2\beta_s$, $\quad B_s^0 \to K^+K^- : 2(\beta_s+\gamma)$

- Flagship analyses at the $B$-factories.

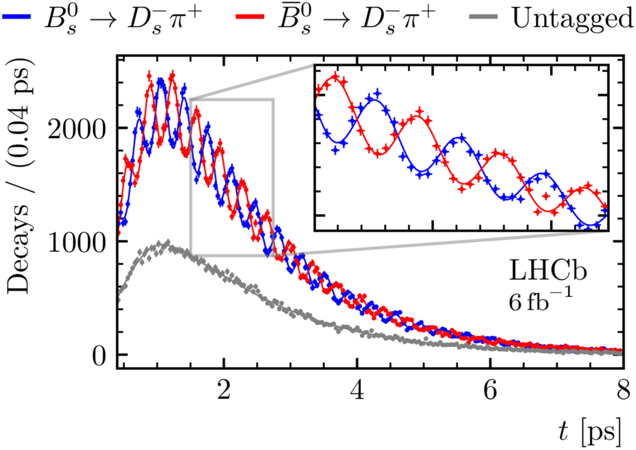
- In the SM, $\beta_s \sim 0$. Very sensitive to penguins. $B_s$ accessible only at the LHC.

# TDCPV IN $B$-DECAYS (CONTD.)

- For $B^0_{(s)}$ decays, $|q/p| \sim 1$. For $B^0$, $\Delta\Gamma \sim 0$ and $\Delta m$ is small (slower oscillations).
- For $B^0_s$, $\Delta m$ is large (faster oscillations).

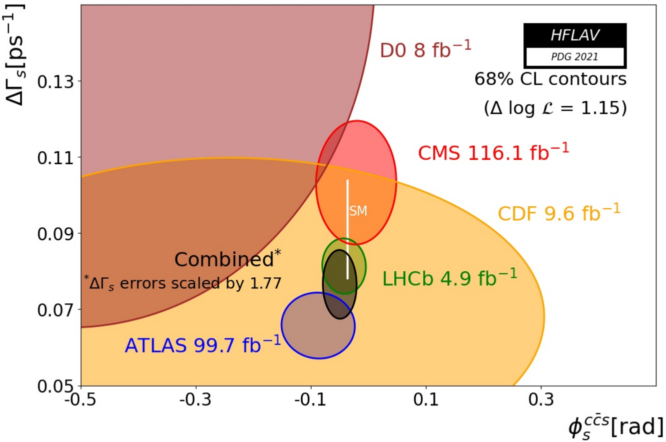


# LHCb data



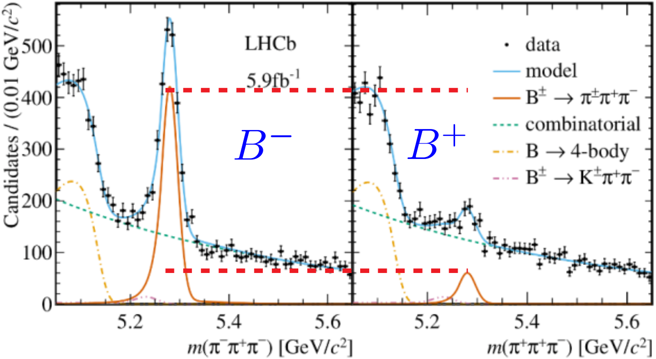
- Beautiful demonstration of matter-antimatter oscillations: interference between mixing and decay in the $B_s^0$ system.

# SM COMPATIBILITY: $B_s^0$ SYSTEM



- Compatible with SM global fits. Mild tension between experiments.

# ALSO, DIRECT $CPV$...



- Direct $CP$ violation in charged $B$ mesons.