Particle physics: lecture 4
Weak interactions II

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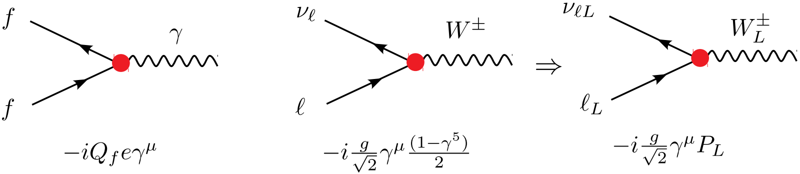
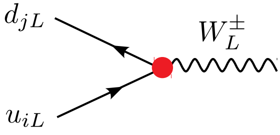
25^{th} October 2022

Electroweak theory: recap



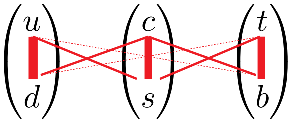
- EM: acts only on charged fermions (neutrinos have no charge) and respects C and P.
- Weak charged current breaks C and P maximally. No RH $W^\pm_R$ bosons seen.
- $g = e/\sin\theta_W \sim 2e$. The weak *coupling* is not weak.
- Till now, no mixing between generations $\Rightarrow$ e^- couples to ν_e and not ν_μ or ν_τ.

Flavor-changing charged weak currents



$$-i\frac{g}{\sqrt{2}}V_{ij}^{\text{CKM}}\gamma^{\mu}P_L$$

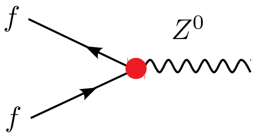
- Changes up-type to down-type quarks.
- Unlike in leptons, generations can change via the 3×3 unitary matrix V_{ij}^{CKM}.



- Up-quark couples strongest to down-quark, but also to the strange- and top-quarks.
- This is called quark-mixing.

- In the lepton sector, the SM predicts no mixing. But this is actually not true $\Rightarrow$ neutrino oscillations.

Weak neutral current



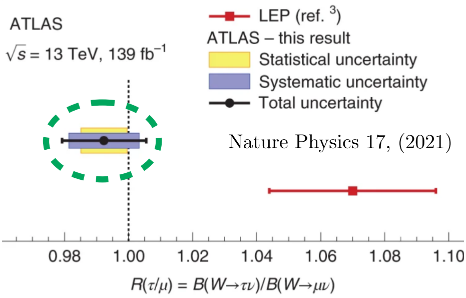
$-i\frac{g'}{2}\gamma^\mu(g_V - g_A\gamma^5)$
$g_V = I_{3f} - 2Q_f \sin^2\theta_W$
$g_A = I_{3f}$

- In the electroweak theory, the photon and Z^0 boson mix. $g' = g\tan\theta_W$.
- The neutral current also breaks parity, but $g_{A,V}$ depends on the fermion species. $I_3 = +1/2$ for neutrinos, etc.

- Generally, whenever a photon can be emitted, a Z^0 can also be emitted. Additionally, $\nu \to \nu Z^0$ weak interaction possible.
- For quarks, flavor changing neutral currents don't appear at tree level. That is, Z^0 does not induce quark-mixing.

Flavor Universality

- A bedrock of the SM formulation is that the couplings $\{e, g, g'\}$ are universal among all fermions.
- EM charge of proton is *exactly* same as that of electron.



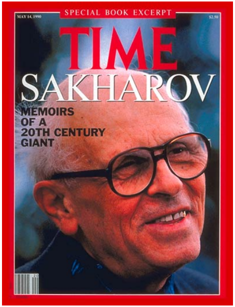
Nature Physics 17, (2021)

- Weak decays: recent test of the W^- coupling to μ^- vs. τ^- lepton.

Matter-antimatter asymmetry

- The Big Bang produced equal amounts of matter and antimatter.
- Outside of particle colliders, in cosmology, today's Universe is completely matter-dominated.
- Where did the antimatter go?

The Sakharov conditions

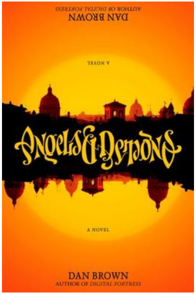


- In 1967, Andrei Sakharov gave three necessary conditions for baryon asymmetry.
- There has to be a process that includes:
  - Baryon number violation
  - Both C and CP violation
  - The process is out of thermal equilibrium.

- SM: EM and strong forces are C and P invariant.
- Only the weak interaction can provide C and CP violation.

What exactly does CP conjugation do?

- Changes LH particles to RH antiparticles.
- Note: the operators $P_{L,R} = (1 \mp \gamma^5)/2$ project out the chiral parts of the full wave function.
- Helicity means projection of spin along the momentum direction.



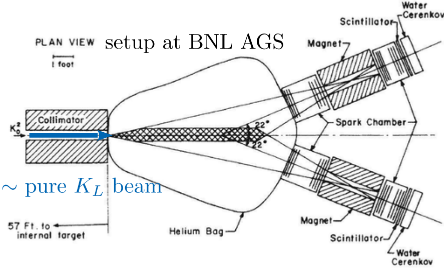
- For massless particles (eg. neutrino), helicity = chirality.
- For massive particles, helicity is frame dependent since a boost can reverse the momentum direction. helicity $\neq$ chirality
- Finally, the spin is an axial vector and does not change under CP.

CP violation: the discovery in kaon systems

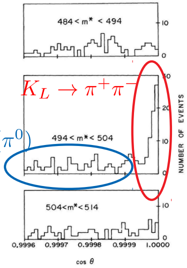
- The kaon system had already shown itself to be potent source of discoveries.
- Existence of both $K^+ \to \pi^+\pi^0$ ($P = +1$) and $K^+ \to \pi^+\pi^-\pi^+$ ($P = -1$) showed parity not conserved (Lee and Yang).
- CP violation was discovered by Cronin and Fitch in 1964 in neutral K^0 mesons at Brookhaven. Came as a great surprise!
- Two neutral K^0 mesons with seeming same mass, spin, parity, except one is long-lived (K_L, $c\tau \sim 2.7$ cm) and the other is short-lived (K_S, $c\tau \sim 15$ m).
- $K_S \to \pi^+\pi^-$ ($CP = +1$) and $K_L \to \pi^+\pi^-\pi^0$ ($CP = -1$). The K_L decay has very little phase-space that explains the long lifetime.

CP VIOLATION: DISCOVERY (CNTD.)

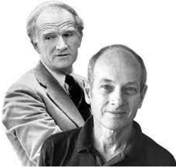
- Quark assignment: $|K^0\rangle = |d\bar{s}\rangle$. $CP|K^0\rangle = |\overline{K^0}\rangle = |\bar{d}s\rangle$
- CP eigenstates are $|K_\pm\rangle = \frac{|K^0\rangle \pm |\overline{K^0}\rangle}{\sqrt{2}}$. W/o CPV, $K_{S,L} \equiv K_{+,-}$.
- Experimentally, $K_{S,L}$ separated by letting the K^0 beam propagate for $\sim$ 17m $\Rightarrow$ almost pure K_L beam.
- C&F found that out of $\sim$ 22700 K_L events, there were 45 $K_L \to \pi^+\pi^-$ candidates. CP violation!



$K_L \to \pi^+\pi^-(\pi^0)$ continuum

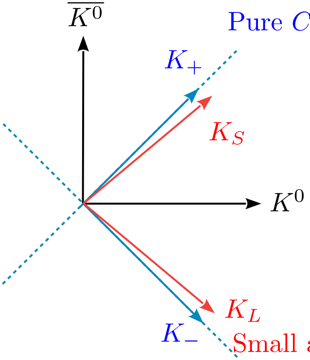


1980



SM picture of the kaon system

- Physical (mass eigenstates) $K_{S,L}$ are not CP eigenstates



Pure CP eigenstates

$$|K_L\rangle = \frac{|K_+\rangle + \varepsilon|K_-\rangle}{\sqrt{1+|\varepsilon|^2}}$$

$$|K_S\rangle = \frac{|K_-\rangle + \varepsilon|K_+\rangle}{\sqrt{1+|\varepsilon|^2}}$$

$$|\varepsilon| \ll 1$$

$$\langle K_S|K_L\rangle = \frac{2\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}$$

Small admixture ε induces CPV

- NB: amount of CPV here is quite small ($|\varepsilon| \sim 10^{-3}$) unlike parity, which is maximally violated.

Neutral meson oscillations

- Let's recast the notation as follows

$$|K_S\rangle = p|K^0\rangle + q|\overline{K^0}\rangle \qquad p = (1+\varepsilon)/(\sqrt{2}\sqrt{1+\varepsilon|^2})$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K^0}\rangle \qquad q = (1-\varepsilon)/(\sqrt{2}\sqrt{1+\varepsilon|^2})$$

- Time-evolution of the mass eigenstates:

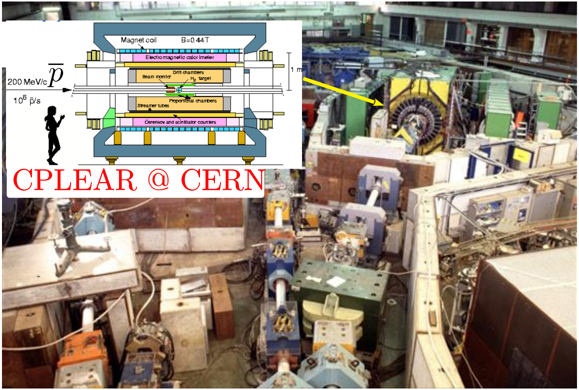
$$\begin{pmatrix} K_S(t) \\ K_L(t) \end{pmatrix} = \begin{pmatrix} e^{-i\omega_S t} & 0 \\ 0 & e^{-i\omega_L t} \end{pmatrix} \begin{pmatrix} K_S(0) \\ K_L(0) \end{pmatrix},\ \omega_i = m_i - \Gamma_i/2$$

- Translate this to time evolution of the flavor eigenstates:

$$\begin{pmatrix} K^0(t) \\ \overline{K^0}(t) \end{pmatrix} = \begin{pmatrix} g_+(t) & \frac{p}{q} g_-(t) \\ \frac{q}{p} g_-(t) & g_+(t) \end{pmatrix} \begin{pmatrix} K^0(0) \\ \overline{K^0}(0) \end{pmatrix},\ g_\pm(t) = \frac{e^{-i\omega_S t} + e^{-i\omega_L t}}{2}$$

- The $g_-(t)$ terms lead to matter-antimatter oscillations!

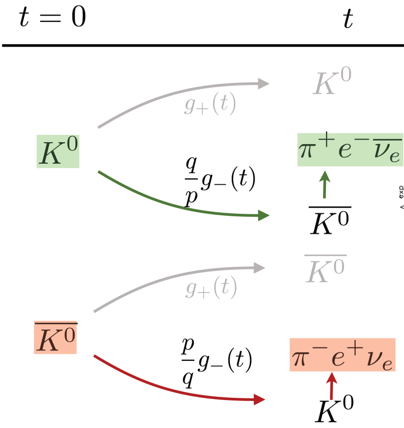
- Flavor tagging: need to know the flavor at $t = 0$.



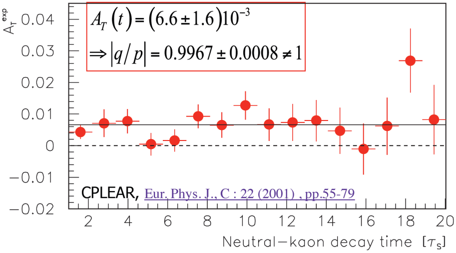
$$p\overline{p} \rightarrow \begin{cases} \pi^- K^+ \overline{K^0} \\ \pi^+ K^- K^0 \end{cases}$$

- The $K^{\pm}$ tags the K^0 flavor at production.

- Electron charge tags the flavor at decay.



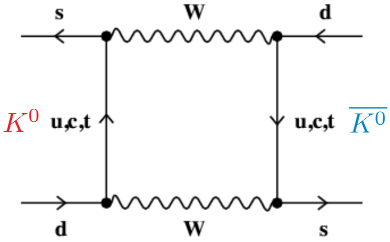
$$A_T(t) = \frac{\overline{I}_{\pi^- e^+ \nu}(t) - I_{\pi^+ e^- \overline{\nu}}(t)}{\overline{I}_{\pi^- e^+ \nu}(t) + I_{\pi^+ e^- \overline{\nu}}(t)} = \frac{1-|q/p|^4}{1+|q/p|^4} = 4\mathcal{R}\epsilon$$



- CP-violation (also T-violation) in mixing ⇒ indirect CPV.

THE "BOX DIAGRAMS"

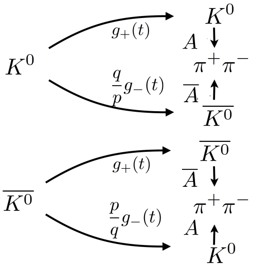
- Microscopic picture of the mixing process:



- Occurs only at the loop level. New heavy particles can be exchanged in the loop.
- Therefore these processes are very sensitive to New Physics (NP) outside the SM.

Three types of CPV for neutral mesons

$t = 0$ t **Amplitude**



$$A\left[g_+(t) + \lambda g_-(t)\right]$$

$$\overline{A}\left[g_+(t) + \tfrac{1}{\lambda}g_-(t)\right]$$

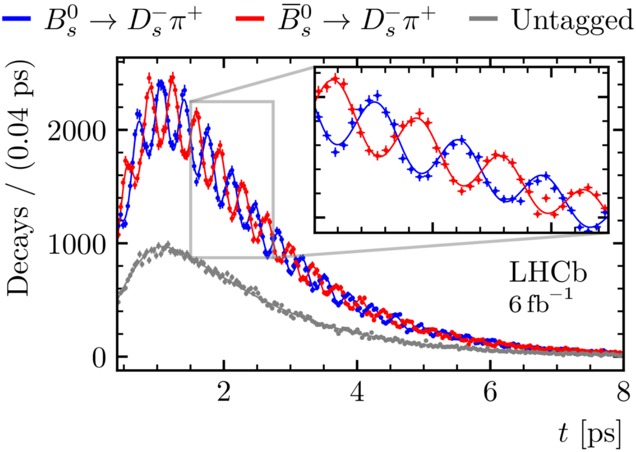
$$A \equiv \langle \pi^+\pi^- | K^0 \rangle,$$
$$\overline{A} \equiv \langle \pi^+\pi^- | \overline{K^0} \rangle,$$
$$\lambda \equiv \frac{q}{p}\frac{\overline{A}}{A}$$

$$\Gamma(K^0 \to \pi^+\pi^-) \propto |A|^2 \left[|g_+(t)|^2 + |\lambda|^2 |g_-(t)|^2 + 2\mathrm{Re}(\lambda g_+^*(t) g_-(t))\right]$$

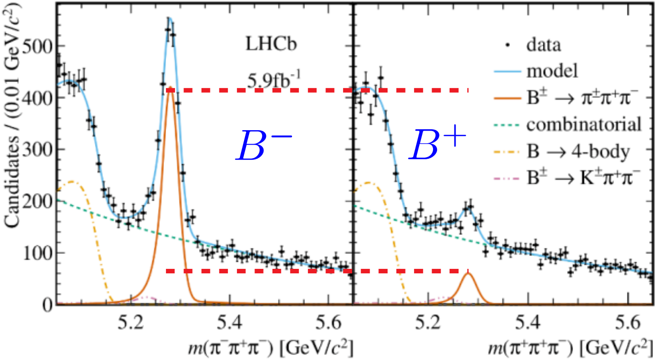
$$\Gamma(\overline{K^0} \to \pi^+\pi^-) \propto |\overline{A}|^2 \left[|g_+(t)|^2 + |\frac{1}{\lambda}|^2 |g_-(t)|^2 + \frac{2}{|\lambda|^2}\mathrm{Re}(\lambda^* g_+^*(t) g_-(t))\right]$$

- CPV in decay (or direct CPV): $\frac{A}{\overline{A}} \neq 1$
- CPV in mixing (or indirect CPV): $\frac{q}{p} \neq 1$
- CPV in interference between mixing and decay: $\mathrm{Im}\lambda \neq 0$

Examples...



- Beautiful demonstration of matter-antimatter oscillations: interference between mixing and decay in the B_s^0 system.



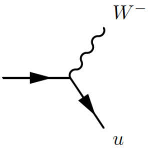
- Direct CP violation in charged B mesons.

GIM mechanism and the charm quark

- By the 60's, it was known that the d and s mixed.

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$d\cos\theta_C + s\sin\theta_C$



UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland

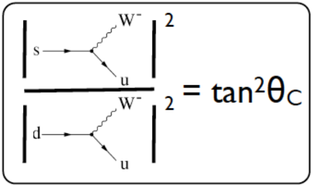
(Received 29 April 1963)

To determine θ, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu) = \tan^2\theta M_K (1 - M_\mu^2/M_K^2)^2 / M_\pi (1 - M_\mu^2/M_\pi^2)^2. \quad (3)$$

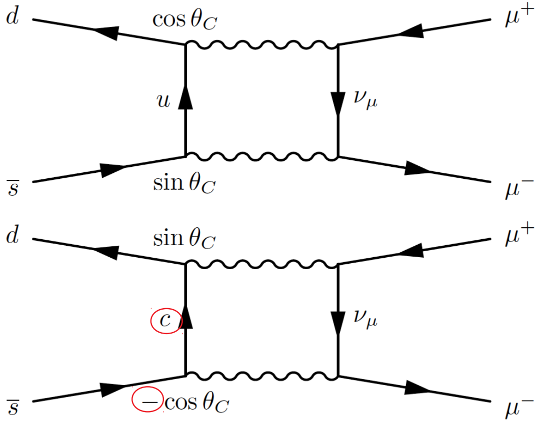
From the experimental data, we then get[5,6]

$$\theta = 0.257. \quad (4)$$



GIM mechanism and the charm quark (cntd.)

- However suppression of $K^0 \to \mu^+\mu^-$ could not be explained.
- GIM mechanism: add a fourth quark, charm.

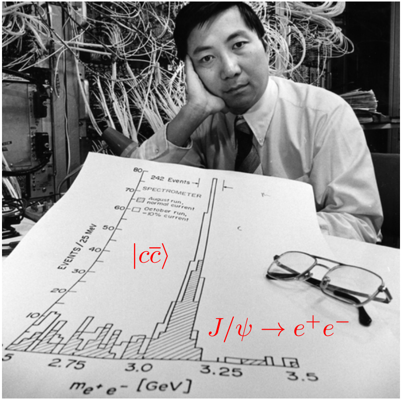


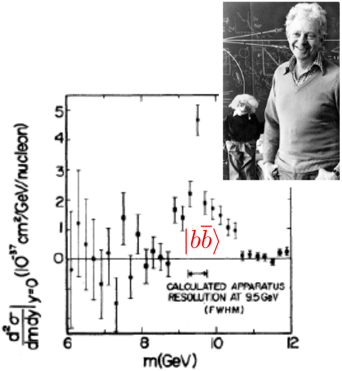
GIM cancellation due to 4th charm quark

Flavor changing neutral currents (FCNC) are suppressed

Heavy quark discoveries

- Charm was simultaneously discovered at SLAC (Richter) and Ting (BNL) in Nov. 1974.
- Beauty was discovered by Lederman *et al.* at FermiLab in 1977.
- Heaviest top quark discovered much later in 1994.

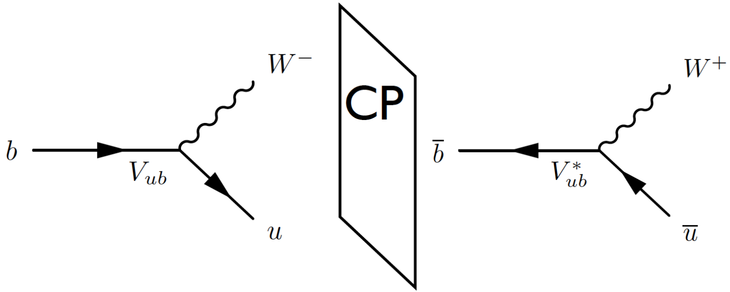




V^{CKM}: source of CPV in the SM

- For N quark generations, $N \times N$ complex matrix ($2N^2$ real parameters)
- $V^\dagger V = 1$ (unitarity condition) removes N^2 parameters. $2N-1$ parameters can be absorbed into the phases of the quarks.
- Left are $(N-1)^2$ physical parameters. $\frac{N}{2}(N-1)$ are quark rotation angles and $\frac{1}{2}(N-1)(N-2)$ are irreducible CPV phases.
- For $N=2$, one rotation angle and no CPV
- For $N=3$, three angles and one weak phase. Beauty and top quarks
- *Need at least three generations to produce CPV.* (Nobel prize to Kobayashi and Maswaka, 2008)

Effect of CP-conjugation



- All weak phases will flip sign under CP-conjugation
- To produce CPV in the SM, need two amplitudes with both a relative strong and weak phase.

$$|A_{\rm tot}|^2 - |\overline{A}_{\rm tot}|^2 \propto A_1 A_2 \sin \delta_W \sin \delta_S$$

CKM HIERARCHY AND THE FLAVOR PUZZLE

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

Parametrization of the Kobayashi-Maskawa Matrix

Lincoln Wolfenstein
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 22 August 1983)

The quark mixing matrix (Kobayashi-Maskawa matrix) is expanded in powers of a small parameter λ equal to $\sin\theta_c = 0.22$. The term of order λ^2 is determined from the recently measured B lifetime. Two remaining parameters, including the *CP*-nonconservation effects, enter only the term of order λ^3 and are poorly constrained. A significant reduction in the limit on ϵ'/ϵ possible in an ongoing experiment would tightly constrain the *CP*-nonconservation parameter and could rule out the hypothesis that the only source of *CP* nonconservation is the Kobayashi-Maskawa mechanism.

PACS numbers: 11.30.Er, 12.10.Ck, 13.25.+m

Strongly hierarchical

CPV

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Why $m_t \sim 173$ GeV, $m_{u,d} \sim$MeV; CKM hierarchy: flavor puzzle.

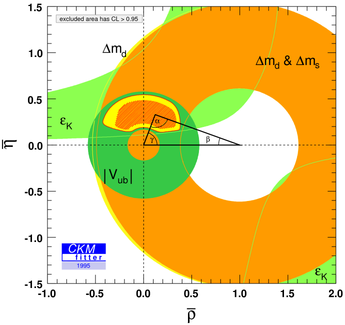
Precision tests: unitarity triangle closure

- V^{CKM} is unitary: $\sum_k V_{ik} V^*_{kj} = 1$, for any combination of ij.
- Results in 6 unitary relations or unitary triangles (UT). Several are highly squashed.
- "The" unitary triangle: $V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$. The three terms are comparable in magnitude.
- Area of the UT's are the same and is a measure of the amount of CPV in the SM.
- SM prediction is 10^{10} smaller than required to explain baryogenesis $\Rightarrow$ new sources of CPV must exist.

UT Closure

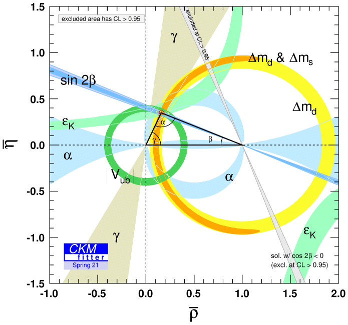
- Measure the sides and angles of the UT in all possible ways (loop-dominated, tree-level)...
- Do they all agree? Does the UT close "exactly"?

1995:



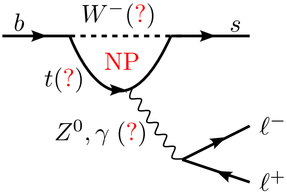
$\Longrightarrow$

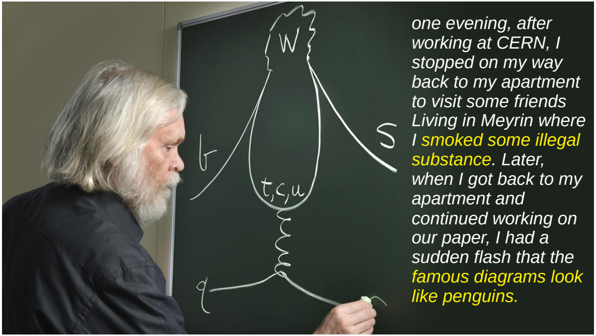
2021:



- Fabulous progress, but still room for NP.

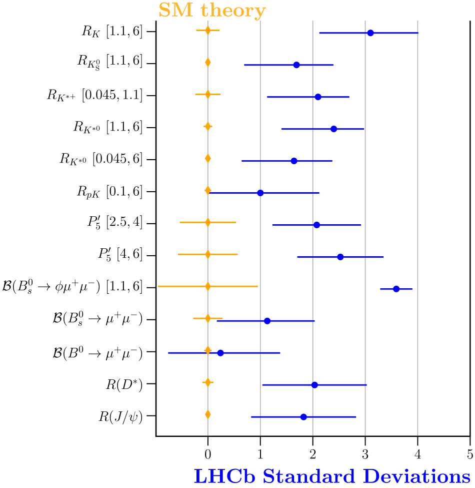
- Flavor changing neutral currents: only via loops in SM





- Special role in NP-hunting in the flavor sector. Effectively a 4-fermion interaction (recall Fermi theory of β-decay).
- New heavy particles (like W boson) can leave imprints.

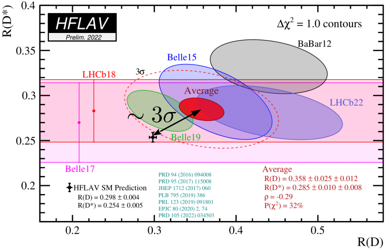
A palette of anomalies in b-quark decays

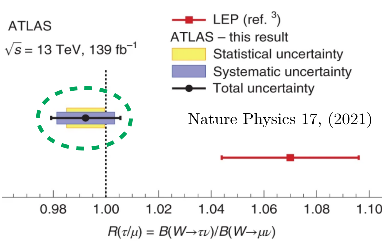


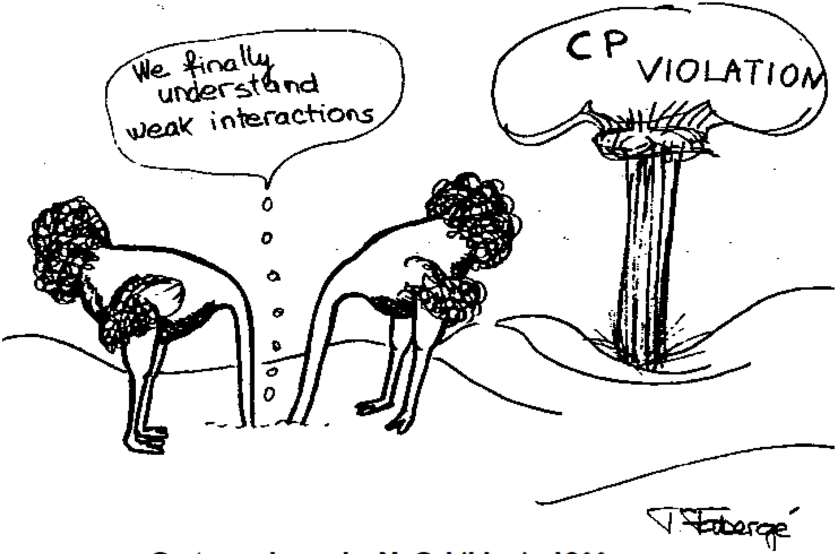
- What do these tell us?
- Possibly new particles/forces beyond TeV scale that can't be produced directly at the LHC, but can be felt indirectly.
- Why b-quarks? New particles can couple to heaviest (third generation), if CKM-like hierarchy.

Back to universality

- In b-decays, more taus seen than muons. Contradicts the ATLAS $W^- \to \ell^- \overline{\nu}_\ell$ data.







Cartoon shown by N. Cabibbo in 1966...