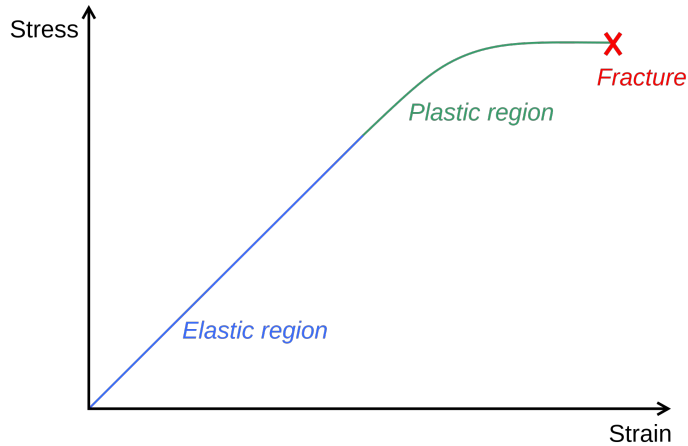


# Modeling Plasticity in the DDD Framework



Dénes Berta  Bolyai Seminar  24 April 2024

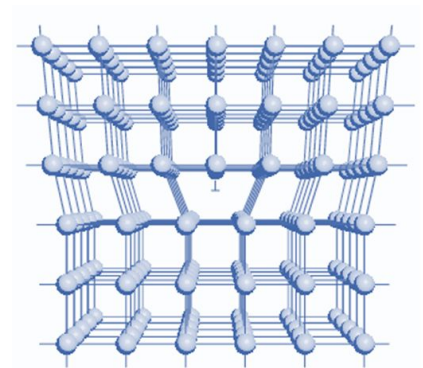
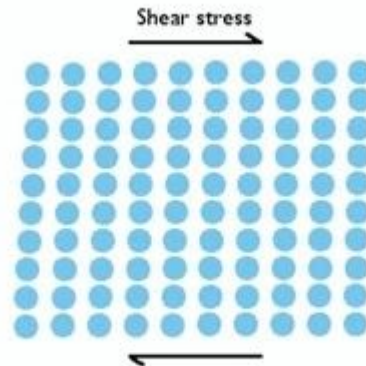
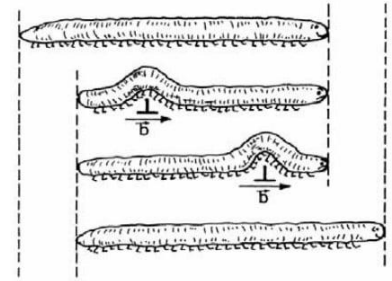
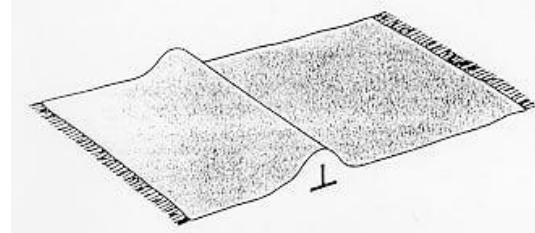
# Plastic deformation of crystalline materials



- ❑ Small strains: elastic deformation
- ❑ Large strains: elasto-plastic deformation

# Conundrum of the critical stress

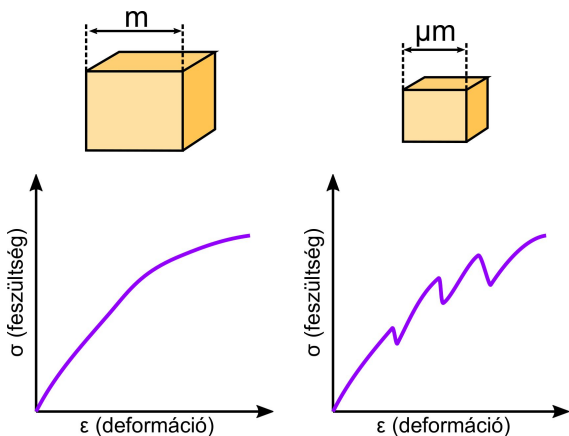
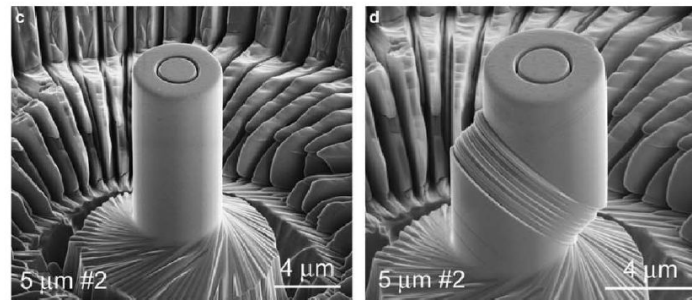
- ❑ Critical stress
  - ❑ Theory:  $\approx 30$  GPa
  - ❑ Experiment:  $\approx 100$  MPa
- ❑ Solution: **dislocations**
- ❑ Orován, Polányi, Taylor



MakeAGIF.com

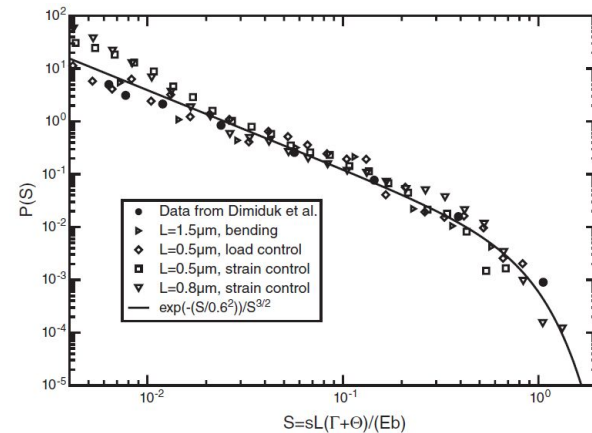
# Plasticity at the (sub)micron scale

- ❑ Size-related hardening
- ❑ Sample-to-sample variation
- ❑ Random strain bursts (scale-free)
- ❑ Dislocation avalanches

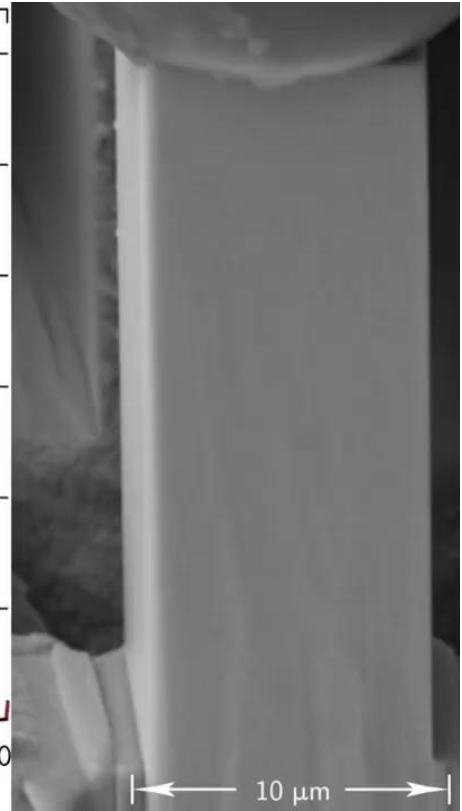
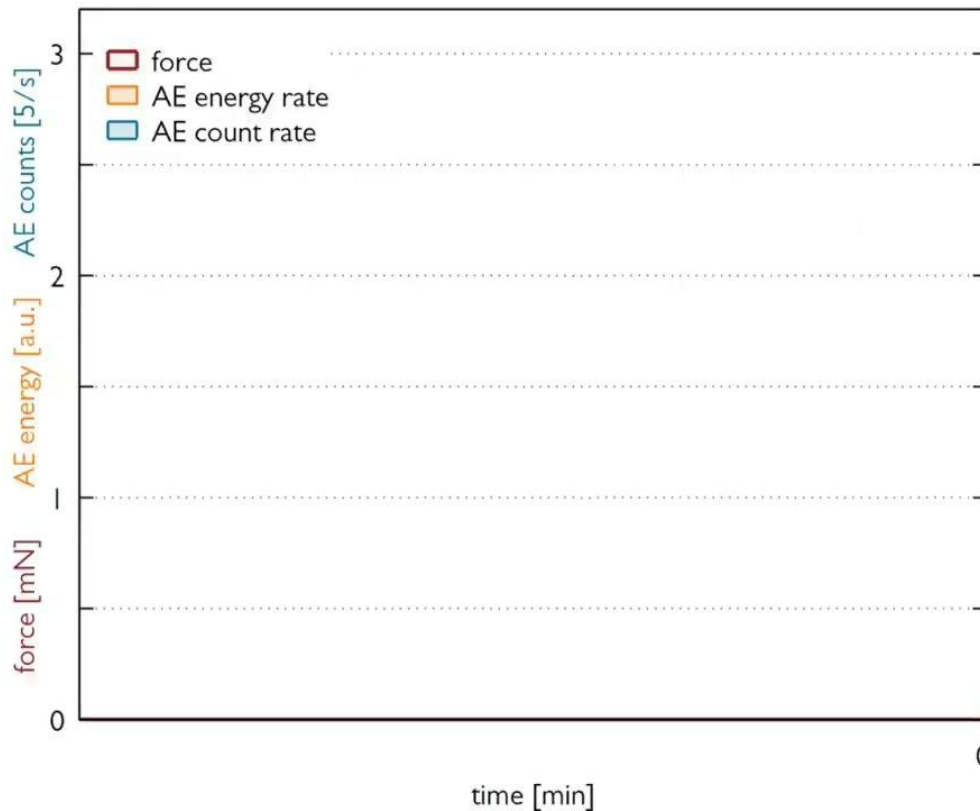


D. M. Dimiduk, C. Woodward, R. LeSar és M. D. Uchic, *Science* **312**, 1188 (2006).

F. F. Csikor, C. Motz, D. Weygand, M. Zaiser és S. Zapperi, *Science* **318**, 251 (2007).

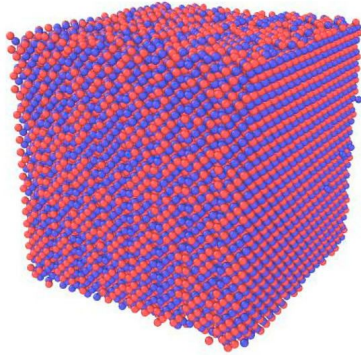


# Micropillar compression test



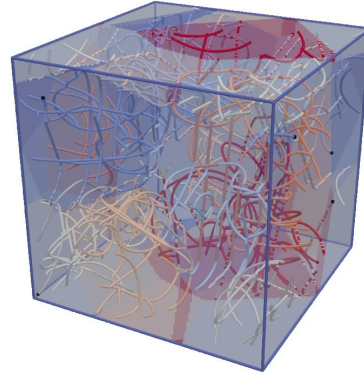
Ispánovity, Péter Dusán, et al., Nature communications 13.1 (2022): 1975.

**MD**



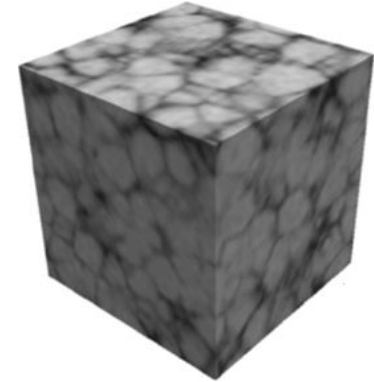
Z. Cui, X. Zhou, Q. Meng  
*Metals* 10.12 (2020): 1660.

**DDD**



R. LeSar and L. Capolungo  
*Theory and Modeling* (2020): 1079-1110.

**CDD**

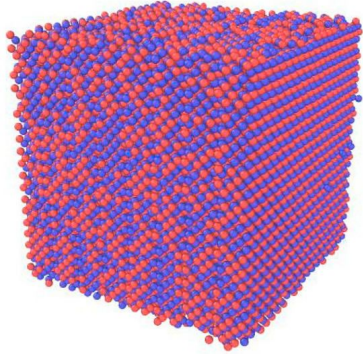


A. El-Azab and G. Po. *Handbook of Materials Modeling: Methods: Theory and Modeling* (2020): 1583-1607.

degrees of freedom, computational cost, realistic description

simulated time and length scale

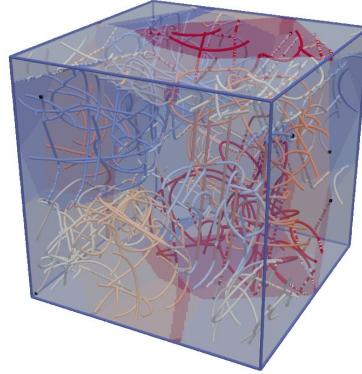
## MD



Z. Cui, X. Zhou, Q. Meng  
*Metals* 10.12 (2020): 1660.

- ❑ unit: atom/molecule
- ❑ interaction potentials

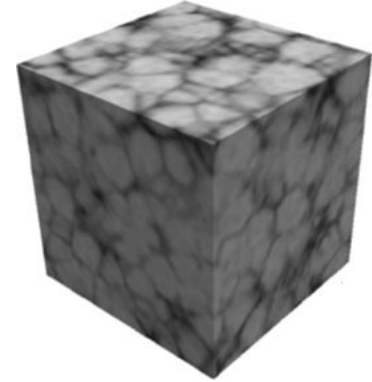
## DDD



R. LeSar and L. Capolungo  
*Theory and Modeling* (2020): 1079-1110.

- ❑ unit: dislocation line
- ❑ stress fields + PK force

## CDD

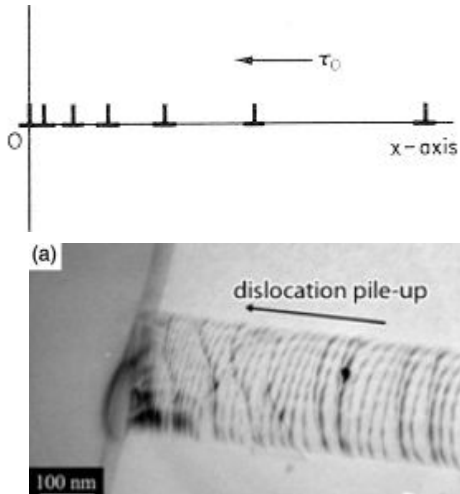


A. El-Azab and G. Po. *Handbook of Materials Modeling: Methods: Theory and Modeling* (2020): 1583-1607.

- ❑ coarse graining
- ❑ PDE for densities

# Dimensional complexity

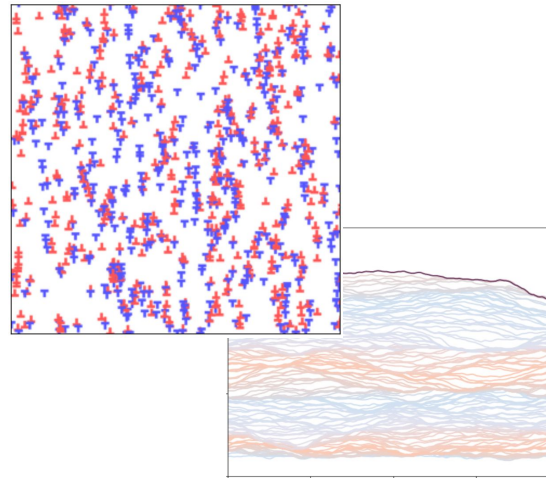
## 1D DDD



- ❑ Dislocation pile-ups

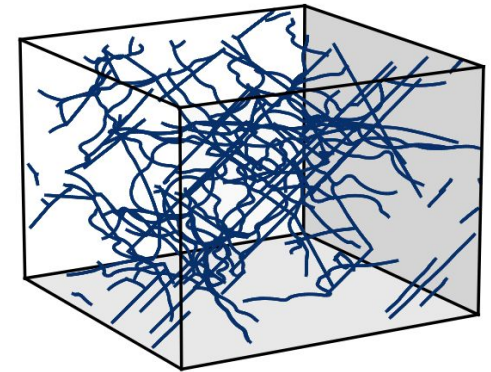
Hull, Derek, and David J. Bacon.  
*Introduction to dislocations.*  
Butterworth-Heinemann, 2001.

## 2D DDD



- ❑ Single-slip systems
- ❑ Dislocation depinning

## 3D DDD

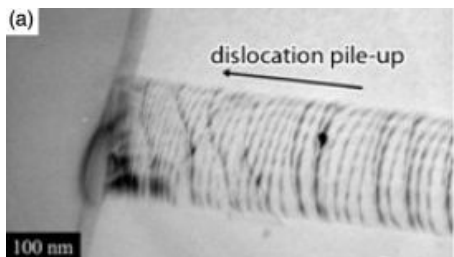
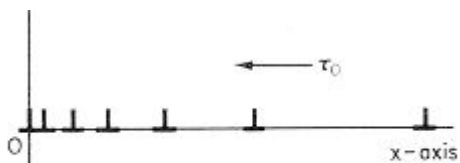


- ❑ Multi-slip systems



# 2D DDD modeling of single-slip dynamics

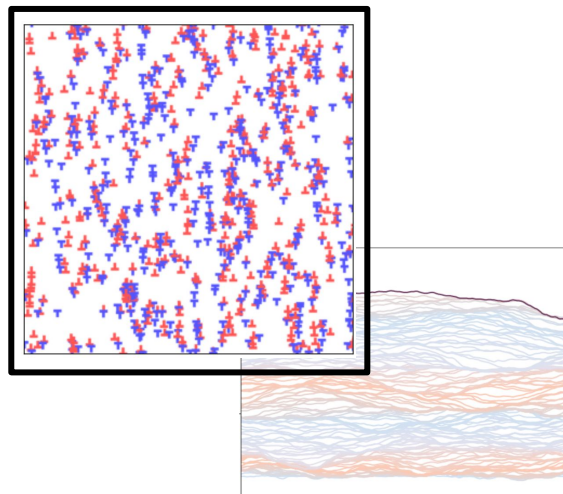
## 1D DDD



- Dislocation pile-ups

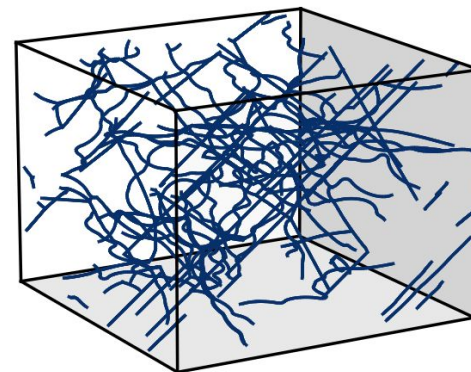
Hull, Derek, and David J. Bacon.  
*Introduction to dislocations.*  
Butterworth-Heinemann, 2001.

## 2D DDD



- Single-slip systems
- Dislocation depinning

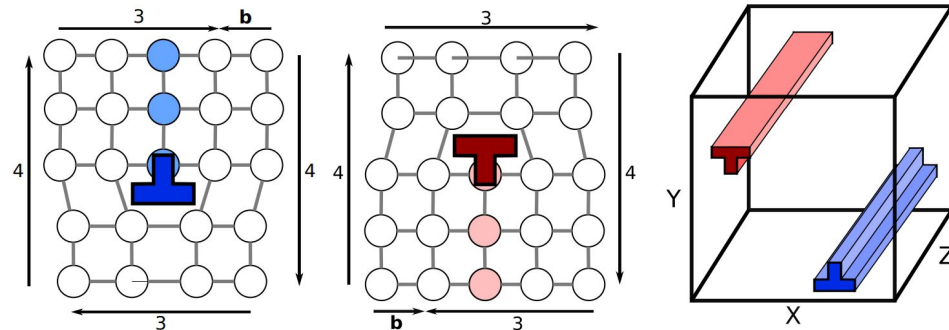
## 3D DDD



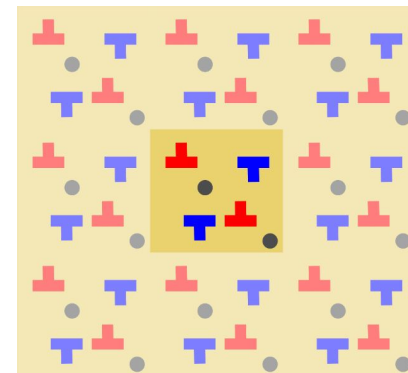
- Multi-slip systems

# Model composition

- ❑ Edge dislocations
- ❑ (Immobile point defects)
- ❑ Dynamics: dislocation glides
- ❑ PBC

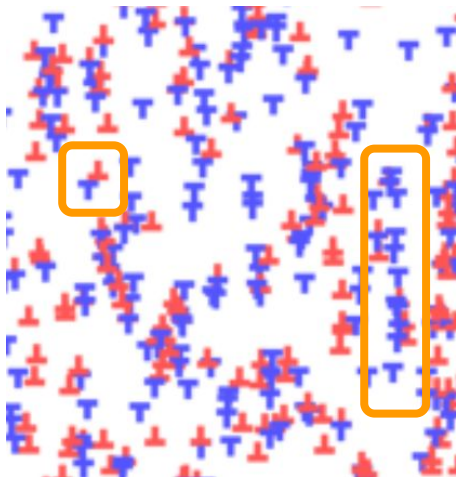


$$B\dot{x}_i = bs_i \left[ \underbrace{\sum_{j=1, j \neq i}^{N_{\text{dis}}} s_j \tau_{\text{dis}}(\mathbf{r}_i - \mathbf{r}_j)}_{\text{disl-disl. int.}} + \underbrace{\sum_{k=1}^{N_{\text{vac}}} \tau_{\text{vac}}(\mathbf{r}_i - \tilde{\mathbf{r}}_k)}_{\text{disl-p.d. int.}} + \underbrace{\tau_{\text{ext}}}_{\text{external stress}} \right] = b \left[ -\frac{\partial}{\partial x_i} (U + s_i \tau_{\text{ext}} x_i) \right]$$

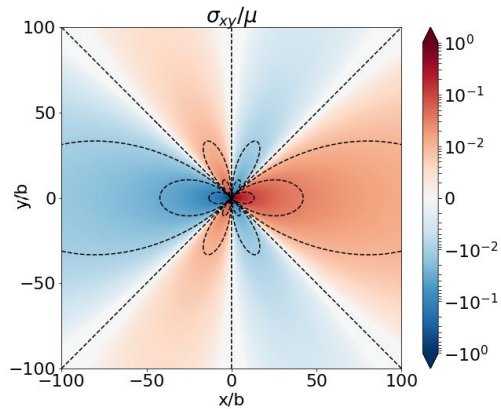
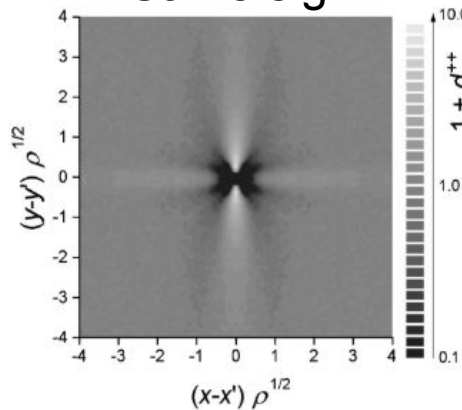


# Dislocation correlations

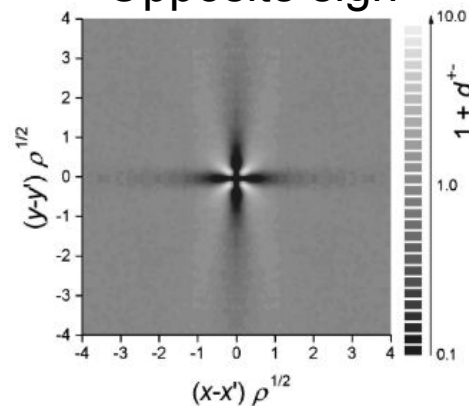
- ❑ Anisotropic stress field
- ❑ Walls
- ❑ Dipoles



Same sign

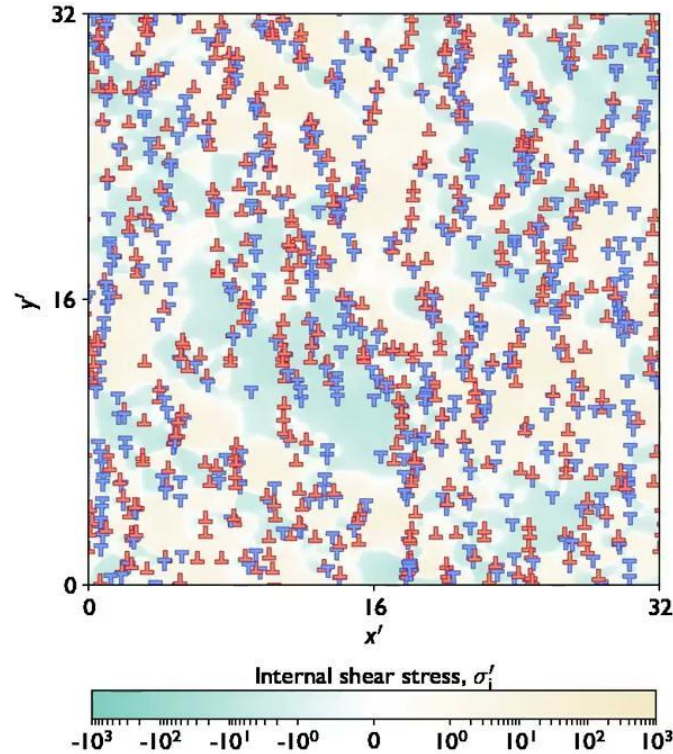
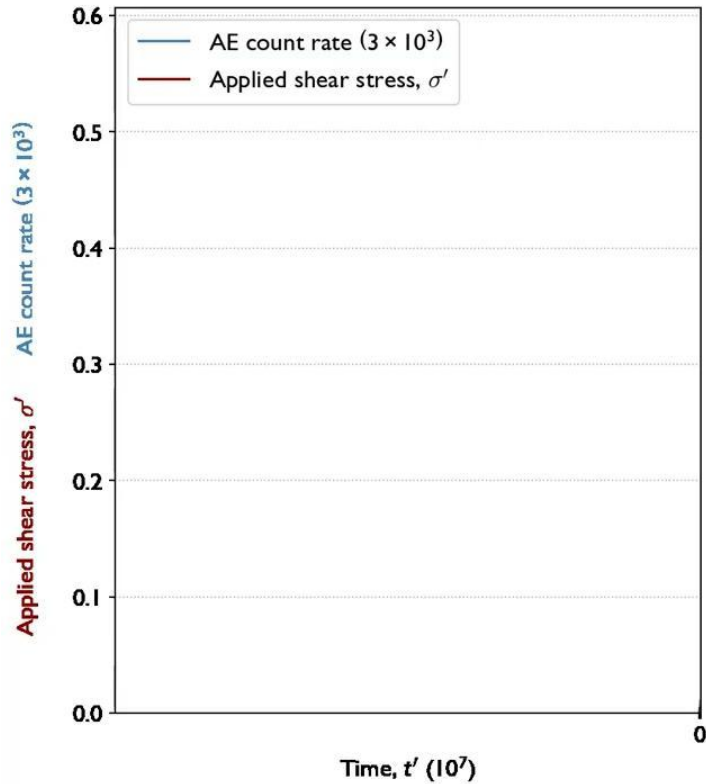


Opposite sign



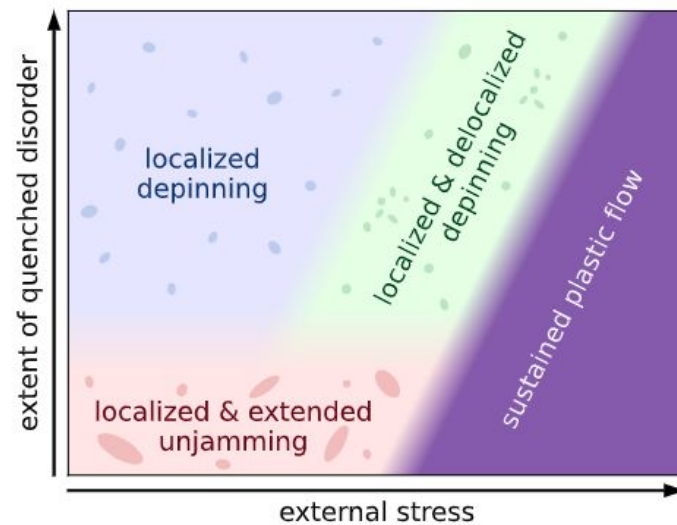
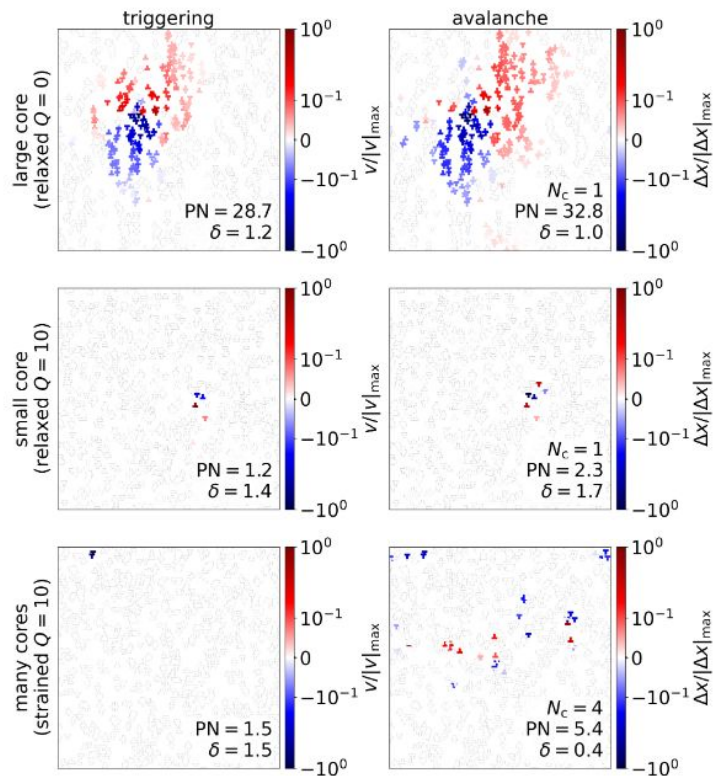
M. Zaiser, M-C. Miguel, and G. István, *Physical Review B* 64.22 (2001): 224102.

# Dislocation avalanches



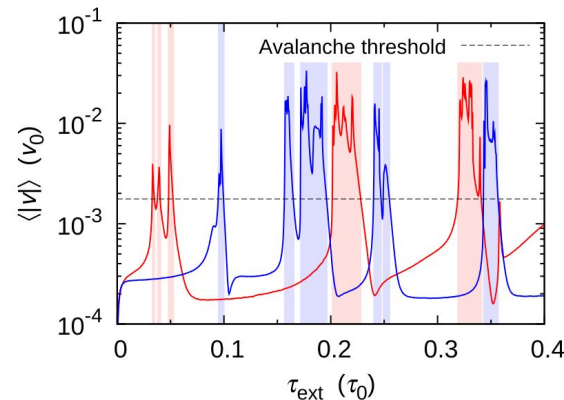
Ispánovity, Péter Dusán, et al., Nature communications 13.1 (2022): 1975.

# Different regimes of avalanche behaviour



# Avalanche statistics

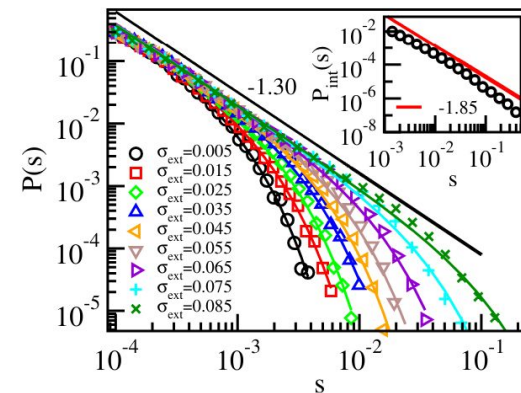
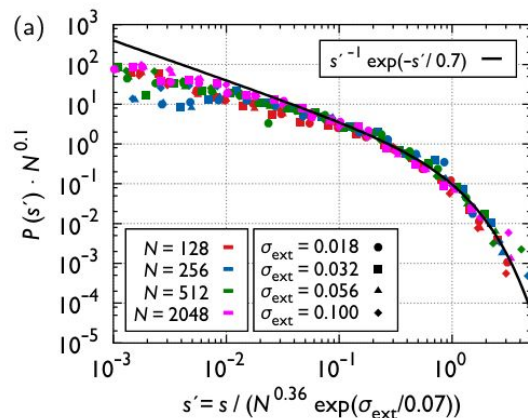
- Scale-free size distribution
- Exponent
  - No point defects: -1
  - Point defects: -1.3



P. D. Ispánovity, et al., *Physical review letters* 112.23 (2014): 235501.

P. D. Ispánovity, et al., *Physical review letters* 105.8 (2010): 085503.

M. Ovaska, L. Laurson, and M. J. Alava, *Scientific reports* 5.1 (2015): 10580.

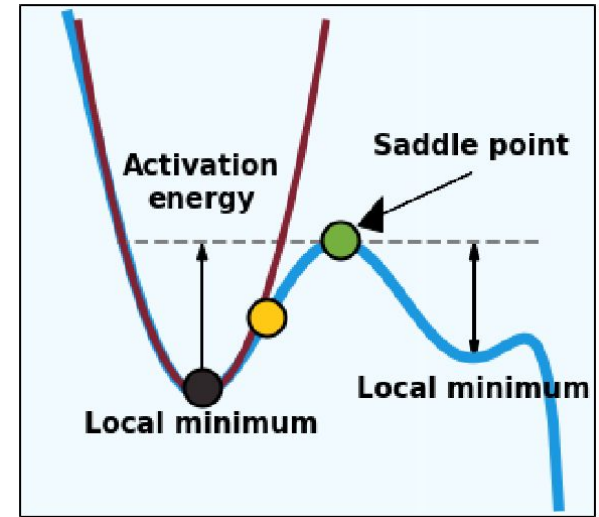


# Linear stability analysis (LSA)

- ❑ At stable equilibrium: we are at a local minimum
- ❑ Series expansion

$$U(\Delta x) = U_0 + \underbrace{\partial_k U \Delta x_k}_{=0} + \partial_{ij}^2 U \Delta x_i \Delta x_j + \mathcal{O}(\Delta x^3)$$

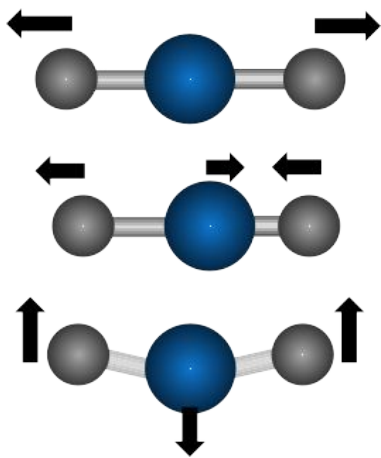
- ❑ Dynamical matrix:  $\Lambda_{ij} = \partial_{ij}^2 U$
- ❑ Harmonic approximation of the energy landscape



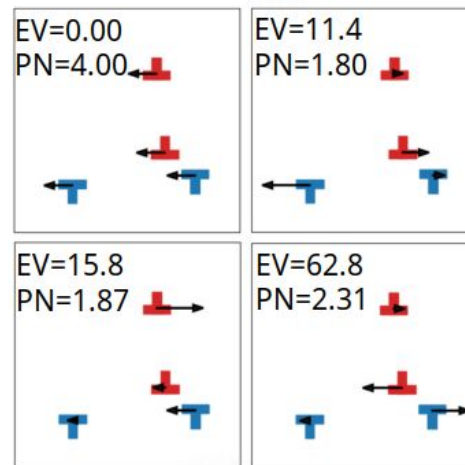
- Eigenvalue problem
  - Eigenvectors: dynamical modes
  - Eigenvalues: steepness of the energy well

EV: eigenvalue  
PN: participation number

## Molecular vibrational modes



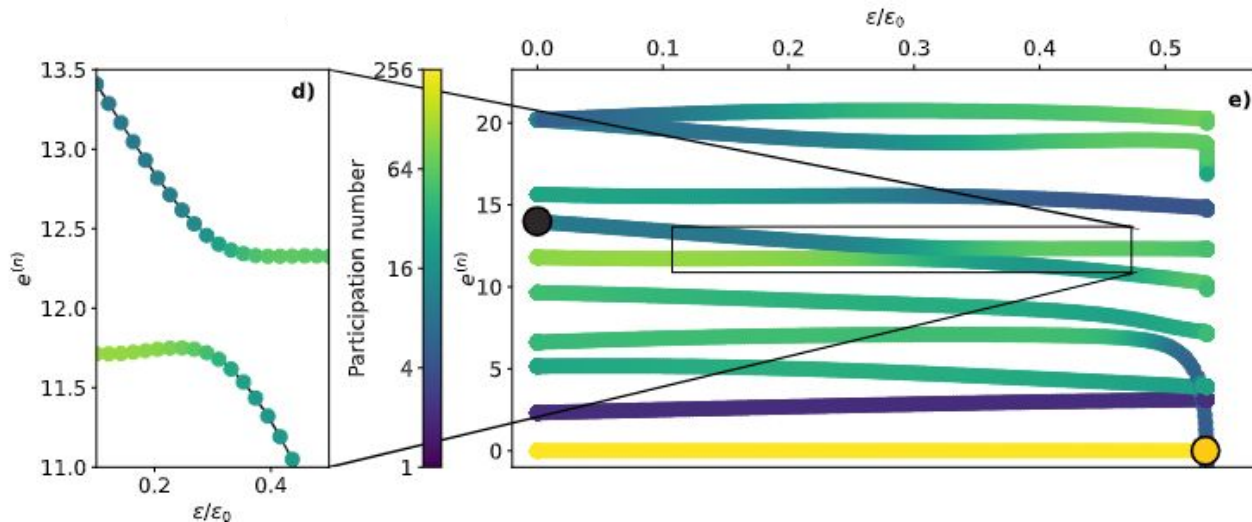
## Eigenmodes of DD systems





# Connection of modes and plastic events

- ❑ Rough energy landscape
- ❑ Instability at the saddle point
- ❑ Dynamic modes evolve into avalanches



# Spectrum of dynamical modes

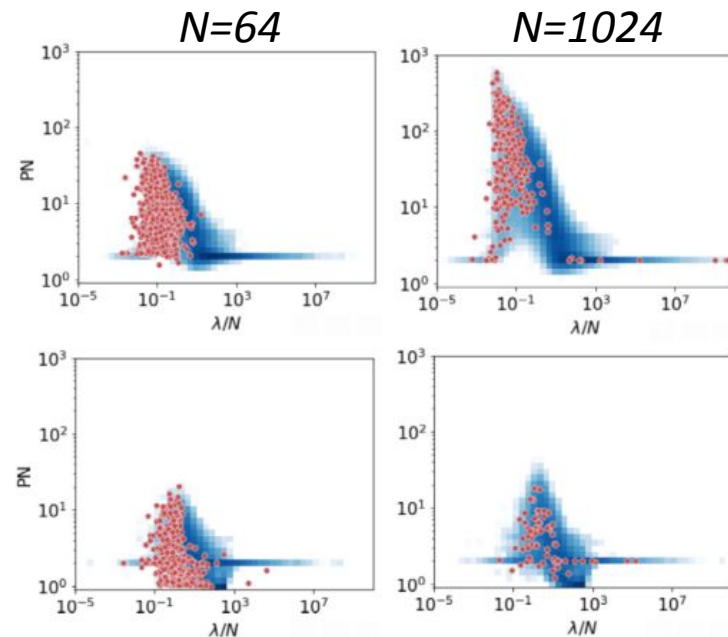
## No quenched disorder

- ❑ Localized modes (dipoles)
- ❑ Large modes (extensive scaling)
- ❑ 'Wall' of low eigenvalues (activation)



## Great extent of quenched disorder

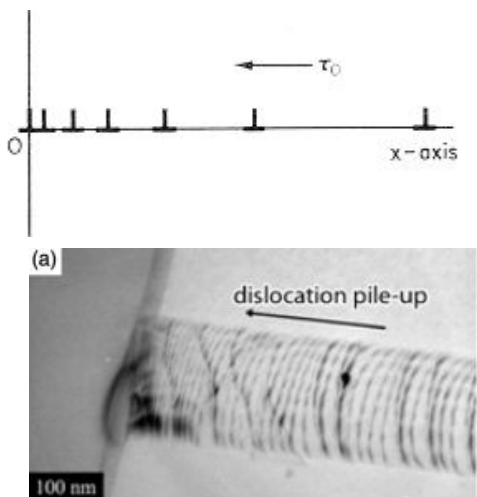
- ❑ Localized modes (dipoles)



$\lambda$ : eigenvalue  
N: number of dislocations  
PN: participation number

# 2D DDD modeling of dislocation depinning

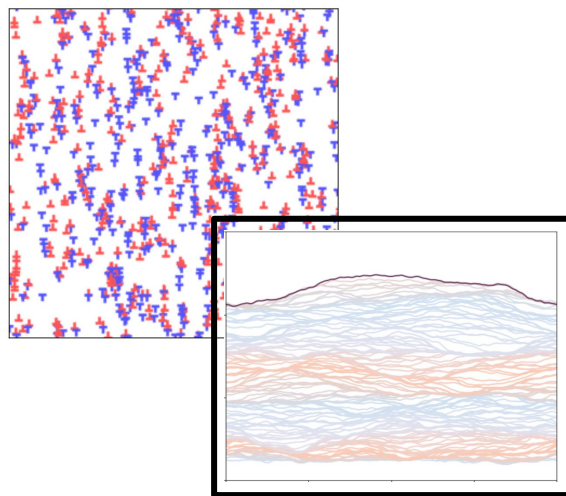
## 1D DDD



- ❑ Dislocation pile-ups

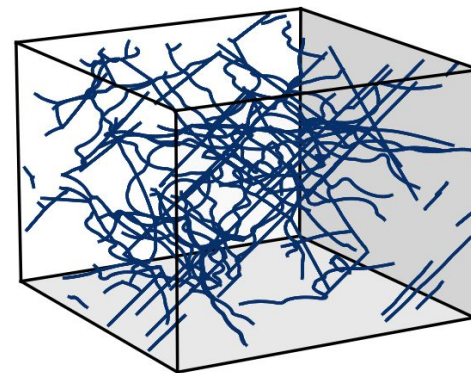
Hull, Derek, and David J. Bacon.  
*Introduction to dislocations.*  
Butterworth-Heinemann, 2001.

## 2D DDD



- ❑ Single-slip systems
- ❑ **Dislocation depinning**

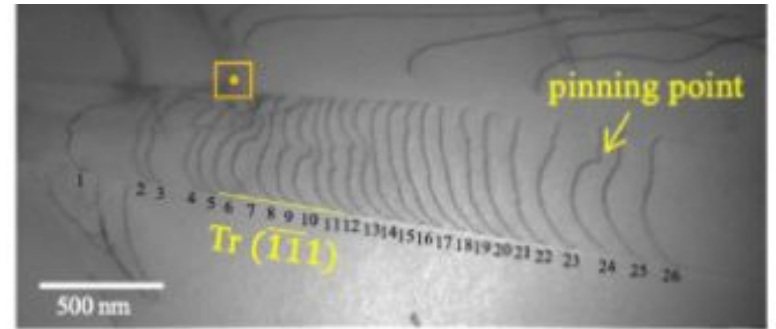
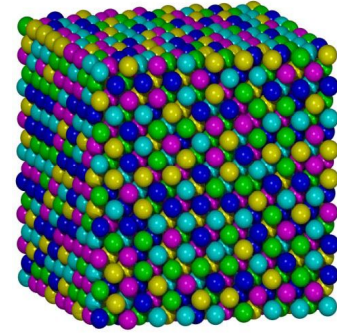
## 3D DDD



- ❑ Multi-slip systems

# Dislocation depinning in high entropy alloys

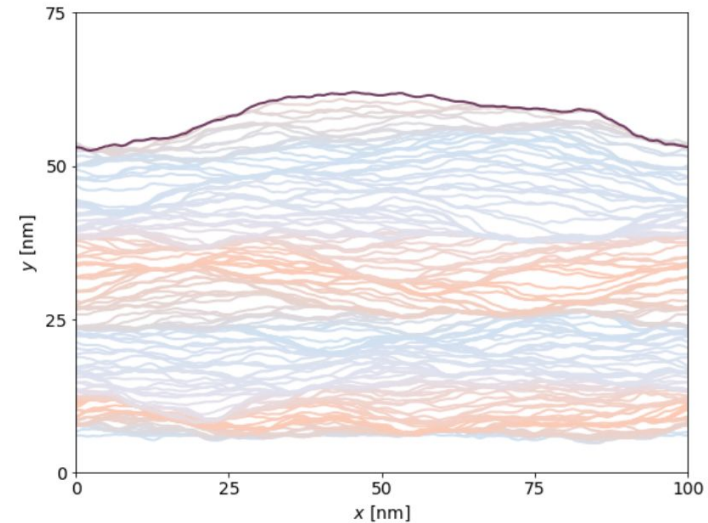
- ❑ High entropy alloys (HEA)
  - ❑ Multi-principle-element (at least 5)
  - ❑ Atomic radius differences
  - ❑ Immense lattice distortion
  - ❑ Heterogeneous internal stress field
- ❑ Impact on dislocations
  - ❑ Rough dislocation lines
  - ❑ Stick-slip dynamics



Zhang, Chen, et al., *Acta materialia* 241 (2022): 118394.

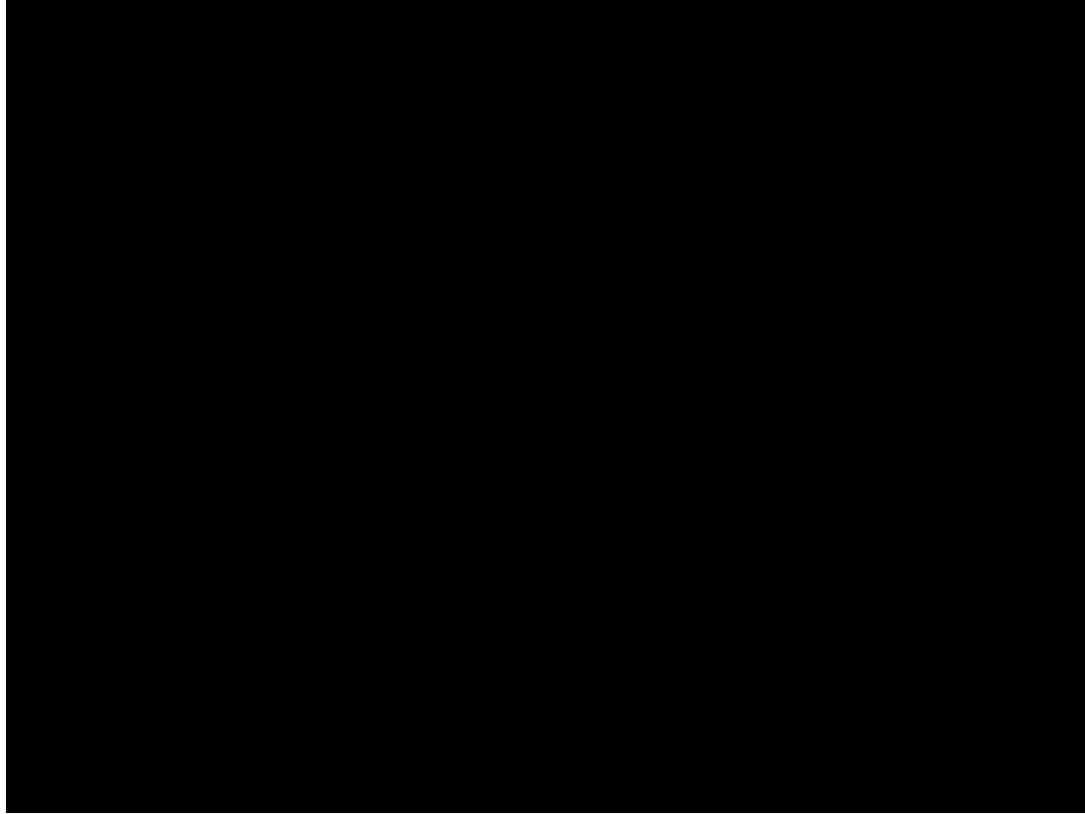
# Model composition

- ❑ Vertical and horizontal dislocation segments
- ❑ Heterogeneous pinning stress field
  - ❑ Gaussian distribution
  - ❑ No spatial correlations
- ❑ Random dynamics



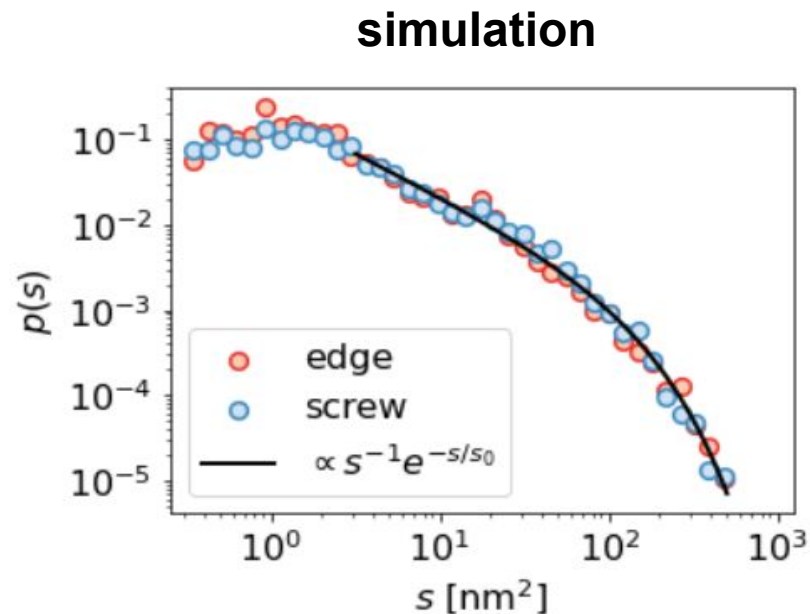
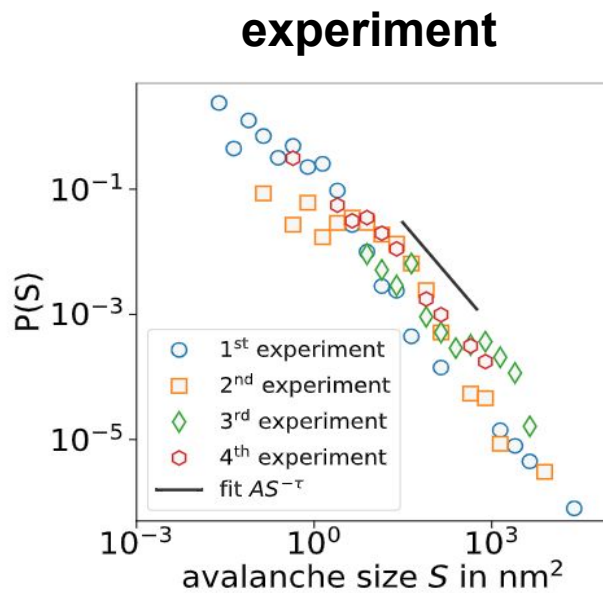
$$\tau(\mathbf{r}) = \tau_{\text{ext}}(\mathbf{r}) + \tau_{\text{self}}(\mathbf{r}) + \tau_{\text{pin}}(\mathbf{r})$$

$$\tau_{\text{self}}(\mathbf{r}_i) = \sum_j^{\{s\}} \tau_s(\mathbf{r}_i - \mathbf{r}_j) + \sum_j^{\{e\}} \tau_e(\mathbf{r}_i - \mathbf{r}_j)$$

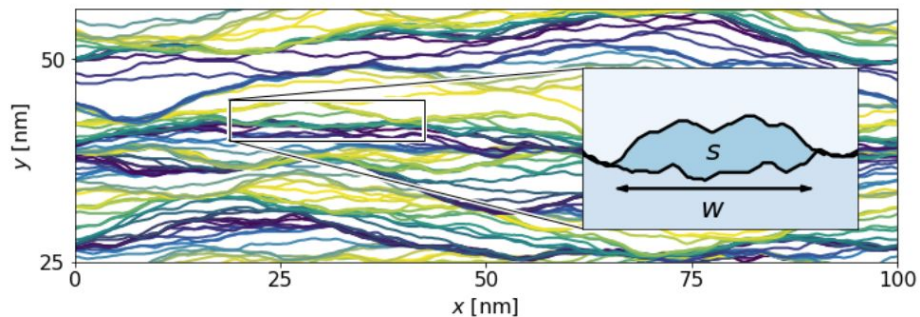


# Slip size distribution

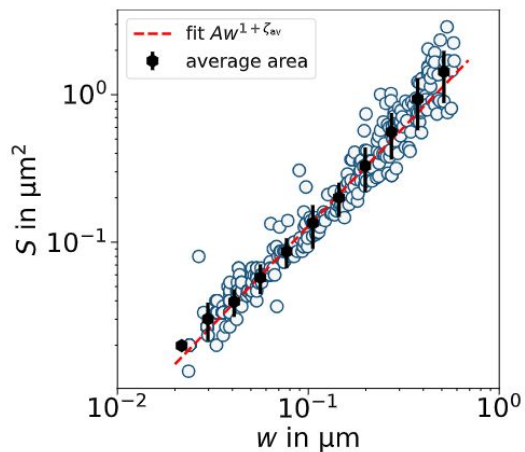
- ❑ Scale-free distribution
- ❑ Exponent: -1



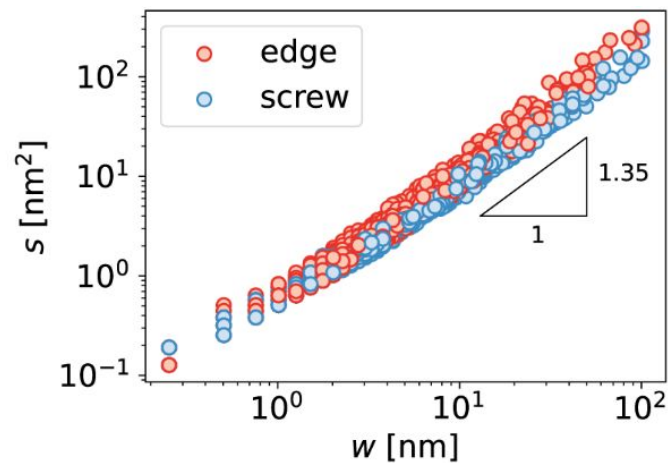
# Fractal dimension of slip events



experiment



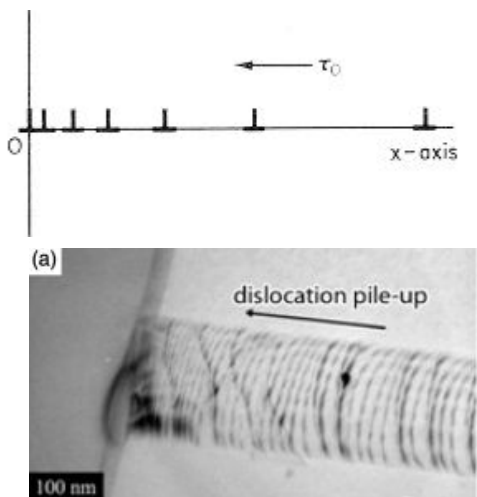
simulation





# 2D DDD modeling of dislocation depinning

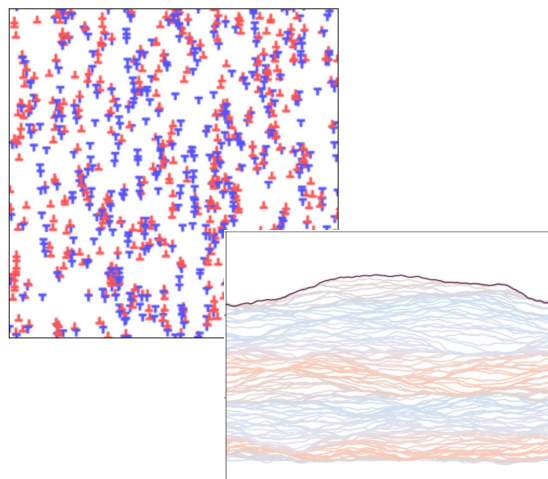
## 1D DDD



- ❑ Dislocation pile-ups

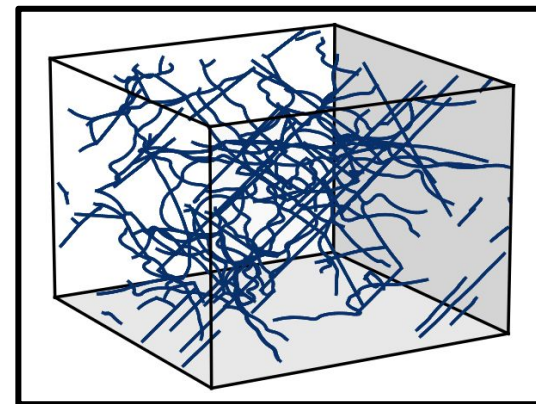
Hull, Derek, and David J. Bacon.  
*Introduction to dislocations.*  
Butterworth-Heinemann, 2001.

## 2D DDD



- ❑ Single-slip systems
- ❑ Dislocation depinning

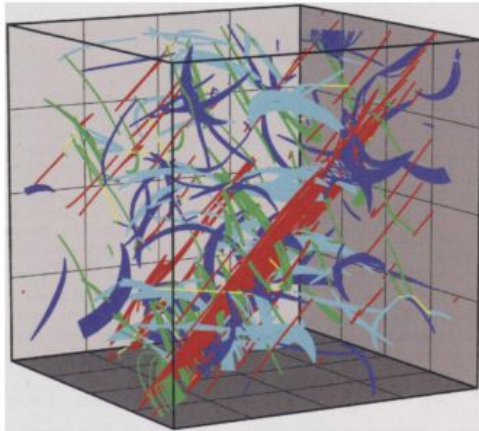
## 3D DDD



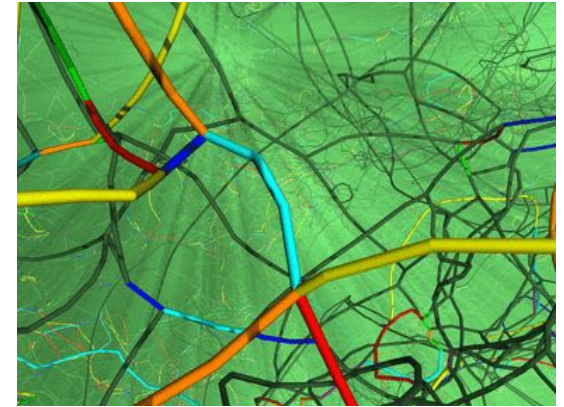
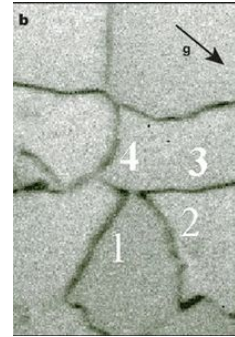
- ❑ Multi-slip systems

# Model composition

- ❑ Non-physical dislocation nodes
- ❑ Straight dislocation segments
- ❑ Dislocation junctions
- ❑ Usually multi-slip



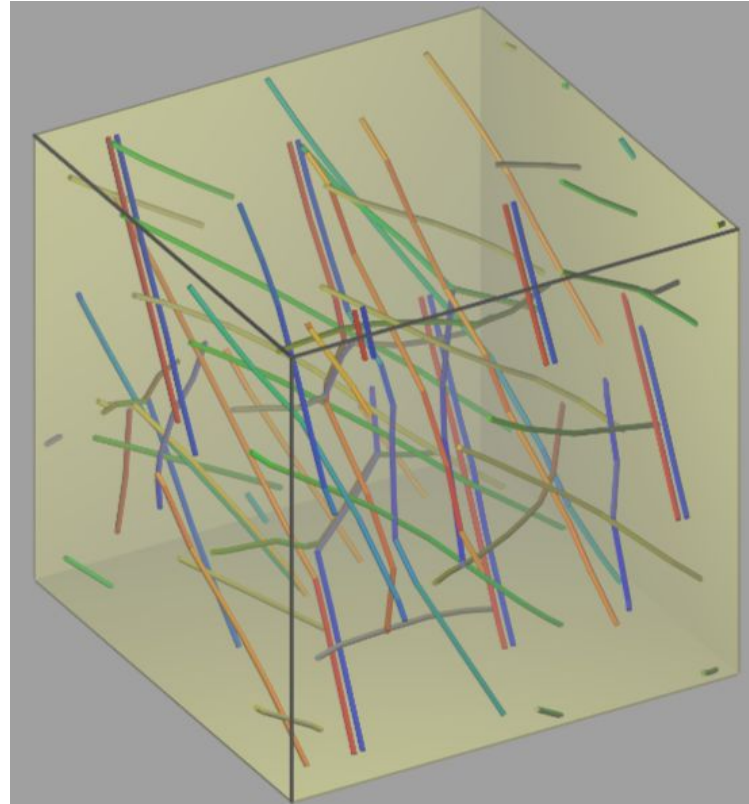
F. F. Csikor et al., *science*  
318.5848 (2007): 251-254.



V. V. Bulatov et al., *Nature* 440.7088  
(2006): 1174-1178.

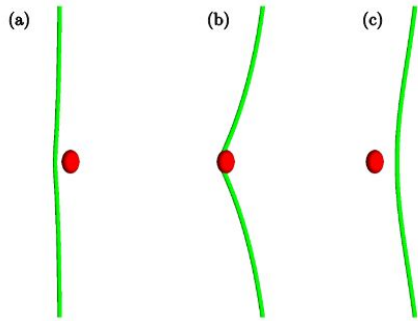
# Special case: single-slip

- ❑ Less complex structure
- ❑ Dipoles
- ❑ Similar to 2D

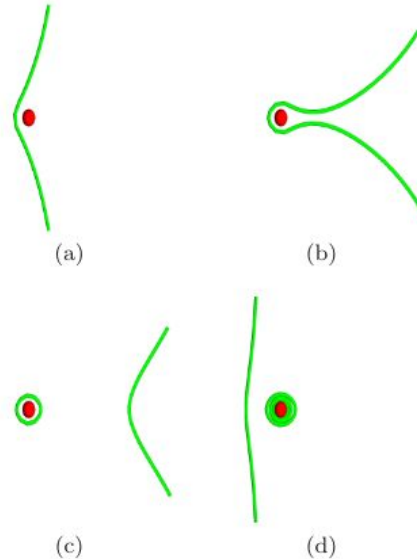


# Interaction with precipitates

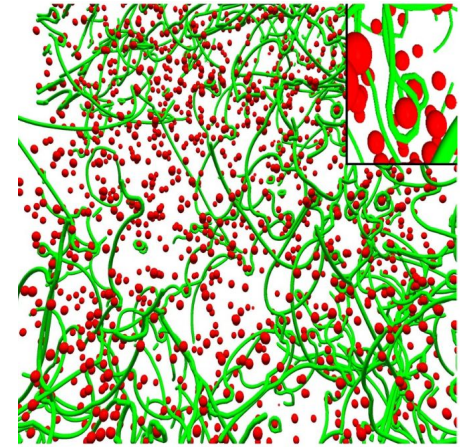
## weak precipitates



## strong precipitates



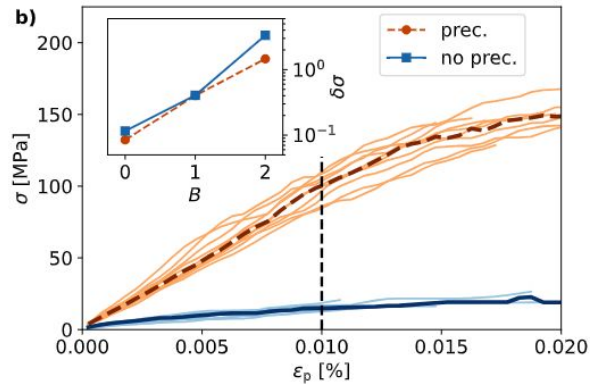
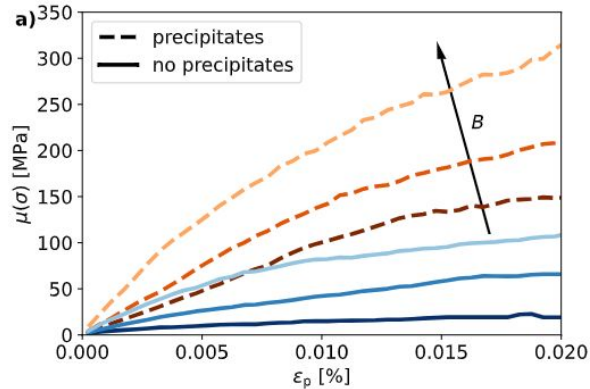
Orowan loop



A. Lehtinen, et al. *Scientific reports* 8.1 (2018): 6914.

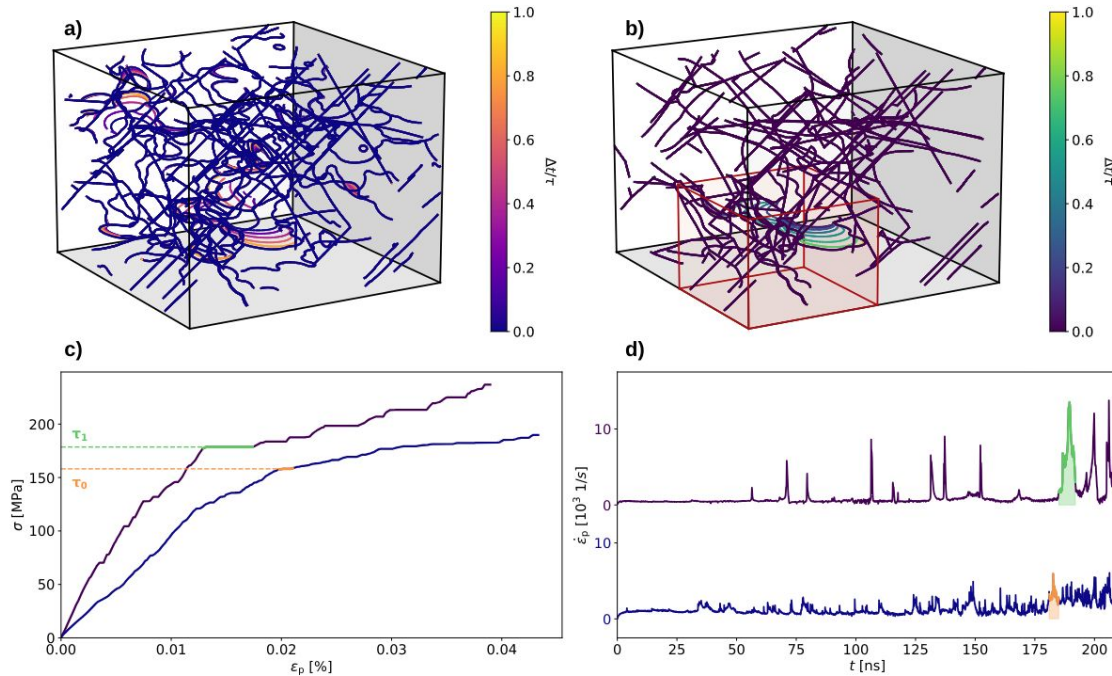
A. Lehtinen, et al., *Physical Review E* 93.1 (2016): 013309.

# Size- and precipitate-related hardening



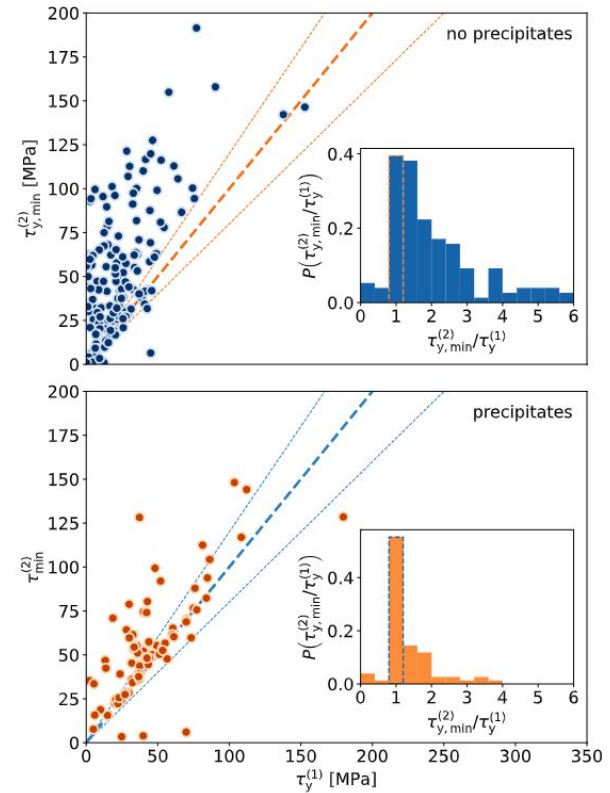
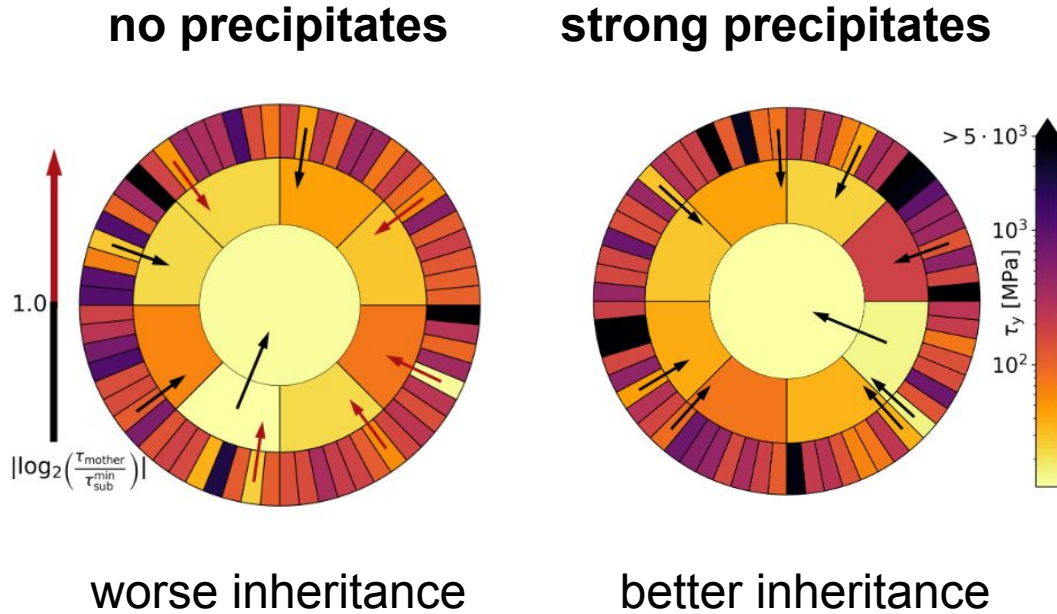
- ❑ Fluctuations at this size regime
- ❑ Smaller subregions are harder
- ❑ Configurations with precipitates are harder

# Weakest-link behaviour

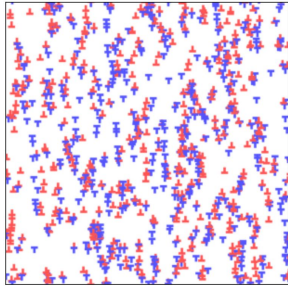


- Similar behaviour during global and local quasi-static loading:
- Similar plastic events
- At similar stresses

# No precipitates vs strong precipitates

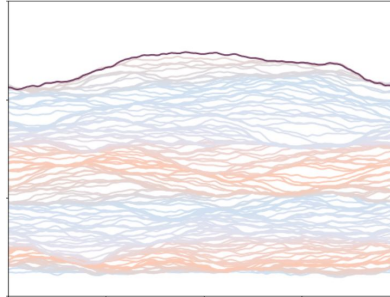


## 2D DDD (straight)



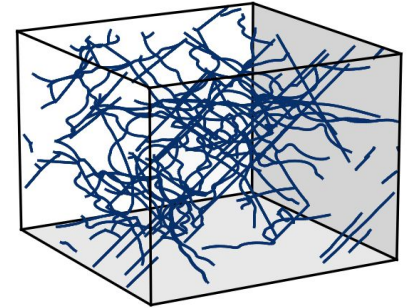
- ❑ Scale-free avalanches
- ❑ Different behaviour w/ and w/o point defects
- ❑ Dynamic modes related to avalanches

## 2D DDD (curved)



- ❑ Depinning dynamics
- ❑ Scale-free plastic slips
- ❑ Non-trivial dislocation shape

## 3D DDD



- ❑ Weakest-link behaviour
- ❑ Better with precipitates
- ❑ Hardening
  - ❑ size-related
  - ❑ precipitate-related



# Thanks to...



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