Mass estimates of exoplanets based on the topology of interacting networks

Bolyai Seminar, 2024 April

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Motivation

Planet masses if we have

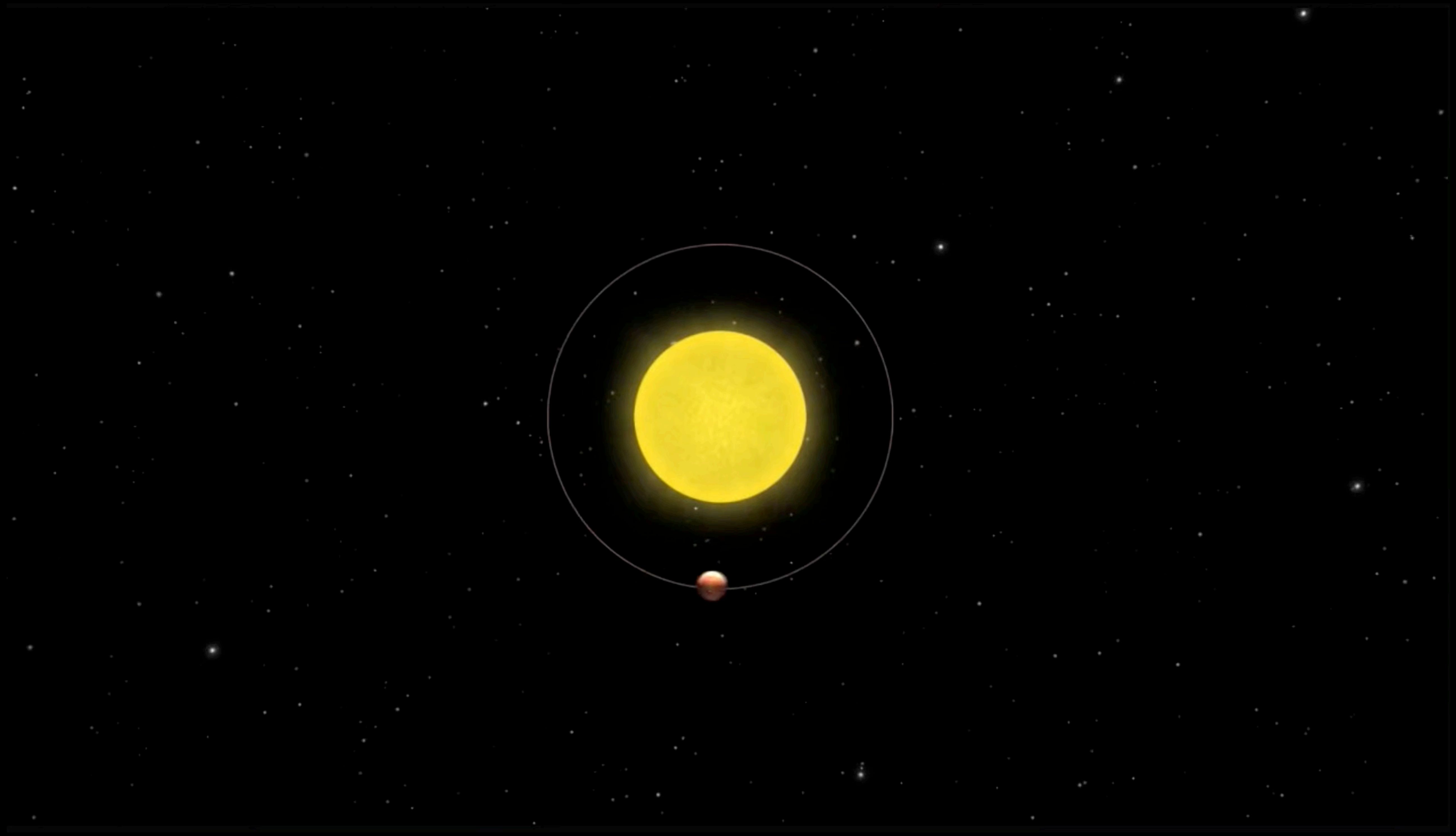
- no equations
- no initial conditions
- no numerical integration
- no best fit model & MCMC

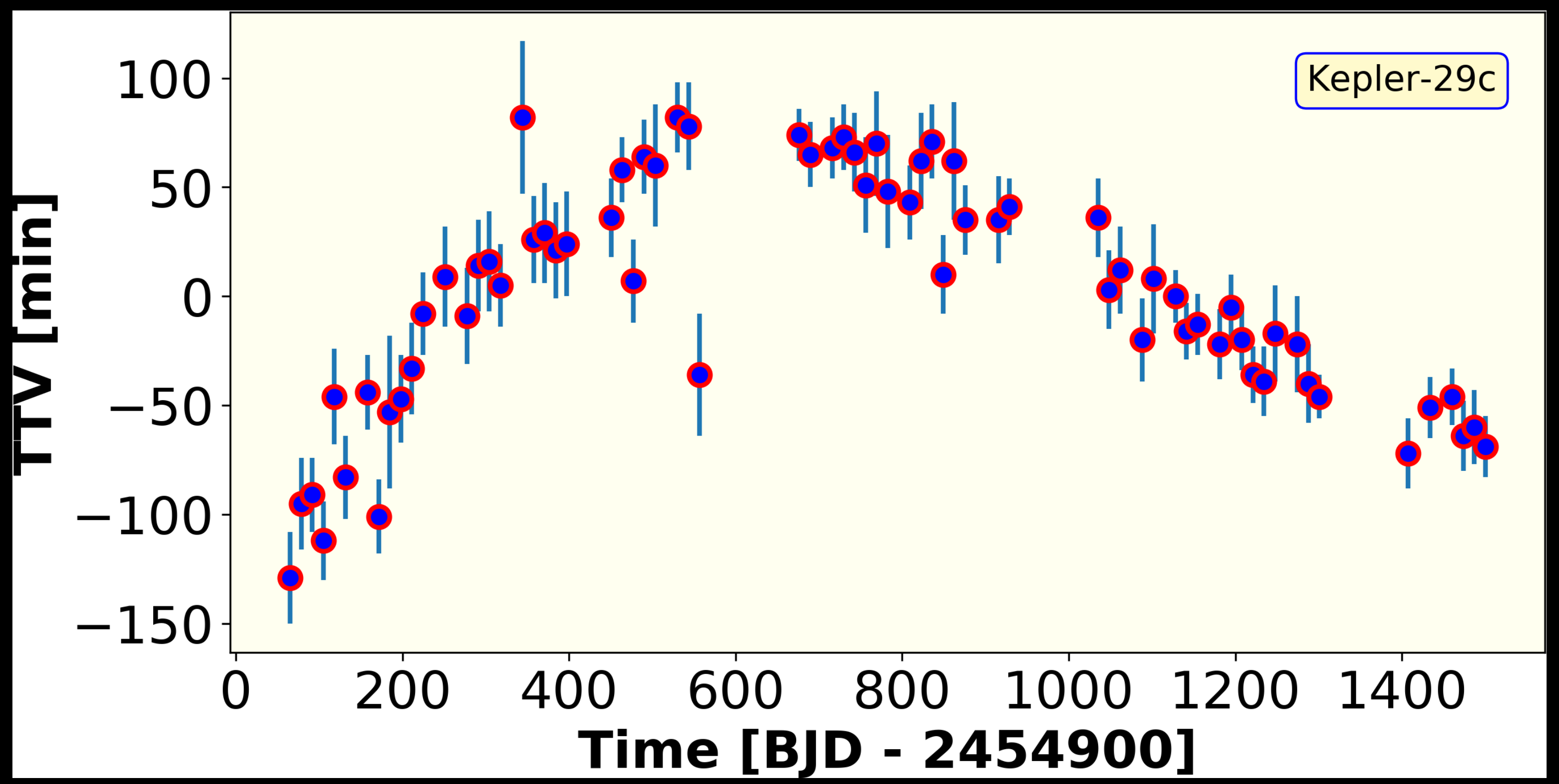
BUT we have

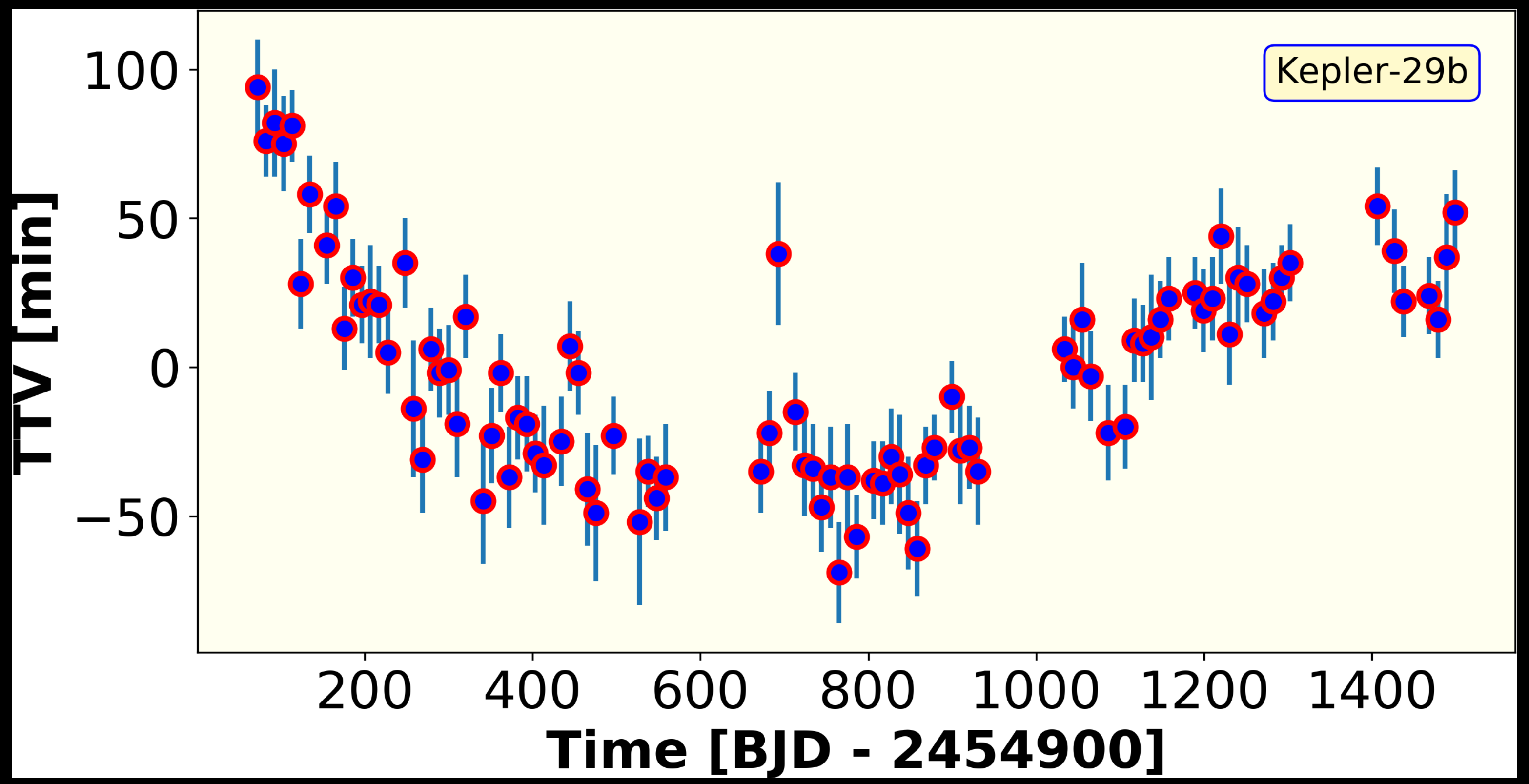
- a lot of data

classical parameter estimation

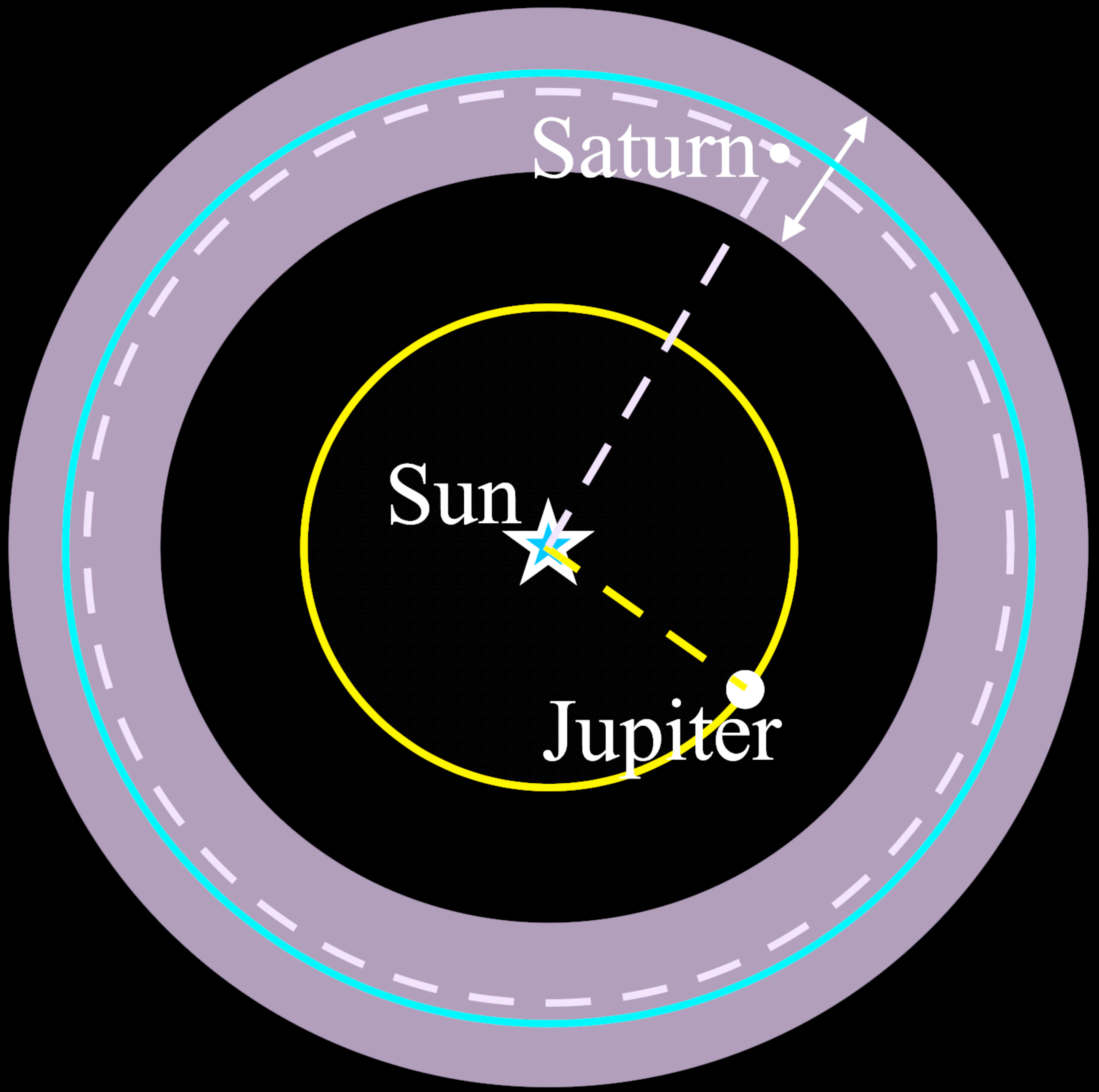
data ⟶ chi^2, MCMC, n-body int. ⟶ system parameters

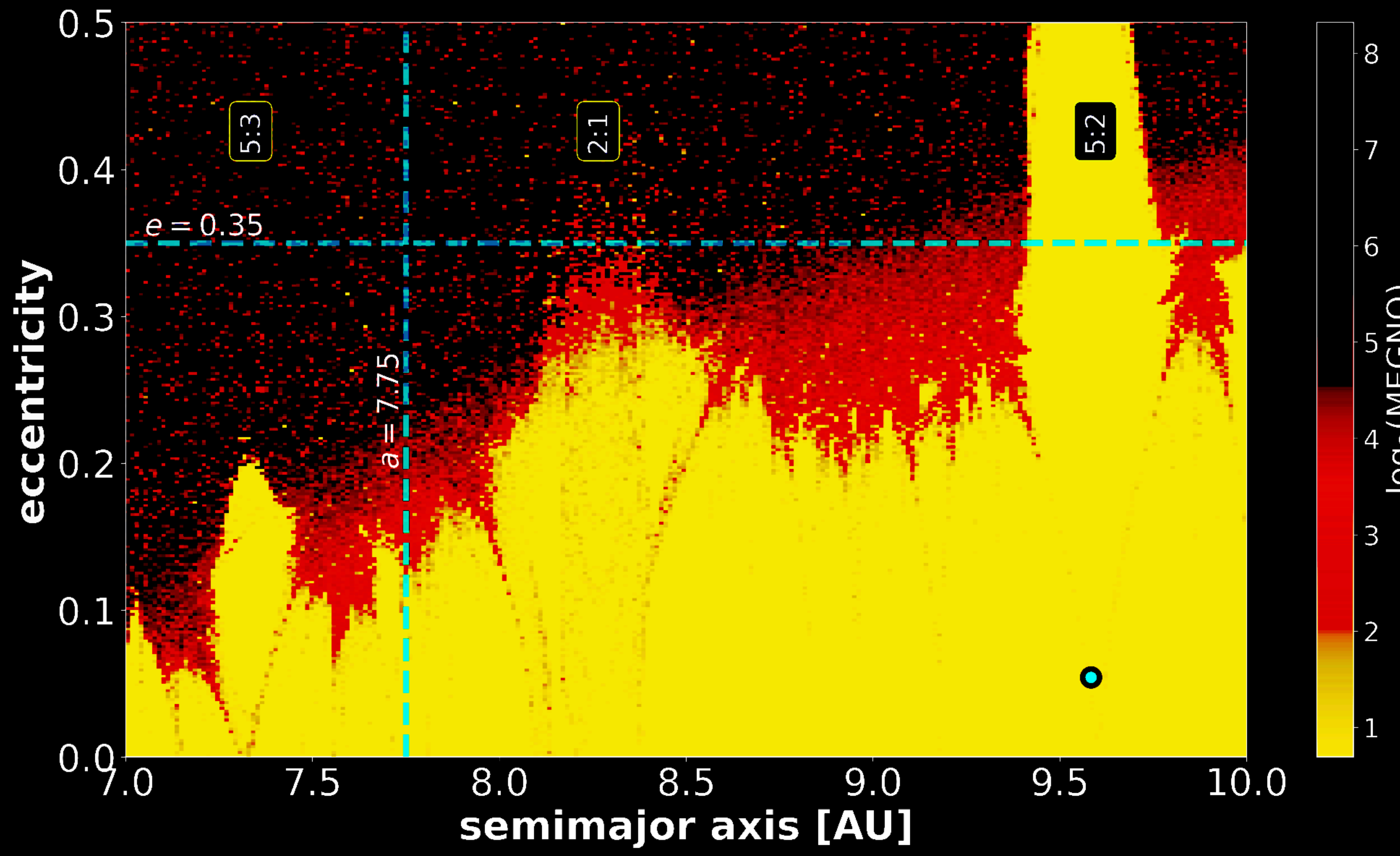






the case study of Jupiter & Saturn





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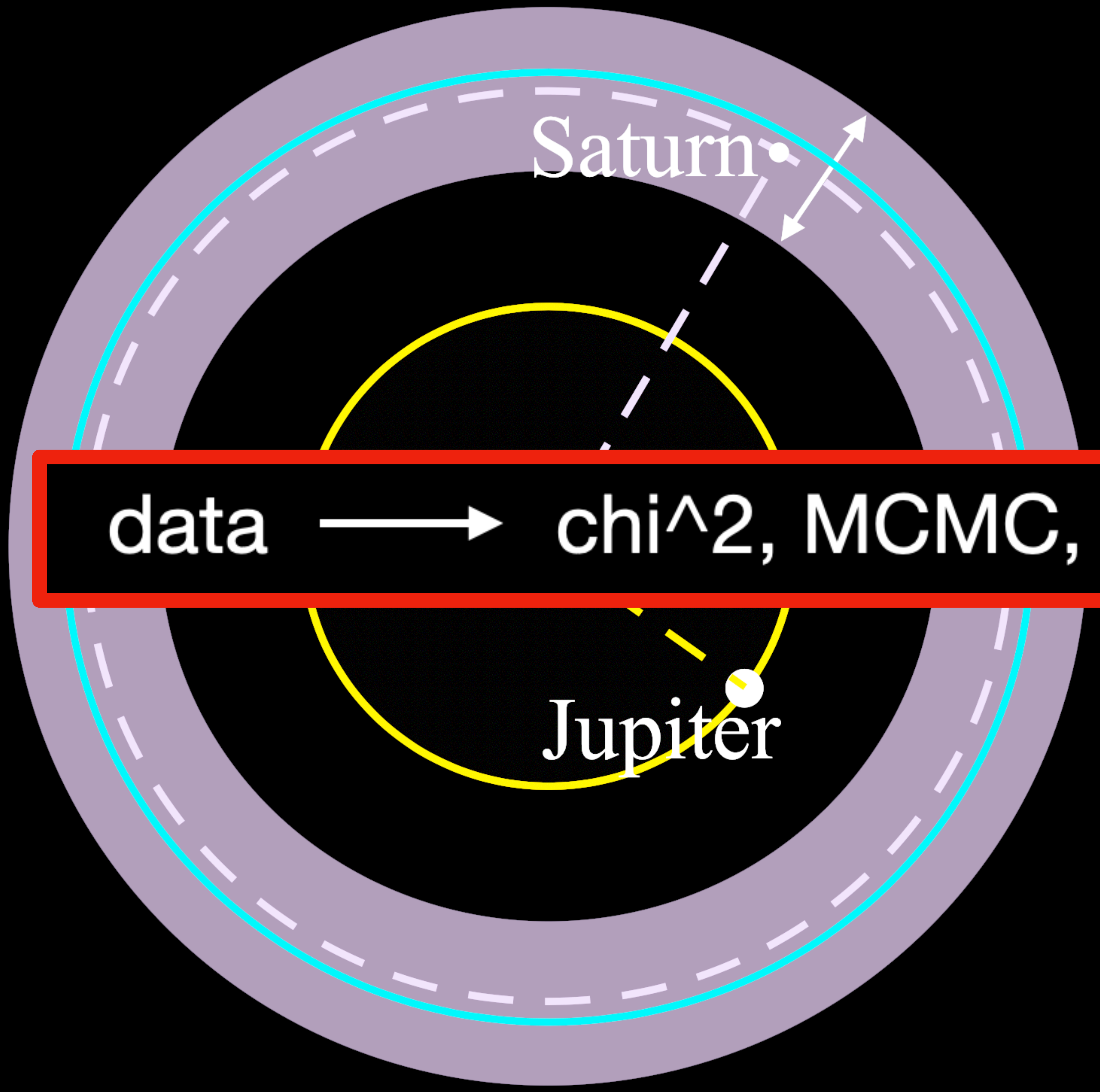
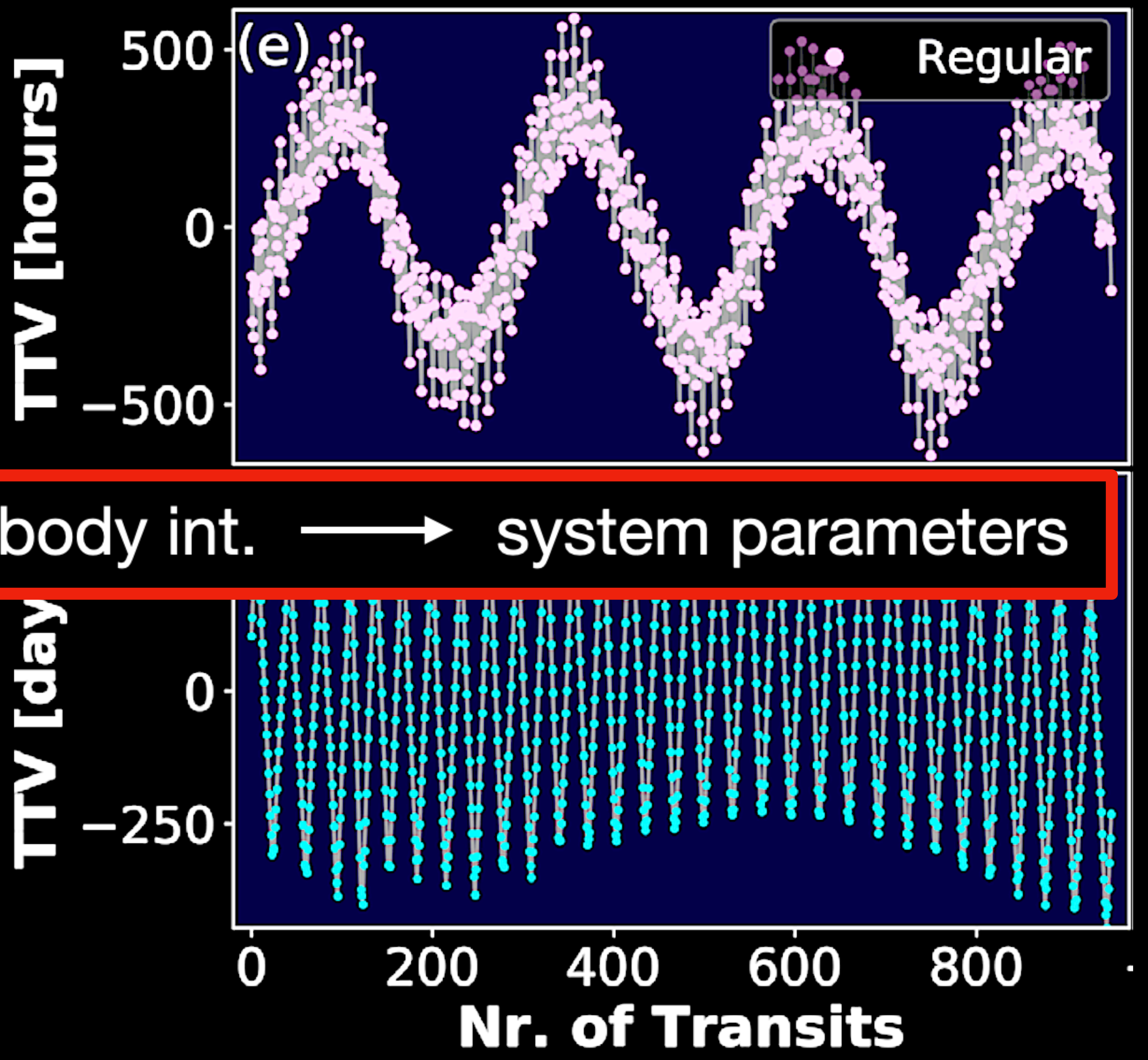


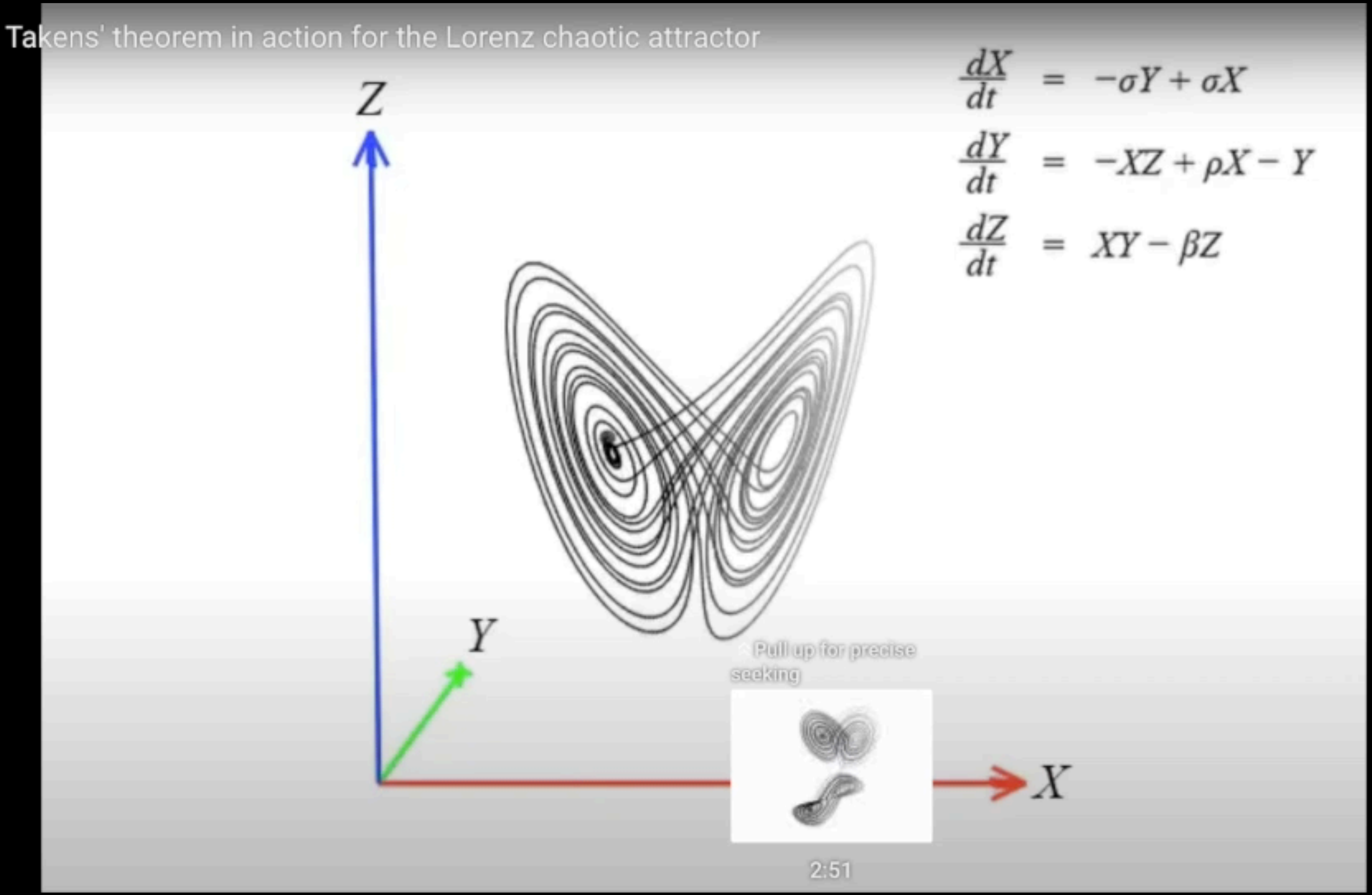


data ⟶ chi^2, MCMC, n-body int. ⟶ system parameters

from time series to dynamics

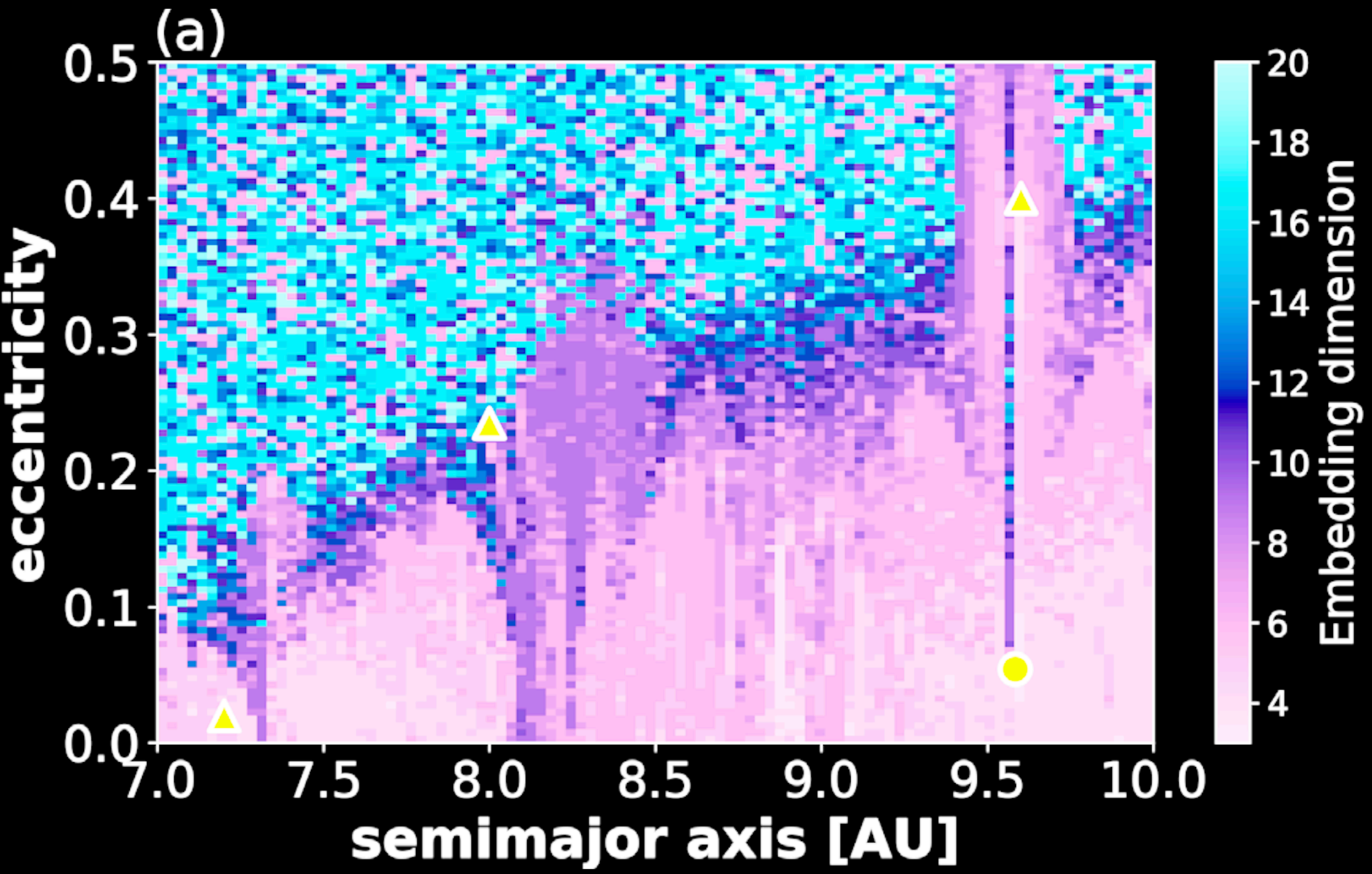
phase space reconstruction

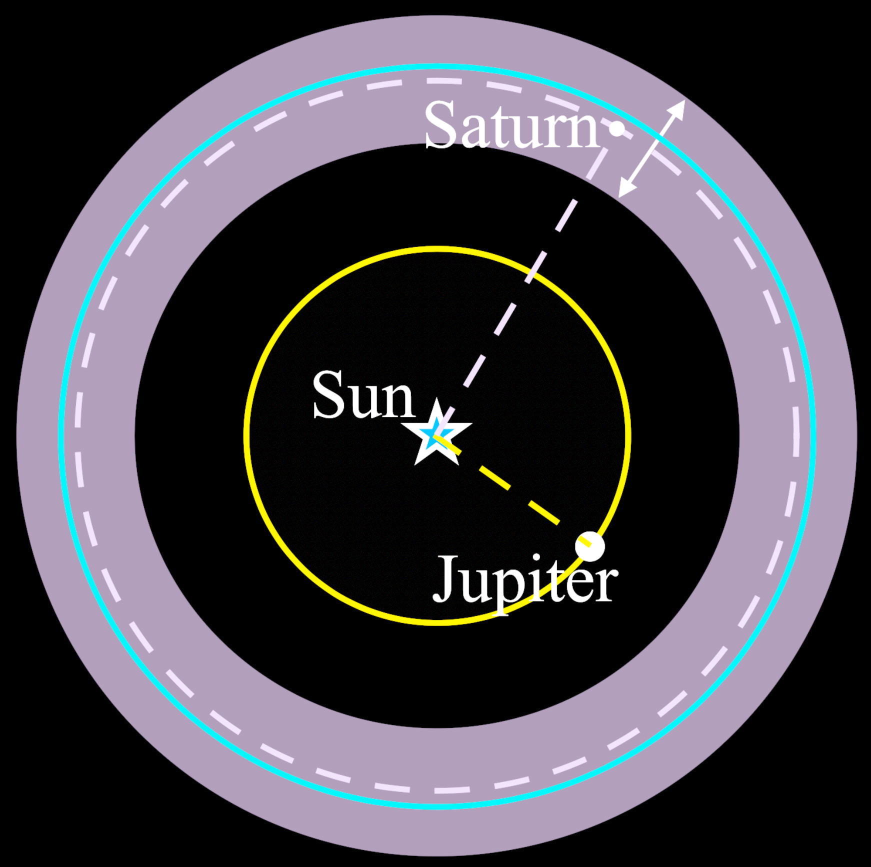
if no all the components of the state vector are known



from time series to dynamics & exoplanets

phase space reconstruction
from TTV data

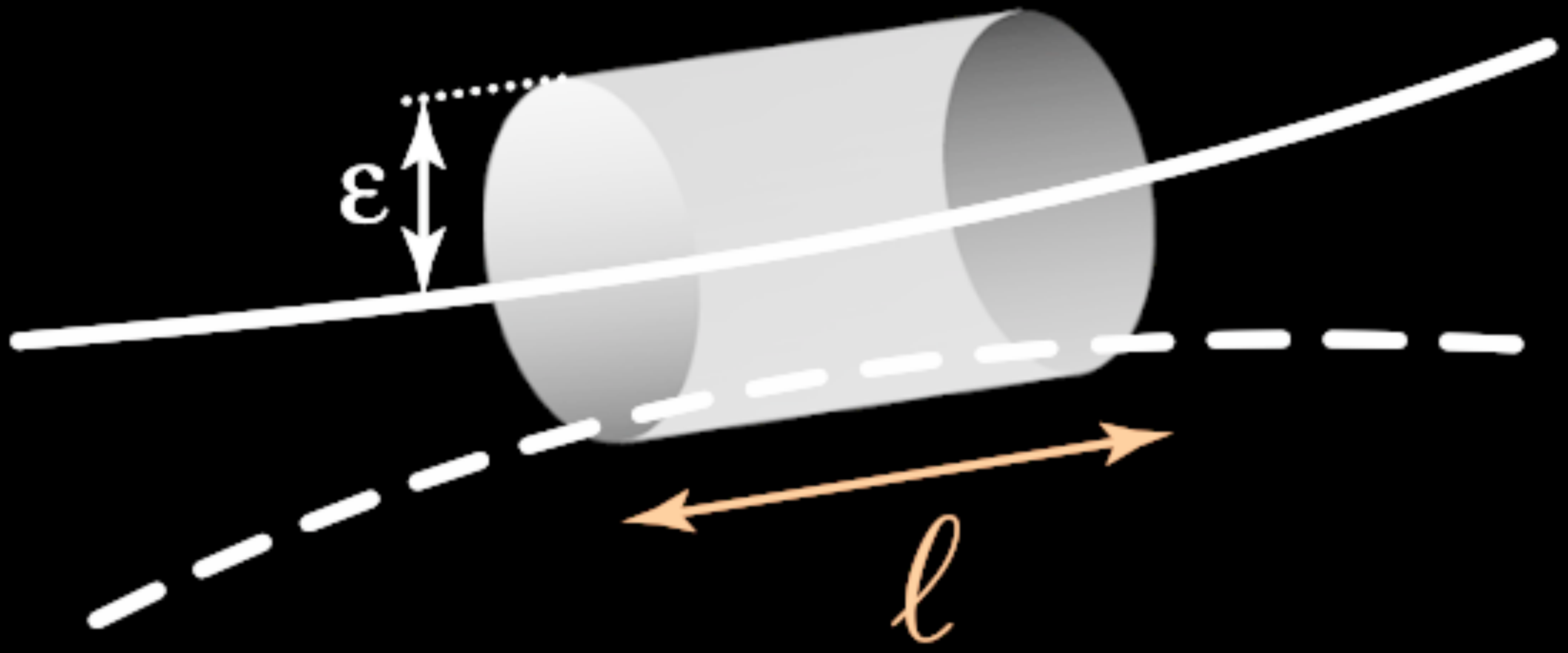




6 DoF system

from time series to dynamics & recurrences

use the reconstructed trajectories from the perspective of the Poincaré recurrence theorem



from time series to dynamics & recurrences

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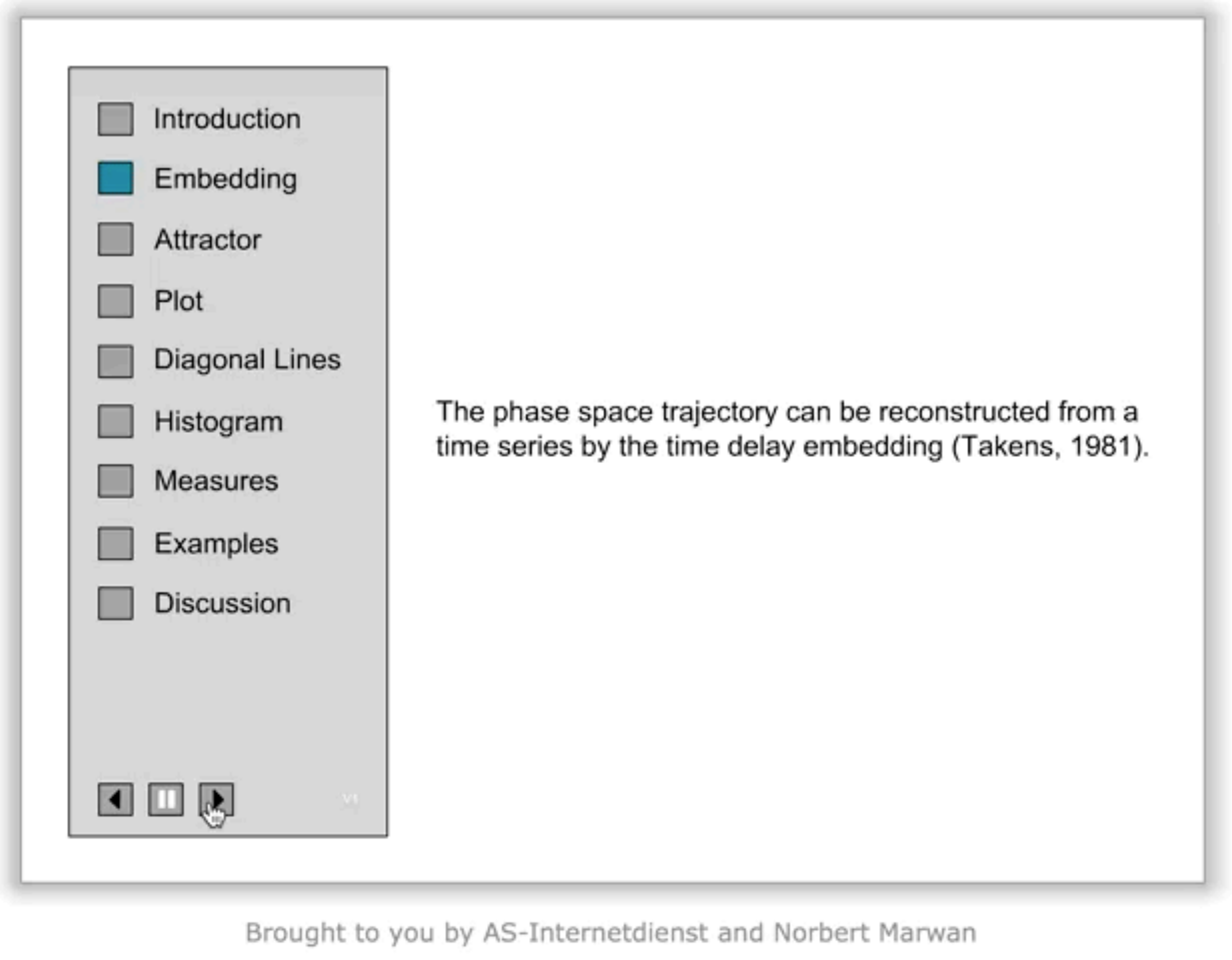
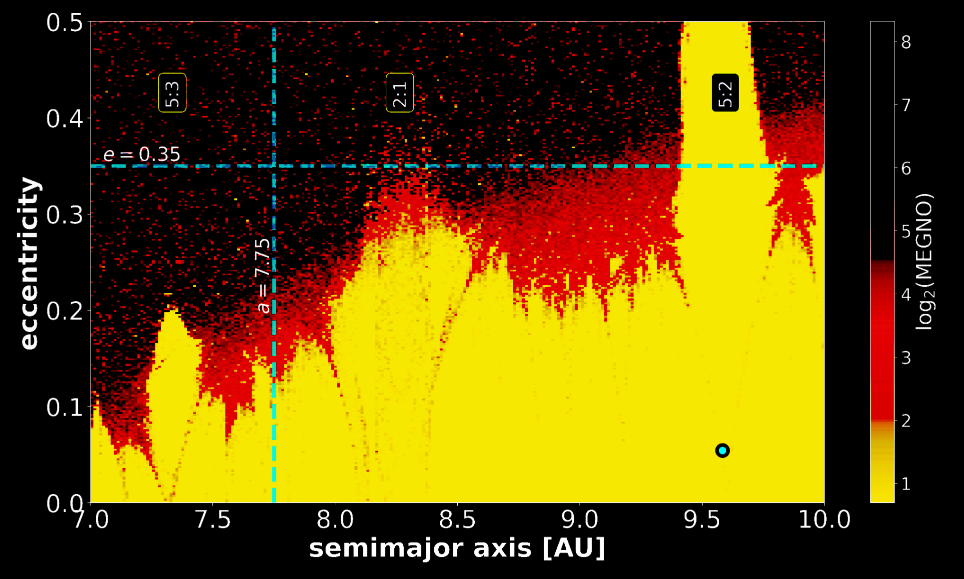
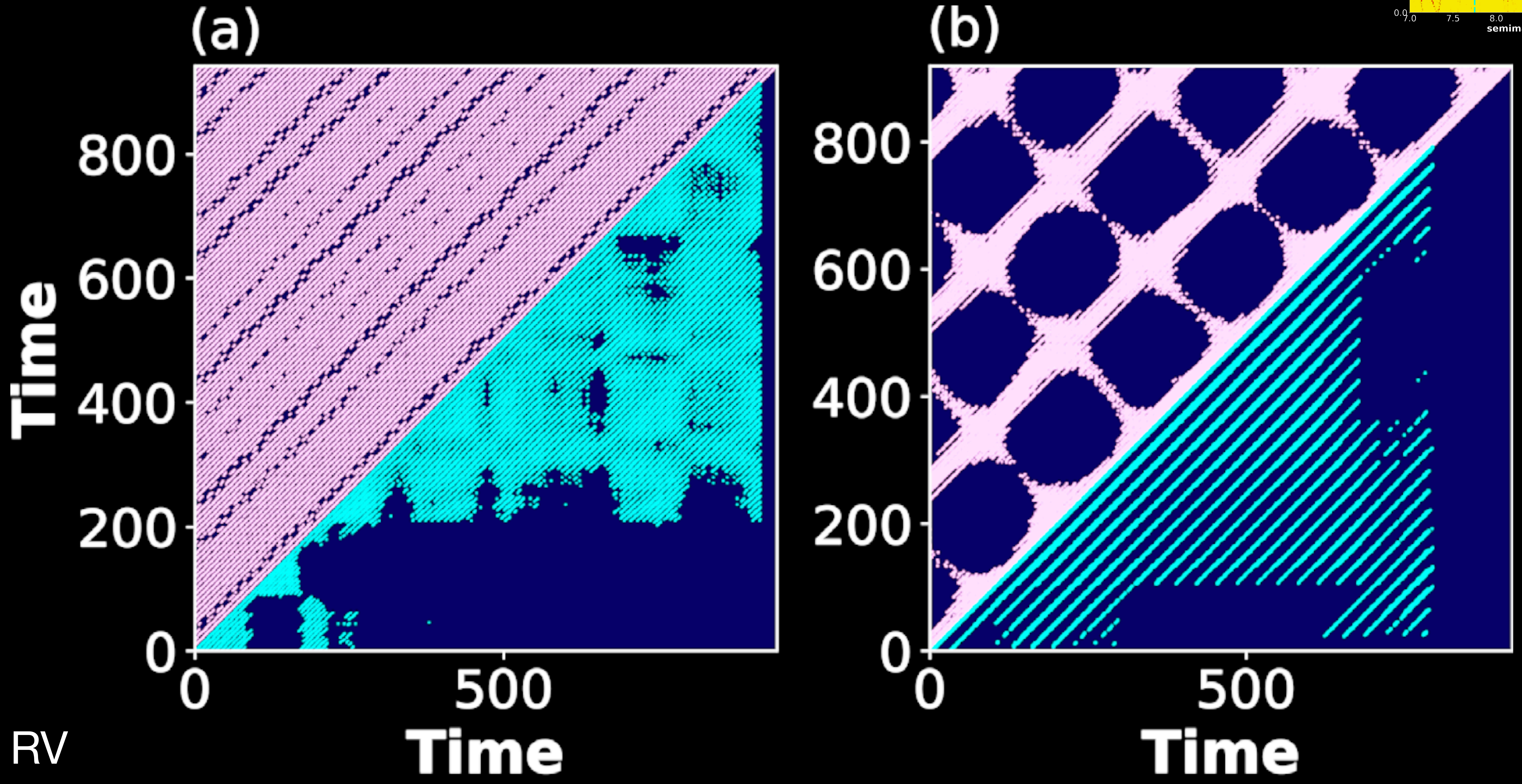


recurrences & exoplanets





RV

TTV

complex networks in brief

a **graph** is a structure amounting to a set of objects in which some of the pairs are in some sense ‘’related”

node/vertices & connections/edges

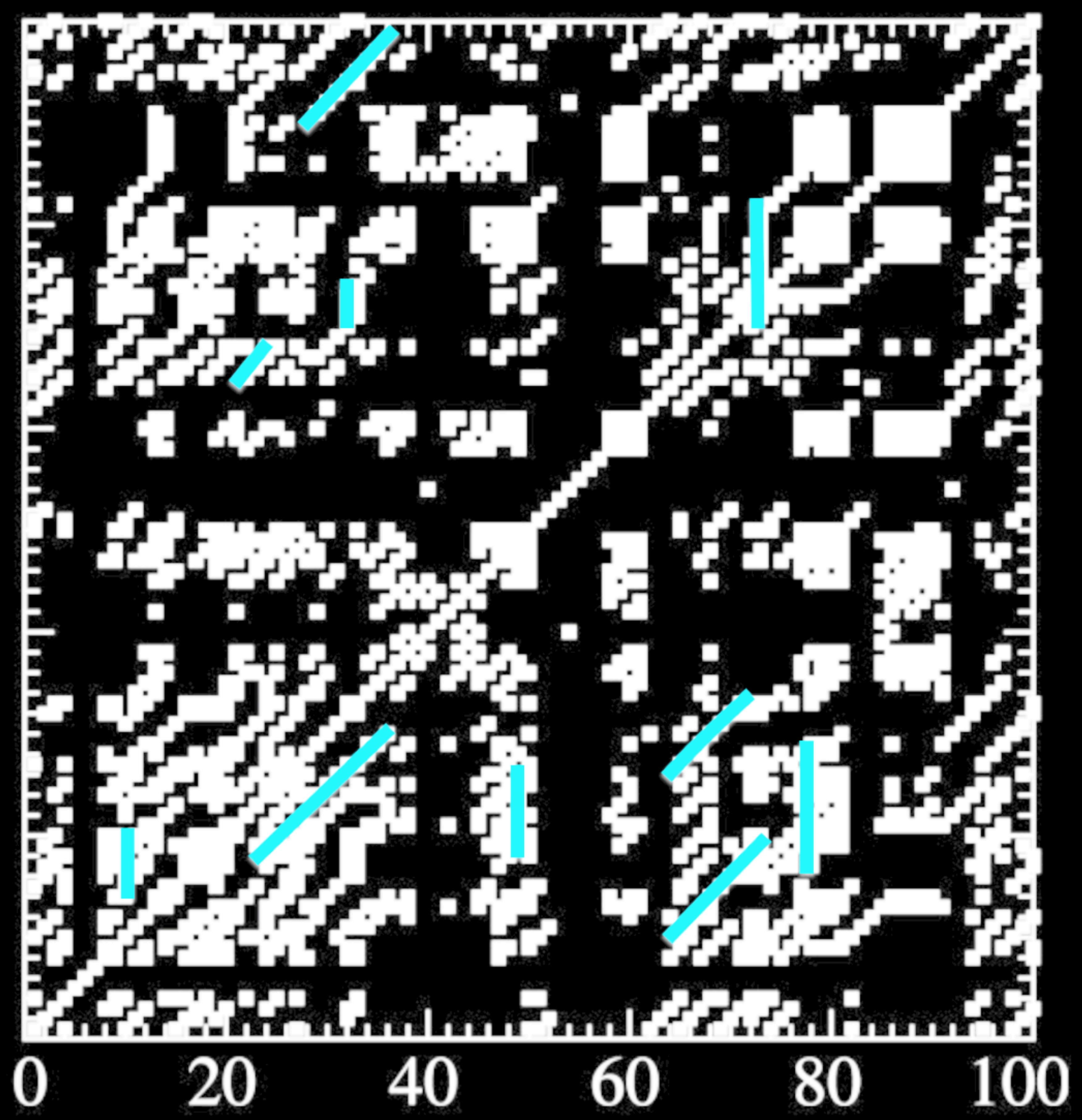
degree list., clustering, transitivity, ave. path length

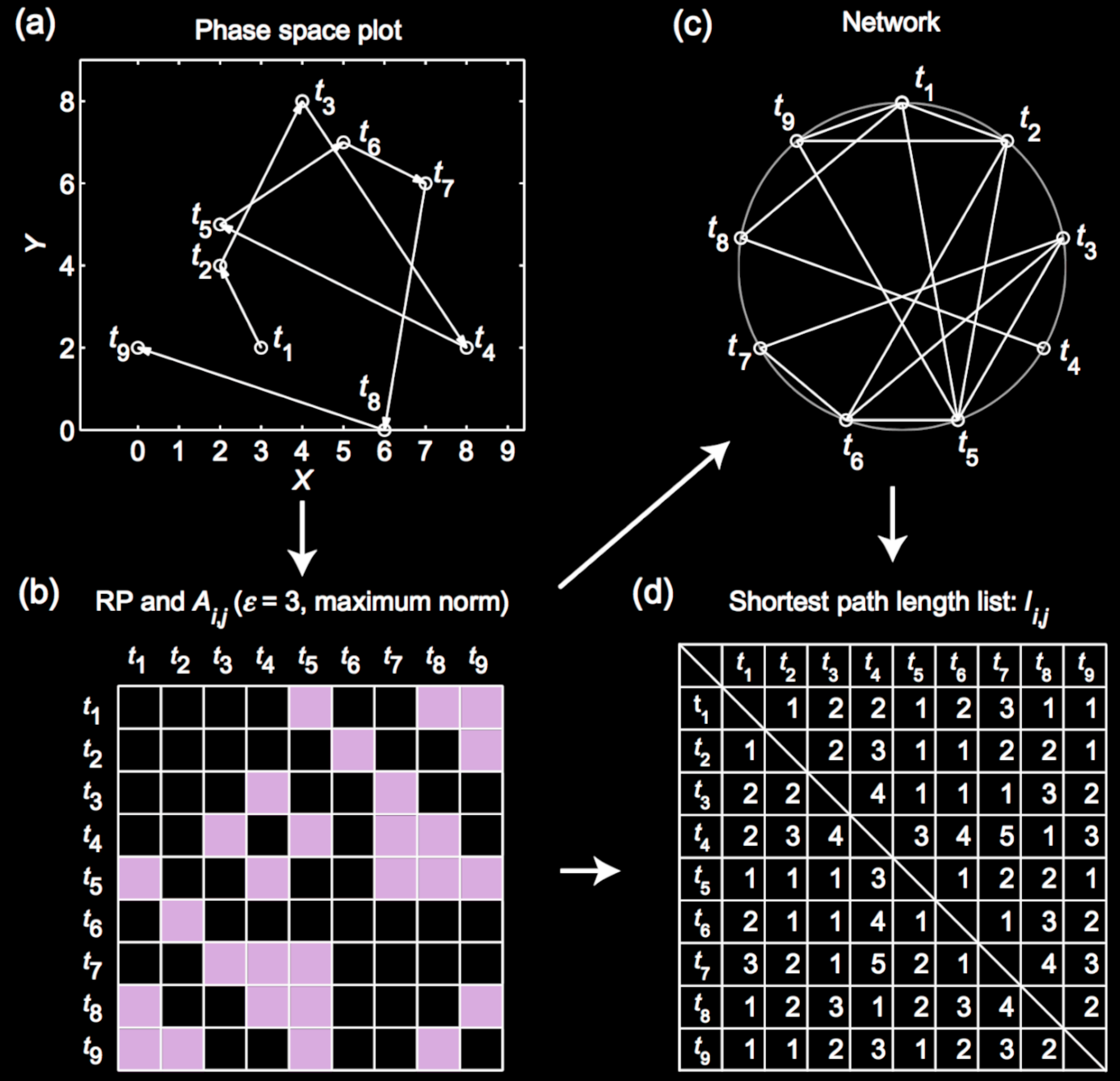
$$\mathbf{A}_{17:27,15:25} = \begin{pmatrix} 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 1&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \end{pmatrix}$$

recurrence plots & networks

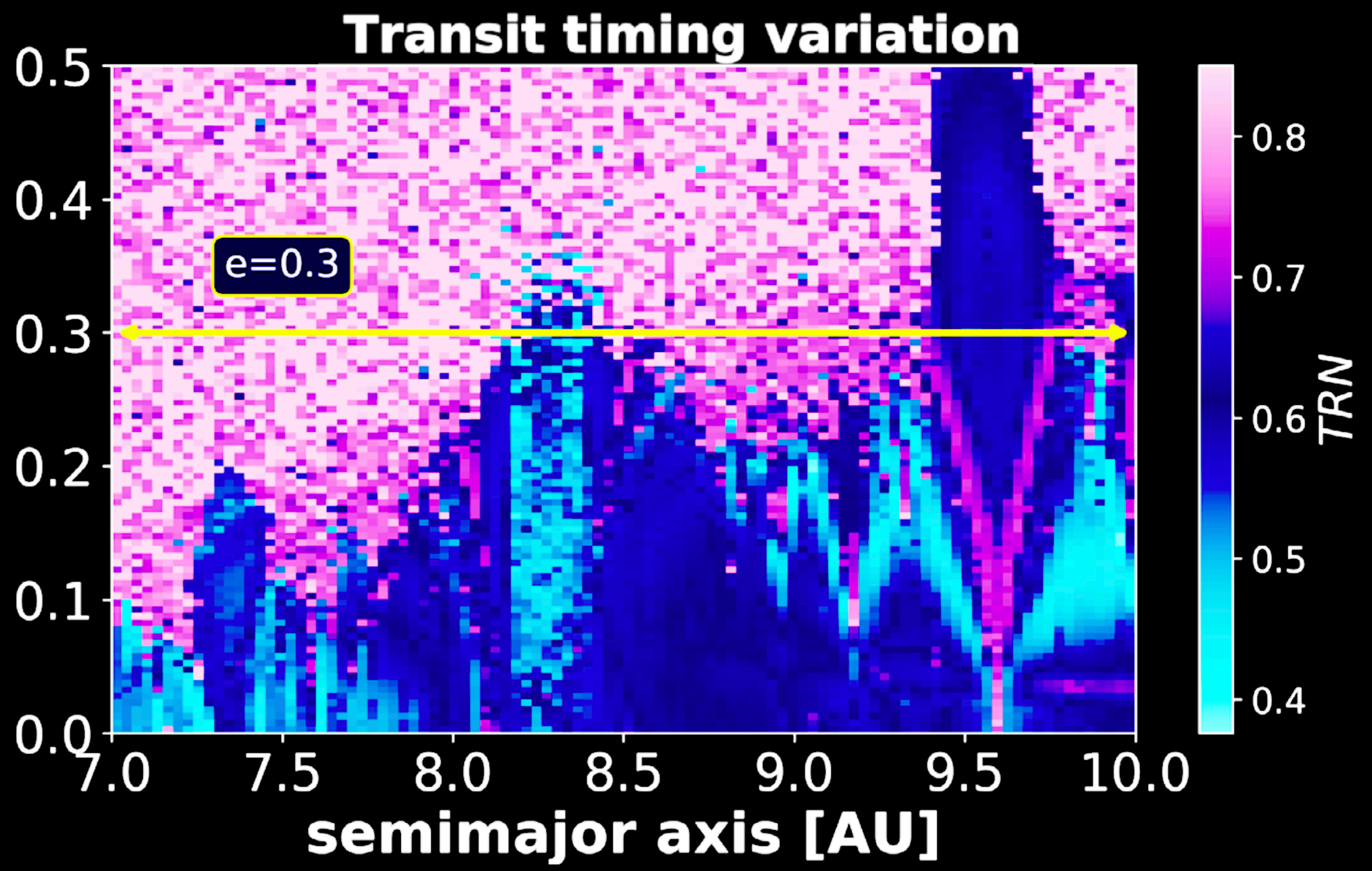
$$\mathbf{R}_{i,j}(\epsilon) = \Theta(\epsilon - ||\boldsymbol{x}_i - \boldsymbol{x}_j||)$$

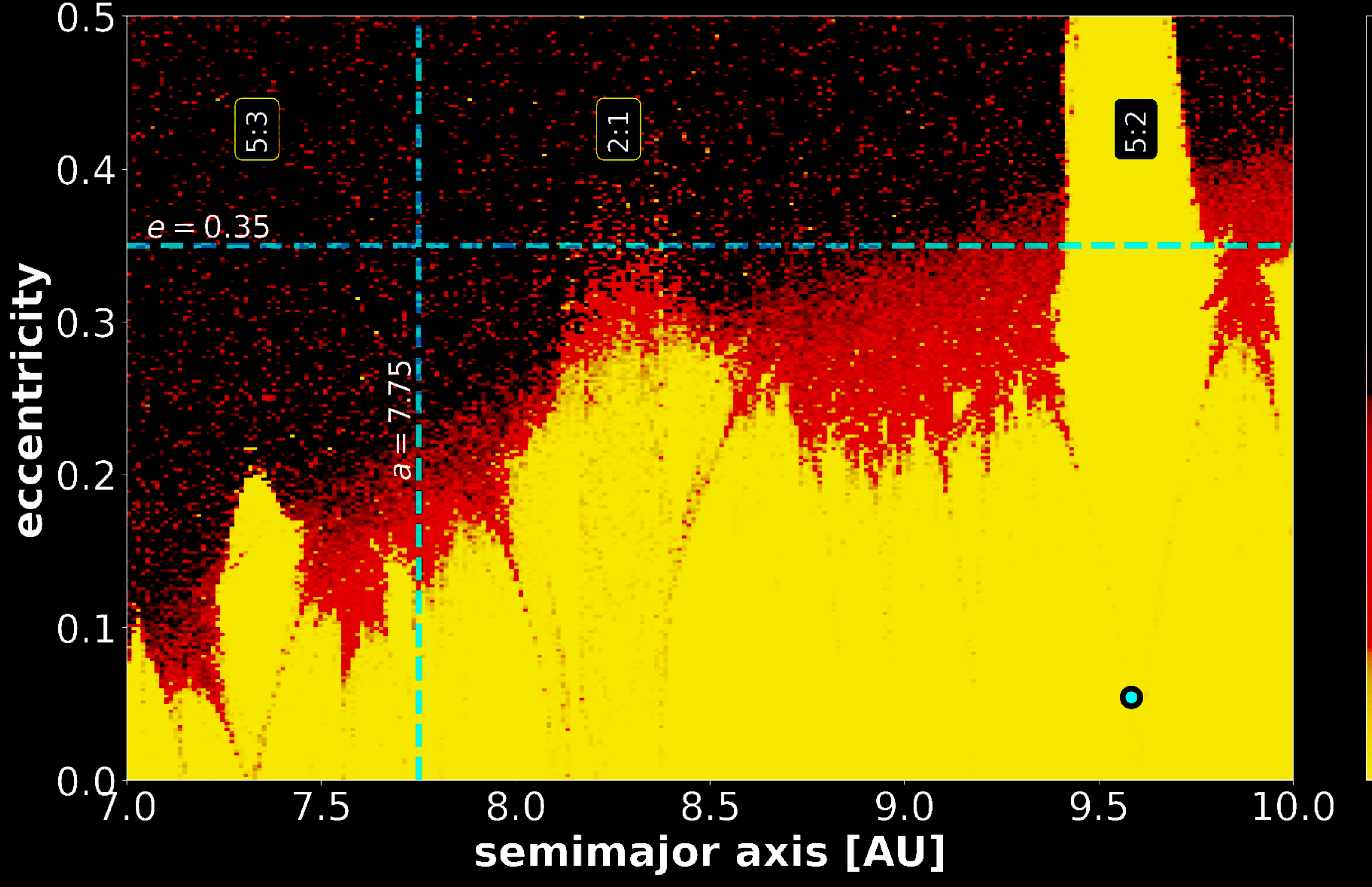
$$\mathbf{A}_{i,j}(\epsilon) = \mathbf{R}_{i,j}(\epsilon) - \delta_{ij}$$



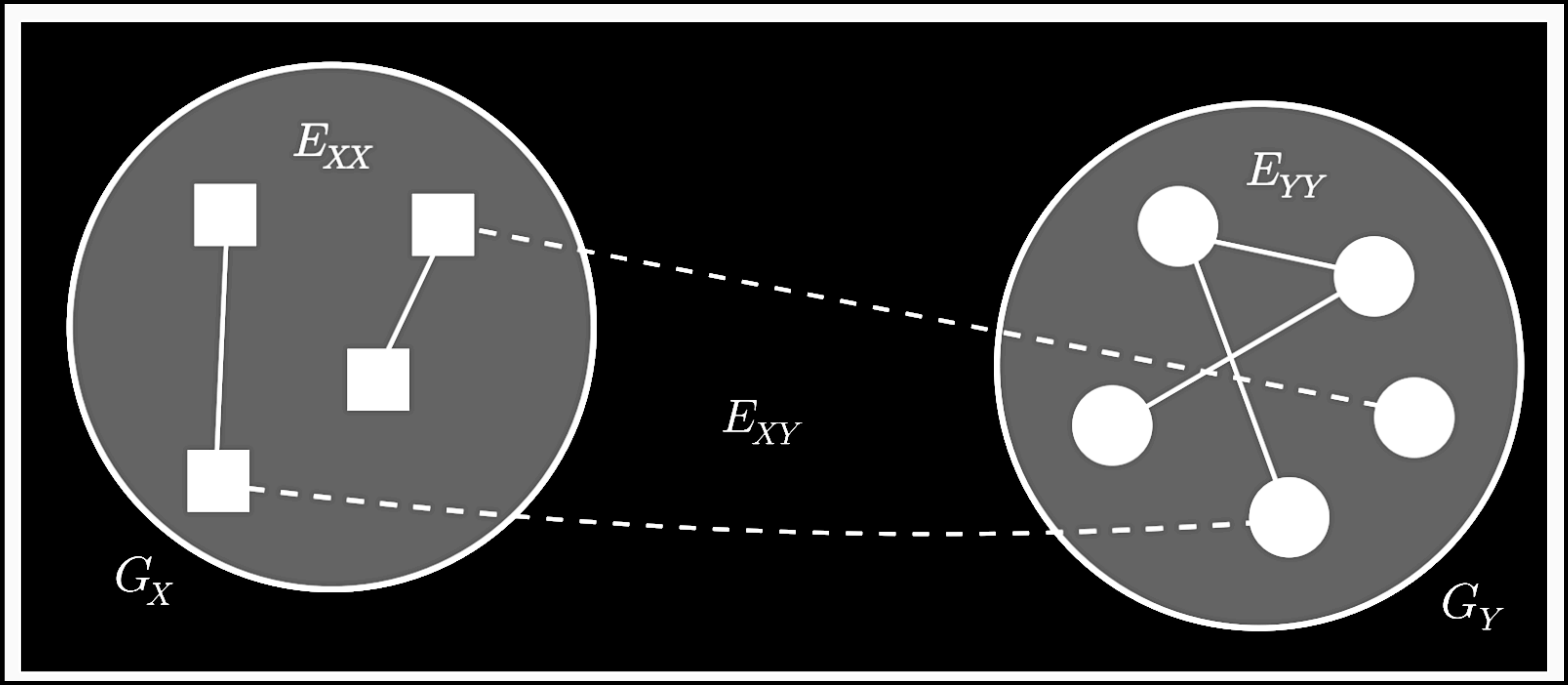


recurrence networks & exoplanet stability

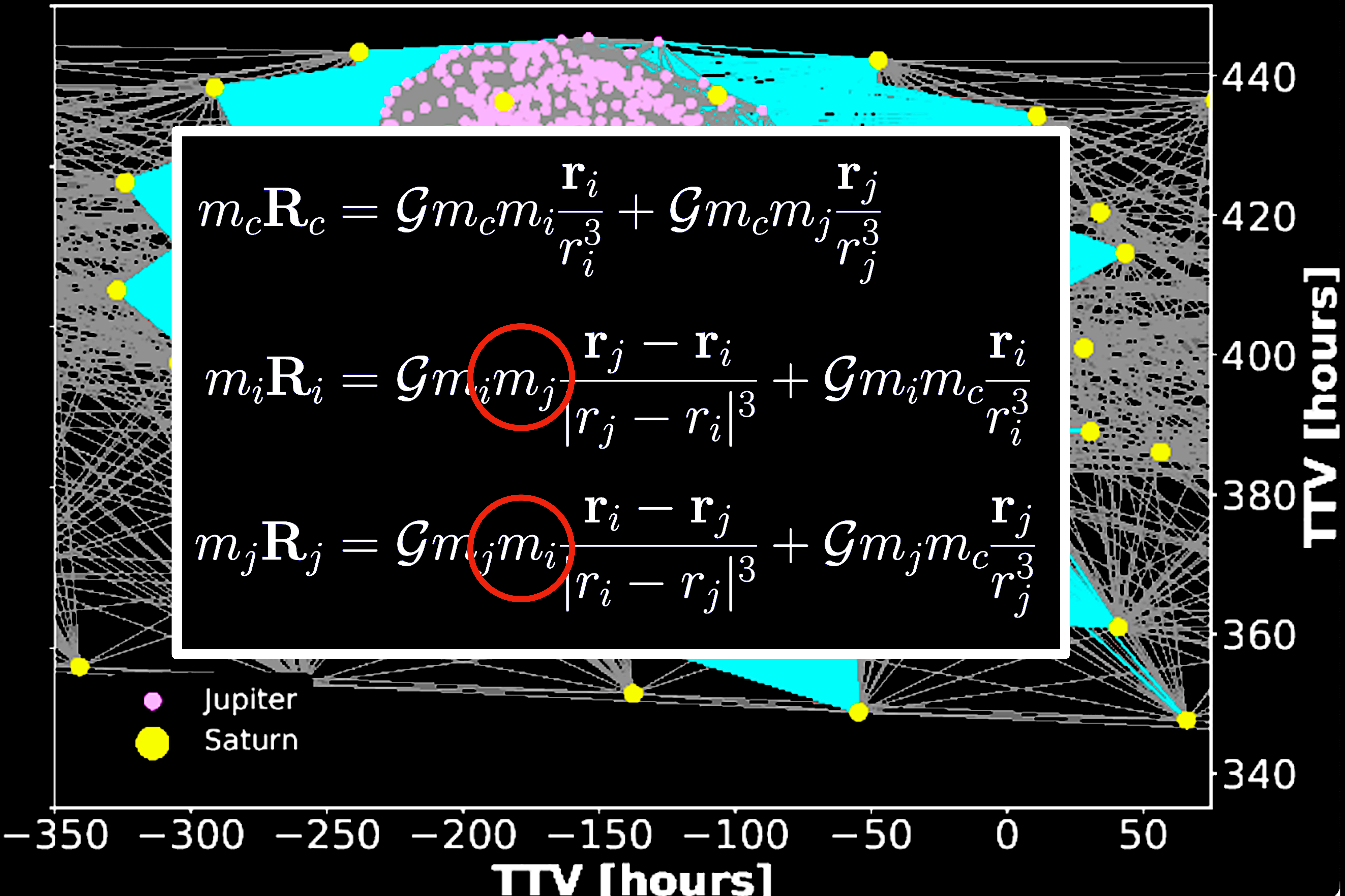




inter-system recurrence networks (isn)



isrn for Jupiter & Saturn

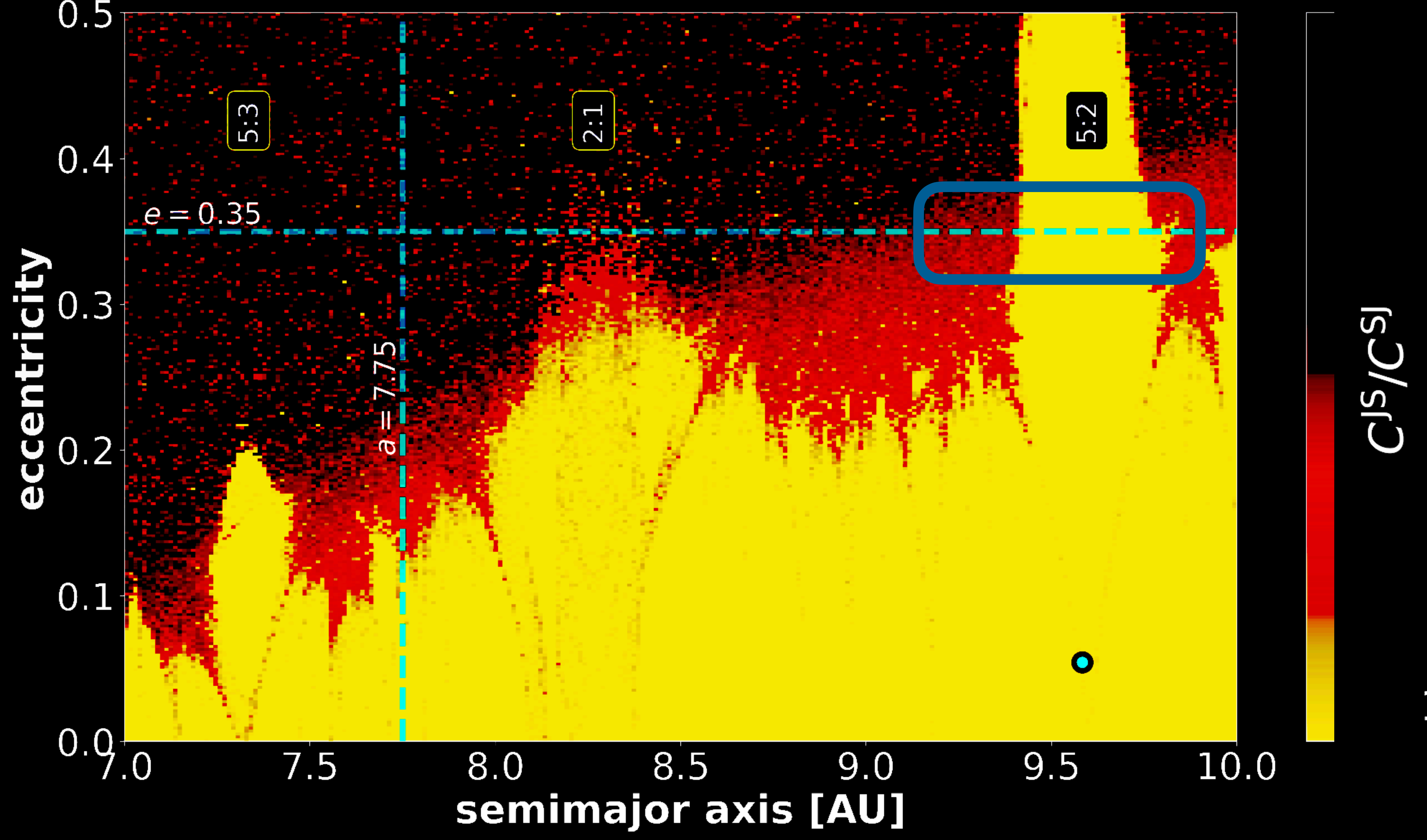


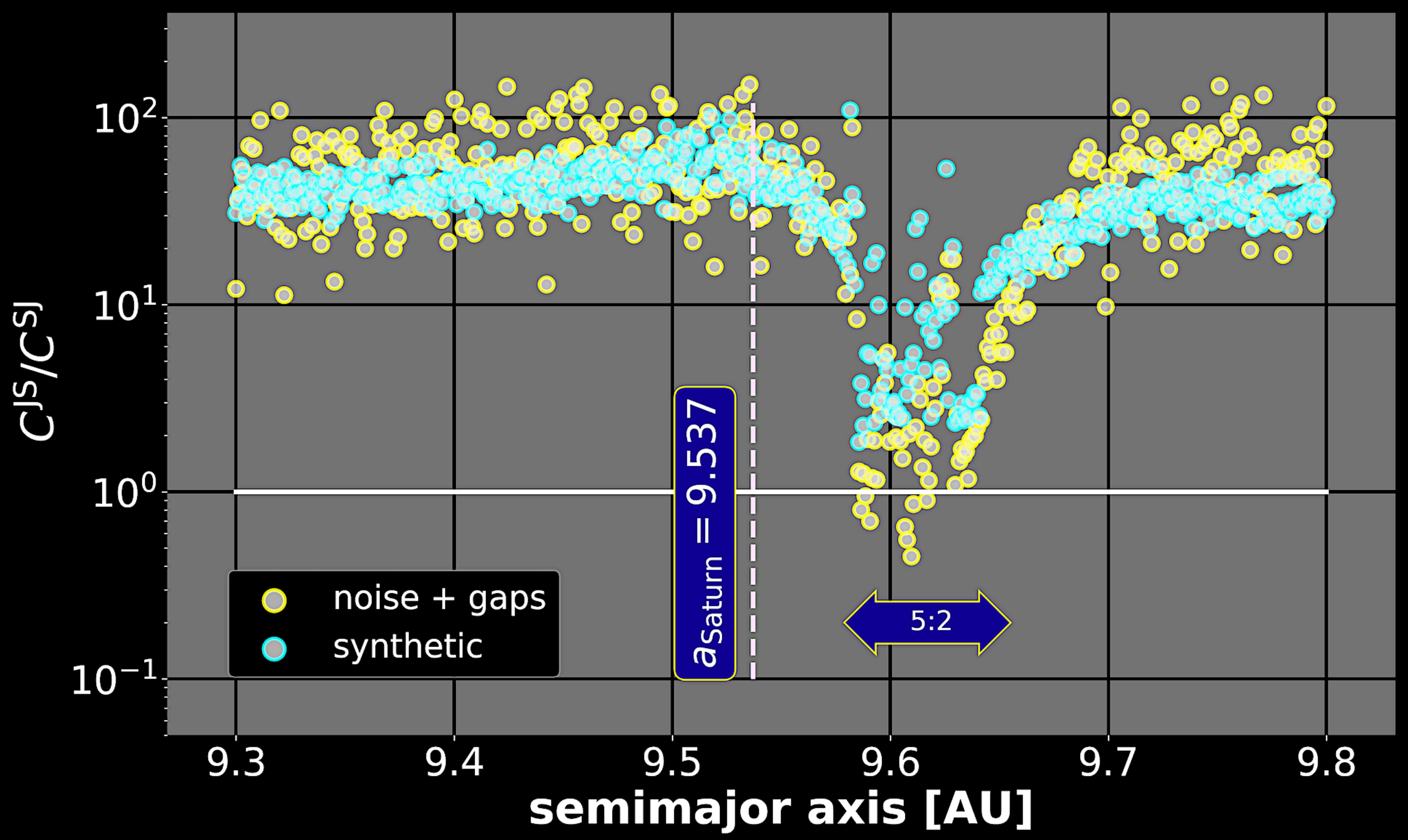
$$m_c\mathbf{R}_c = \mathcal{G}m_c m_i \frac{\mathbf{r}_i}{r_i^3} + \mathcal{G}m_c m_j \frac{\mathbf{r}_j}{r_j^3}$$

$$m_i\mathbf{R}_i = \mathcal{G}m_i m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|r_j - r_i|^3} + \mathcal{G}m_i m_c \frac{\mathbf{r}_i}{r_i^3}$$

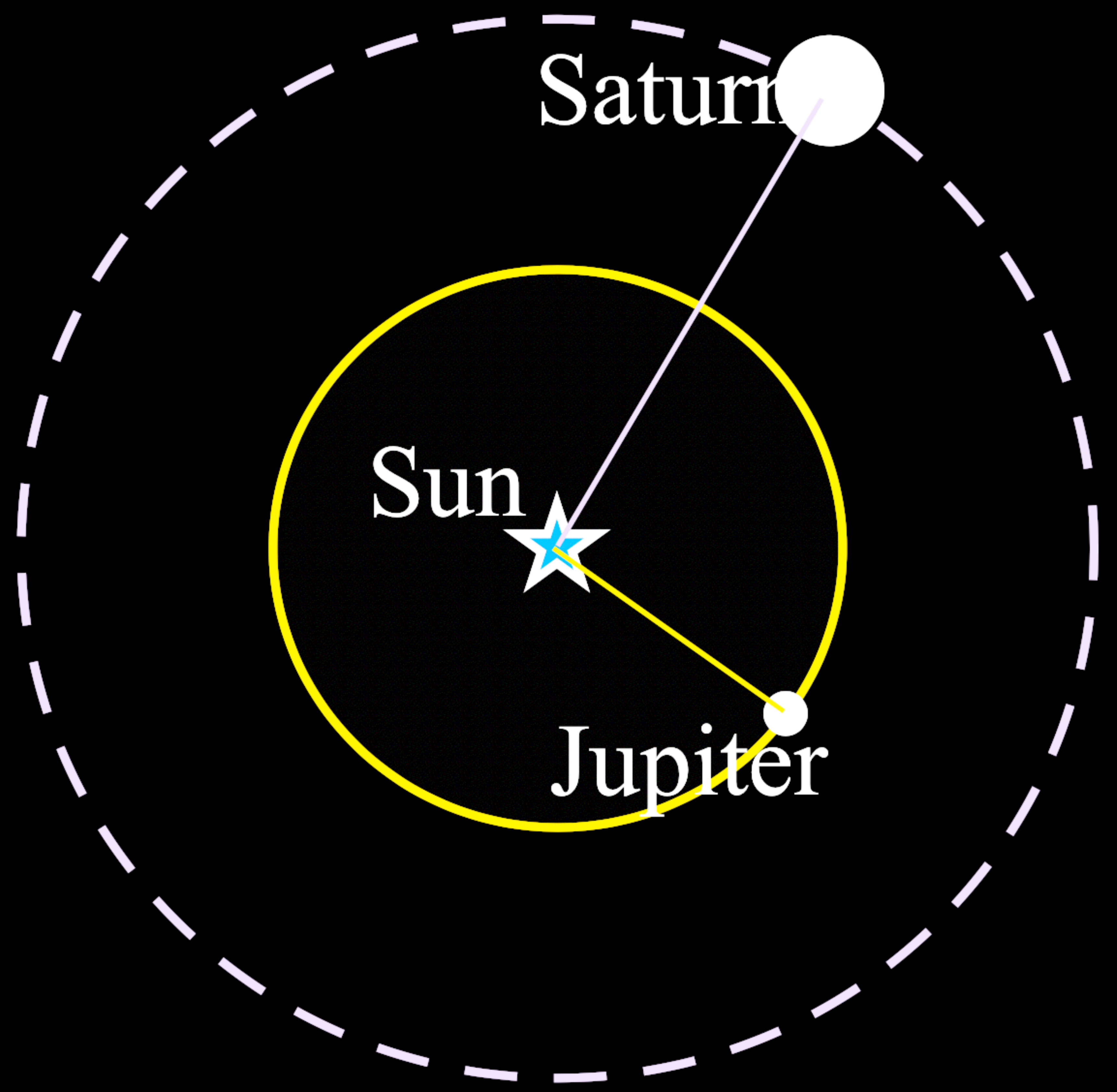
$$m_j\mathbf{R}_j = \mathcal{G}m_j m_i \frac{\mathbf{r}_i - \mathbf{r}_j}{|r_i - r_j|^3} + \mathcal{G}m_j m_c \frac{\mathbf{r}_j}{r_j^3}$$

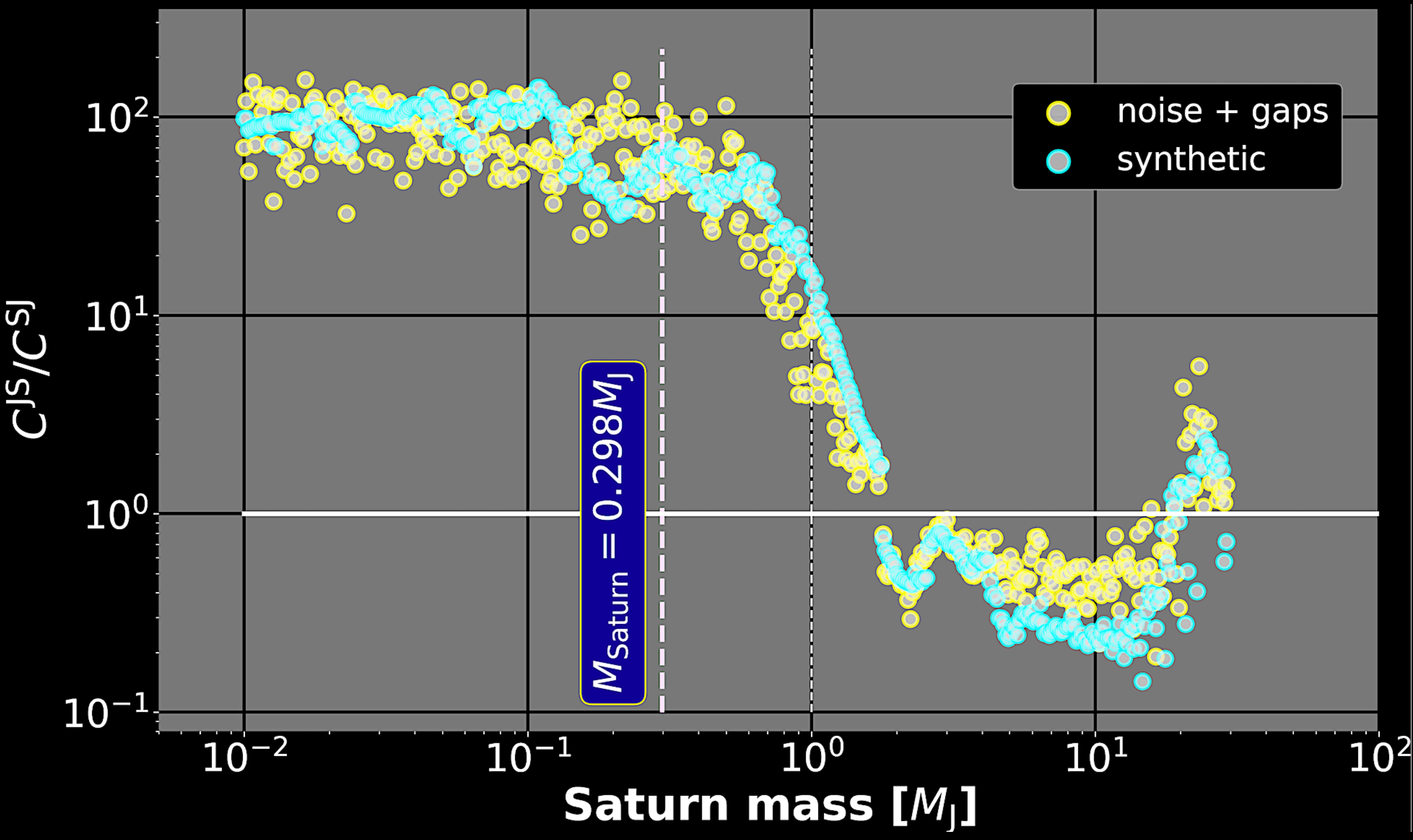
isrn for Jupiter & Saturn





isrn for Jupiter & Saturn





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inter-system recurrence networks

Coupling direction	Expected relation in network measures
no coupling	$\mathcal{T}^{XY} \approx \mathcal{T}^{YX}$, $\mathcal{C}^{XY} \approx \mathcal{C}^{YX}$
$X \rightarrow Y$	$\mathcal{T}^{XY} < \mathcal{T}^{YX}$, $\mathcal{C}^{XY} < \mathcal{C}^{YX}$
$Y \rightarrow X$	$\mathcal{T}^{XY} > \mathcal{T}^{YX}$, $\mathcal{C}^{XY} > \mathcal{C}^{YX}$
$X \leftrightarrow Y$	$\mathcal{T}^{XY} \approx \mathcal{T}^{YX}$, $\mathcal{C}^{XY} \approx \mathcal{C}^{YX}$

$$\mathbf{A}(\varepsilon) = \mathbf{IR}(\varepsilon) - \mathbb{I}_N$$

$$\begin{aligned} CR_{ij}\left(\varepsilon^{XY}\right) &= CR^{XY}\left(x_i, y_j \mid \varepsilon^{XY}\right) \\ &= \Theta\left(\varepsilon^{XY} - d^{XY}(x_i, y_j)\right) \end{aligned}$$

$$\mathbf{IR}(\varepsilon) = \begin{pmatrix} \mathbf{R}^X(\varepsilon) & \mathbf{CR}^{XY}(\varepsilon) \\ \mathbf{CR}^{YX}(\varepsilon) & \mathbf{R}^Y(\varepsilon) \end{pmatrix}$$

isrn for real exoplanets

$$\mathcal{T}^{bc}/\mathcal{T}^{cb} \approx 2.28$$

	Kepler-29 b	Kepler-29 c
Alternative planet names	KOI-738.01, KOI-738 b, KIC 10358759 b	KOI-738.02, KOI-738 c, KIC 10358759 c
Lists	Confirmed planets	
Mass [M_{jup}]	$0.014^{+0.004}_{-0.005}$ edit	0.013±0.004 edit
Mass [M_{earth}]	$4.5^{+1.4}_{-1.5}$	$4.0^{+1.2}_{-1.3}$
Radius [R_{jup}]	0.328 edit	0.264 edit
Radius [R_{earth}]	3.68	2.96
Orbital period [days]	10.33 edit	13.29 edit
Semi-major axis [AU]	0.0900 edit	0.1100 edit
Eccentricity	N/A	N/A
Equilibrium temperature [K]	N/A	N/A
Discovery method	transit	
Discovery year	2012	

openexoplanetcatalogue.com