

LECTURE 5:

(BARE) PERTURBATION THEORY

(IN THE SYMM. PHASE)

PERTURBATION THY (IN THE SYMM. PHASE)

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$$S = S_0 + S_I$$

$$S_0 = \sum_x \frac{1}{2} \Delta_\mu^F \phi \Delta_\mu^F \phi + \frac{m_0^2}{2} \phi^2 = \frac{1}{2} \sum_{xy} \phi_x S_{xy} \phi_y$$

$$S_I = \frac{g_0}{4!} \sum_x \phi(x)^4$$

Expansion in g_0 around $g_0 = 0$

$$G(x_1, \dots, x_n) = \frac{1}{Z} \int \prod_x d\phi(x) \phi(x_1) \dots \phi(x_n) \sum_{m=0}^{\infty} \frac{1}{m!} \left[\frac{-g_0}{4!} \sum_y \phi(y)^4 \right]^m e^{-S_0}$$

$$= \frac{1}{Z} \sum_{m=0}^{\infty} \frac{(-g_0)^m}{m! (4!)^m} \sum_{y_1} \dots \sum_{y_m} \underbrace{\int \prod_x d\phi(x) \phi(x_1) \dots \phi(x_n) \phi(y_1)^4 \dots \phi(y_m)^4 e^{-S_0}}_{\propto \text{an expectation value in the free theory}}$$

$\propto$ a moment of a Gaussian distribution

EXPECTATION VALUES IN THE FREE THY

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(reminder of things in lecture 2)

$$Z_0 \equiv \int \prod_x d\phi(x) \exp\left(-\frac{1}{2} \phi_x S_{xy} \phi_y\right)$$

$$Z_0(\vartheta) = \frac{1}{Z_0} \int \prod_x d\phi(x) \exp\left(-\frac{1}{2} \phi_x S_{xy} \phi_y + \vartheta_x \phi_x\right) = \exp\left[\frac{1}{2} \vartheta_x (S^{-1})_{xy} \vartheta_y\right] = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{2} \left[\vartheta_x (S^{-1})_{xy} \vartheta_y\right]^n$$

free propagator $(S^{-1})_{xy} = G_{xy}^{(0)} = \int \frac{e^{ip(x-y)}}{m_0^2 + \tilde{p}^2}$

where $\tilde{p}_\mu = \frac{2}{a} \sin \frac{p_\mu a}{2}$

and

$$\int \frac{1}{\tilde{p}^2} = \frac{1}{(2\pi)^4} \int_{-\pi/a}^{\pi/a} dp_1 \dots \int_{-\pi/a}^{\pi/a} dp_4$$

integral
over the
Brillouin
zone

$$\langle \phi_{x_1} \dots \phi_{x_{2n}} \rangle_0 = \frac{\partial}{\partial \vartheta_{x_1}} \dots \frac{\partial}{\partial \vartheta_{x_{2n}}} Z_0(\vartheta) \Big|_{\vartheta=0}$$

$$= \sum_{\substack{\text{pairings} \\ \text{of } x_1 \dots x_{2n} \text{ to} \\ \text{pairs } (y_1, z_1) \dots (y_n, z_n)}} G_{y_1 z_1}^{(0)} G_{y_2 z_2}^{(0)} \dots G_{y_n z_n}^{(0)}$$

Wick's theorem

EXAMPLES

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$$\begin{aligned}
 \langle \phi_x \phi_y \phi_z \phi_v \rangle_0 &= \langle \phi_x \phi_y \rangle_0 \langle \phi_z \phi_v \rangle_0 + \langle \phi_x \phi_z \rangle_0 \langle \phi_y \phi_v \rangle_0 + \langle \phi_x \phi_v \rangle_0 \langle \phi_y \phi_z \rangle_0 \\
 &= G_{xy}^{(0)} G_{zv}^{(0)} + G_{xz}^{(0)} G_{yv}^{(0)} + G_{xv}^{(0)} G_{yz}^{(0)} \\
 &= \begin{array}{c} y \\ \uparrow \\ x \end{array} \begin{array}{c} z \\ \uparrow \\ v \end{array} + \begin{array}{c} y \quad z \\ \diagdown \quad \diagup \\ x \quad v \end{array} + \begin{array}{c} y \quad z \\ \text{---} \end{array} \\
 \langle \phi_x \phi_x \phi_x \phi_x \rangle_0 &= 3 \langle \phi_x \phi_x \rangle_0 \langle \phi_x \phi_x \rangle_0 = 3 G_{xx}^{(0)} G_{xx}^{(0)} \\
 &= 3 \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ x \end{array}
 \end{aligned}$$

etc.

these types of expectation values will be the building blocks for perturbation theory

TWO-POINT FN. TO ORDER g_0

$$S_I \sim O(g_0)$$

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$$\frac{\langle \phi(x) \phi(y) e^{-S_I} \rangle_0}{\langle e^{-S_I} \rangle_0} = \frac{\langle \phi(x) \phi(y) \rangle_0 - \langle \phi(x) \phi(y) S_I \rangle_0 + O(g_0^2)}{1 - \langle S_I \rangle_0 + O(g_0^2)}$$

$$= \langle \phi(x) \phi(y) \rangle_0 + \overbrace{\left(\langle \phi(x) \phi(y) \rangle_0 \langle S_I \rangle_0 - \langle \phi(x) \phi(y) S_I \rangle_0 \right)}^{O(g_0^2)} + O(g_0^2)$$

$$= G_0(x, y) + \left(G_0(x, y) \boxed{\langle S_I \rangle_0} - \boxed{\langle \phi(x) \phi(y) S_I \rangle_0} \right) + O(g_0^2)$$

we want to calculate
these expectation values in
the free theory

$$\langle S_I \rangle_0 = - \frac{g_0}{4!} \sum_{\mathbf{z}} \langle \underbrace{\phi(\mathbf{z}) \phi(\mathbf{z}) \phi(\mathbf{z})}_3 \phi(\mathbf{z}) \rangle_0 = \frac{1}{8} \left(-g_0 \sum_{\mathbf{z}} G_0(\mathbf{z}, \mathbf{z}) G_0(\mathbf{z}, \mathbf{z}) \right)$$

$$= \frac{1}{8} \quad \text{[Diagram: A tadpole diagram consisting of a loop with a vertex labeled } \mathbf{z} \text{ at the bottom.]}$$

implicit in the picture: a factor $-g_0$ for an internal pt.
a sum over where the internal pt. is

$$\langle \phi(x) \phi(y) S_I \rangle_0 = - \frac{g_0}{4!} \sum_{\mathbf{z}} \langle \phi(x) \phi(y) \phi(\mathbf{z}) \phi(\mathbf{z}) \phi(\mathbf{z}) \phi(\mathbf{z}) \rangle_0$$

$$= - \frac{g_0}{4!} \sum_{\mathbf{z}} \langle \phi(x) \phi(y) \underbrace{\phi(\mathbf{z}) \phi(\mathbf{z}) \phi(\mathbf{z}) \phi(\mathbf{z})}_4 \rangle_0 - \frac{g_0}{4!} \sum_{\mathbf{z}} \langle \phi(x) \phi(y) \underbrace{\phi(\mathbf{z}) \phi(\mathbf{z}) \phi(\mathbf{z})}_3 \phi(\mathbf{z}) \rangle_0$$

$$= \frac{1}{8} \left[-g_0 G_0(x, y) \sum_{\mathbf{z}} G_0(\mathbf{z}, \mathbf{z}) G_0(\mathbf{z}, \mathbf{z}) \right] + \frac{1}{2} \left[-g_0 \sum_{\mathbf{z}} G_0(x, \mathbf{z}) G_0(\mathbf{z}, \mathbf{z}) G_0(\mathbf{z}, y) \right]$$

cancels against $\langle S_I \rangle_0 \langle \phi(x) \phi(y) \rangle_0$

$$\frac{1}{2} \quad \text{[Diagram: A tadpole diagram with external legs x and y, and a loop vertex labeled } \mathbf{z} \text{ at the top.]}$$

TADPOLE

GOING TO MOMENTUM SPACE

$$\frac{1}{\Omega} \sum_{p \in B} \xrightarrow{L \rightarrow \infty} \frac{1}{(2\pi)^4} \int_{-\pi/a}^{\pi/a} dp_1 \dots \int_{-\pi/a}^{\pi/a} dp_4 \equiv \int_{\vec{p}} \quad \text{to make notation short}$$

$$\sum_x e^{i(p-p')x} = \Omega \delta_{pp'} \rightarrow \delta_{\vec{p} \vec{p}'}$$

$$-\frac{g_0}{2} \sum_z G_0(x, z) G_0(z, z) G_0(z, y) = -\frac{g_0}{2} \int_{\vec{p}_1} \int_{\vec{p}_2} \int_{\vec{p}_3} \sum_z e^{ip_1(x-z)} 1 e^{ip_3(z-y)} G_0(p_1) G_0(p_2) G_0(p_3)$$

$$\begin{aligned} & \xrightarrow{\uparrow} -\frac{g_0}{2} \int_{\vec{p}_1} \int_{\vec{p}_2} \int_{\vec{p}_3} \delta_{\vec{p}_1 \vec{p}_3} \frac{e^{ip_1 x}}{m_0^2 + \vec{p}_1^2} \frac{1}{m_0^2 + \vec{p}_2^2} \frac{e^{-ip_3 y}}{m_0^2 + \vec{p}_3^2} = -\frac{g_0}{2} \int_{\vec{p}} \frac{e^{ip(x-y)}}{(m_0^2 + \vec{p}^2)^2} \int_{\vec{q}} \frac{1}{m_0^2 + \vec{q}^2} \end{aligned}$$

$$\vec{p}_M \equiv \frac{2}{a} \sin \frac{p_M a}{2}$$

ADD TO THE TREE LEVEL TERM

$$\langle \phi(x) \phi(y) \rangle = \int_{\vec{p}} e^{ip(x-y)} \left[\underbrace{\frac{1}{m_0^2 + \vec{p}^2}}_{G^{(0)}} + \underbrace{\frac{1}{m_0^2 + \vec{p}^2}}_{G^{(0)}} \underbrace{\left(-\frac{g_0}{2} \int_{\vec{q}} \frac{1}{m_0^2 + \vec{q}^2} \right)}_{-T} \underbrace{\frac{1}{m_0^2 + \vec{p}^2}}_{G^{(0)}} \right] + O(g_0^2)$$

SUM AN ∞ # OF DIAGRAMS

$$\text{---} + \text{---} \text{ (loop) } + \text{---} \text{ (2 loops) } + \text{---} \text{ (3 loops) } + \dots$$

$$G^{(0)} + G^{(0)}(-T)G^{(0)} + G^{(0)}(-T)G^{(0)}(-T)G^{(0)} + \dots$$

$$= G^{(0)} [1 + (-T)G^{(0)} + (-T)G^{(0)}^2 + \dots]$$

$$= G^{(0)} \frac{1}{1 + G^{(0)}T}$$

$$= \frac{1}{m_0^2 + \vec{p}^2} \frac{1}{1 + \frac{T}{m_0^2 + \vec{p}^2}} = \frac{1}{m_0^2 + T + \vec{p}^2}$$

THE MASS TO ORDER g_0

$$\langle \phi(x) \phi(y) \rangle = \int_{\bar{p}} e^{ip(x-y)} \frac{1}{m_0^2 + \frac{g_0}{2} \int \frac{1}{\bar{q}} \frac{1}{m_0^2 + \tilde{q}^2} + \tilde{p}^2} + O(g_0^2)$$

$$\frac{1}{\xi^2} = m^2 = m_0^2 + \frac{g_0}{2} \int \frac{1}{\bar{q}} \frac{1}{m_0^2 + \tilde{q}^2} + O(g_0^2)$$

RESTORE FACTORS OF LATTICE SPACING

$$\int_{-\pi/a}^{\pi/a} \frac{dq_1}{(2\pi)} \int_{-\pi/a}^{\pi/a} \frac{dq_2}{(2\pi)} \int_{-\pi/a}^{\pi/a} \frac{dq_3}{(2\pi)} \int_{-\pi/a}^{\pi/a} \frac{dq_4}{(2\pi)} \frac{1}{m_0^2 + \sum_n \left(\frac{2}{a} \sin \frac{q_n a}{2} \right)^2}$$

naive $a \rightarrow 0$
i.e. $a \rightarrow 0$ while $m_0 = \text{fixed}$ $g_0 = \text{fixed}$

$$\rightarrow \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{1}{m_0^2 + q^2} \propto \int q^3 dq \frac{1}{q^2} \propto \int q dq$$

DIVERGENT
?????

WHAT HAPPENS AT HIGHER ORDERS

$$\langle \phi(x) \phi(y) \rangle = \int_{\bar{p}} e^{ip(x-y)}$$

at small momenta:

$$G(p) \simeq \left(\frac{Z}{m_R^2} \right) - \left(\frac{Z}{m_R^4} \right) p^2 + O(p^4)$$

coefficients (#) finite for $a = \text{finite}$, but
diverge for $m_0, g_0 = \text{fixed}$ $a \rightarrow 0$

$Z \leftarrow$ WAVEFN.

RENORMALIZATION

$$Z = 1 + (\#)(g_0)^2 + \dots$$

$$m_R^2 + p^2 + O(p^4)$$



RENORMALIZED

MASS

define

$$\phi = \sqrt{Z} \phi_R$$

renormalized field

$$m_R^2 = m_0^2 + (\#)g_0 + (\#)g_0^2 + \dots$$

not quite the physical/pole mass as the $O(p^4)$ term shifts where the pole is in p_4

HOW TO CALC. Z and m_R FROM A SIMULATION?

"magnetization" $M = \sum_x \phi_x \Rightarrow \frac{1}{\Omega} \langle M^2 \rangle = \frac{1}{\Omega} \sum_{x,y} \langle \phi_x \phi_y \rangle = \chi_M$ ($\langle M \rangle = 0$ in symm. phase)

"magnetic" susceptibility $\chi_M = \sum_x \langle \phi_0 \phi_x \rangle = G(\vec{p}=0) = Z/m_R^2$

second moment of the correlation fn.

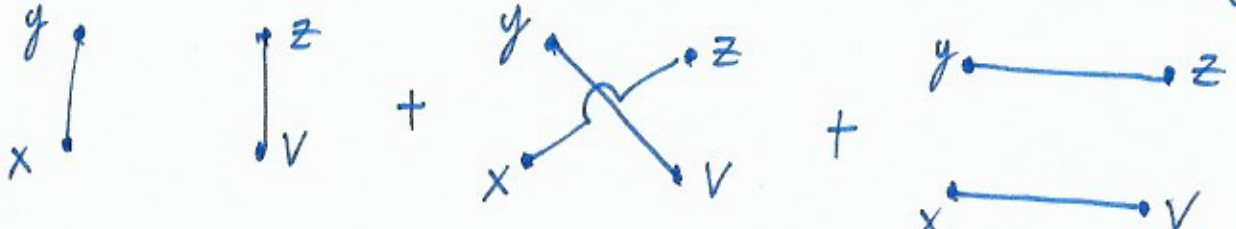
$$M_2 \equiv \sum_x x^2 \langle \phi_0 \phi_x \rangle = - \frac{\partial}{\partial p_x} \frac{\partial}{\partial p_x} G(p) \Big|_{p=0} = \frac{2Z}{m_R^4}$$

(integration by parts)

4PT FN TO ORDER g_0

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$$\langle \phi(x) \phi(y) \phi(z) \phi(v) \rangle_0 \Big|_{O(g_0)} = \underbrace{\langle \phi(x) \phi(y) \phi(z) \phi(v) \rangle_0}_{G_0(x,y) G_0(z,v) + G_0(x,z) G_0(y,v) + G_0(x,v) G_0(y,z)} \langle \int_I \rangle_0 - \langle \phi(x) \phi(y) \phi(z) \phi(v) \int_I \rangle_0$$



again terms like

$$\langle \overbrace{\phi(x)}^1 \overbrace{\phi(y)}^1 \overbrace{\phi(z)}^1 \overbrace{\phi(v)}^1 \rangle \left(-\frac{g_0}{4!} \right) \sum_w \langle \overbrace{\phi(w)}^1 \overbrace{\phi(w)}^1 \overbrace{\phi(w)}^1 \overbrace{\phi(w)}^1 \rangle$$

cancel with

$$\langle \phi \phi \phi \phi \rangle_0 \langle \int_I \rangle_0$$

remaining terms:

$$\sum_w -\frac{g_0}{4!} \langle \overbrace{\phi(x) \phi(y) \phi(z) \phi(v)}^4 \underbrace{\phi(w) \phi(w) \phi(w) \phi(w)}_3 \rangle_0 \Rightarrow -g_0 \sum_w G_0(x,w) G_0(y,w) G_0(z,w) G_0(v,w)$$

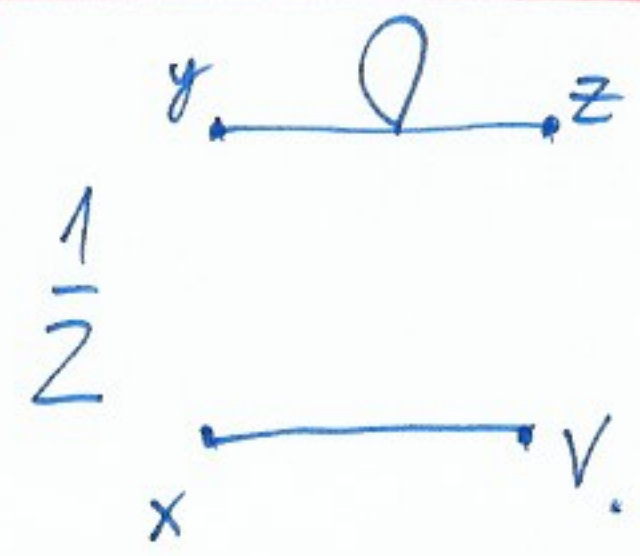
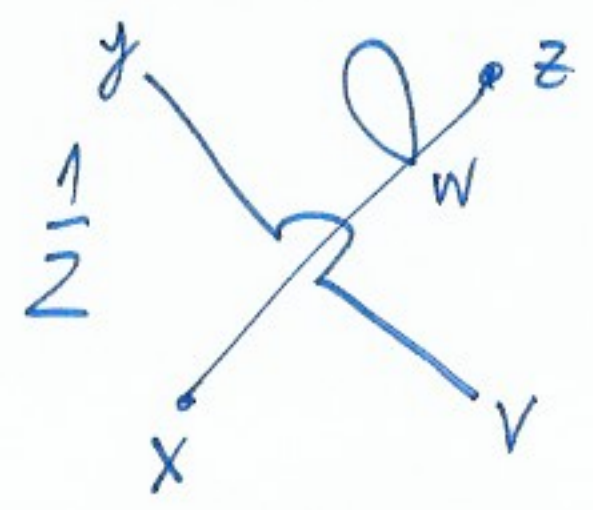
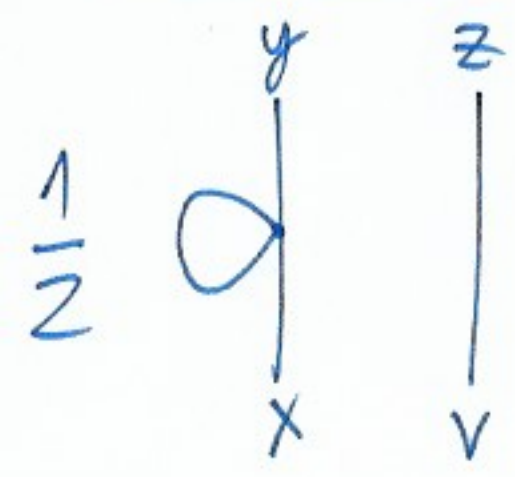
TREE LEVEL COUPLING



$$\sum_w - \frac{g_0}{4!} \langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(w) \phi(w) \phi(w) \phi(w) \phi(w) \rangle_0 = \frac{1}{2} \begin{array}{c} y \\ | \\ x \end{array} \begin{array}{c} z \\ | \\ w \\ | \\ v \end{array}$$

$$\sum_w - \frac{g_0}{4!} \langle \phi(x) \phi(y) \phi(z) \phi(v) \phi(w) \phi(w) \phi(w) \phi(w) \phi(w) \rangle_0 = \frac{1}{2} \begin{array}{c} y \quad z \\ \diagdown \quad \diagup \\ \quad w \\ \diagup \quad \diagdown \\ x \quad v \end{array}$$

$$\sum_w - \frac{g_0}{4!} \langle \phi(x) \phi(y) \phi(z) \phi(v) \phi(w) \phi(w) \phi(w) \phi(w) \phi(w) \rangle_0 = \frac{1}{2} \begin{array}{c} y \quad z \\ \text{---} w \text{---} \\ \quad \uparrow \\ \quad \text{loop} \\ \text{---} x \text{---} v \end{array}$$



NOT
CORRECTIONS
TO THE
COUPLING

JUST
CORRECTIONS
TO THE
MASS
TAKEN
INTO
ACCOUNT
THE PARTICLES
DON'T EVEN
INTERACT

CONNECTED GREEN FUNCTIONS

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$$G(x_1, x_2, \dots, x_n) = \sum_{\mathcal{P}} G_c(x_{i_1}, \dots, x_{j_1}) \dots G_c(x_{i_k}, \dots, x_{e_k})$$

$\mathcal{P}$: partition of indices $\{1, 2, \dots, n\}$ into non-empty subsets

$$G(x) = G_c(x)$$

$$G(x, y) = G_c(x) G_c(y) + G_c(x, y)$$

$$\begin{aligned} G(x, y, z) = & G_c(x) G_c(y) G_c(z) + \\ & G_c(x) G_c(y, z) + G_c(y) G_c(x, z) + \\ & G_c(z) G_c(x, y) + \\ & G_c(x, y, z) \end{aligned}$$

...

INVERSE:

$$G_c(x_1, \dots, x_n) = \sum_{\mathcal{P}} (-1)^{k-1} (k-1)! G(x_{i_1}, \dots, x_{j_1}) G(x_{i_2}, \dots, x_{e_2})$$

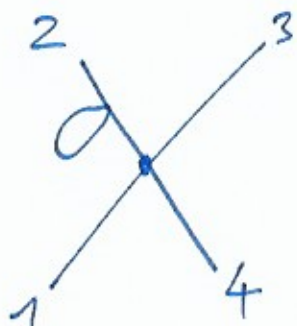
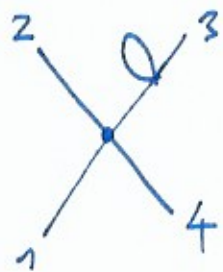
k : # factors in term

$$G_c(x, y) = G(x, y) - G(x) G(y)$$

$$\begin{aligned} G_c(x, y, z) = & G(x, y, z) - G(x) G(y, z) - G(y) G(x, z) \\ & - G(z) G(x, y) \\ & + 2 G(x) G(y) G(z) \end{aligned}$$

... FOR A FREE FIELD
 $G_c(x_1, \dots, x_n) = 0$ for $n \geq 3$

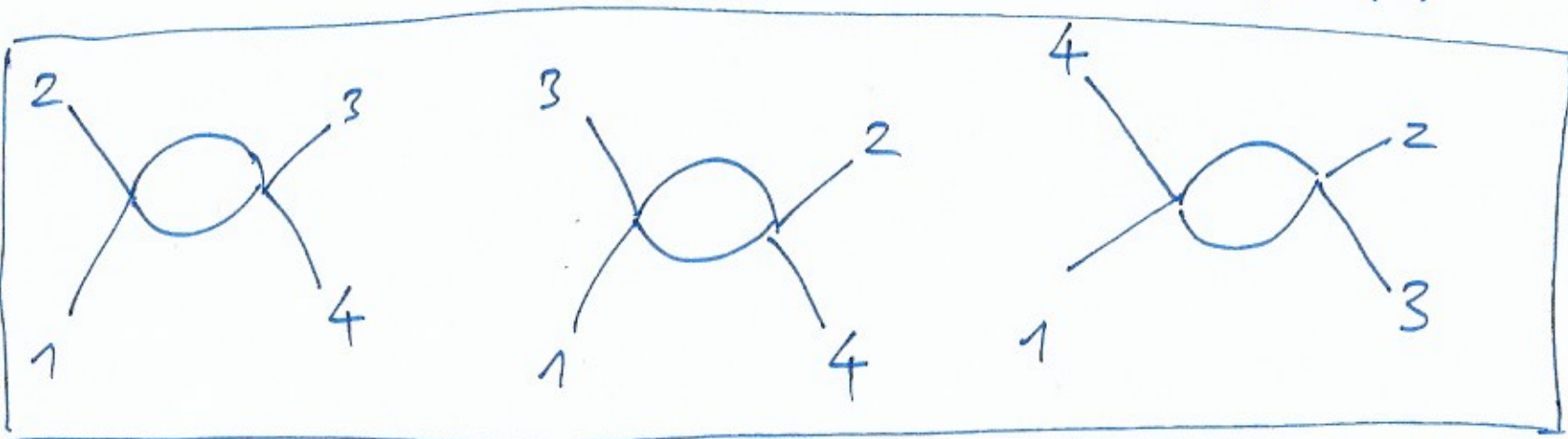
order $(g_0)^2$ corrections to $G_C(x_1, x_2, x_3, x_4)$



...

again, boring...

not a genuine correction to the strength of the interactions, but a correction to the mass of the incoming/outgoing particles



genuine corrections to the interaction strength

VERTEX FUNCTIONS

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get rid of the propagators at the legs

$$T_{x_1' x_2' x_3' x_4'} G_{x_1' x_1}^{\text{conn}} G_{x_2' x_2}^{\text{conn}} G_{x_3' x_3}^{\text{conn}} G_{x_4' x_4}^{\text{conn}} = G_{x_1 x_2 x_3 x_4}^{\text{conn}}$$

at order g_0 (tree level)

$$T_{x_1' x_2' x_3' x_4'} G_{x_1' x_1}^{(0)} G_{x_2' x_2}^{(0)} G_{x_3' x_3}^{(0)} G_{x_4' x_4}^{(0)} = -g_0 \sum_w G_{wx_1}^{(0)} G_{wx_2}^{(0)} G_{wx_3}^{(0)} G_{wx_4}^{(0)}$$

$$\Rightarrow T_{x_1' x_2' x_3' x_4'} = -g_0 \sum_w \delta_{wx_1'} \delta_{wx_2'} \delta_{wx_3'} \delta_{wx_4'} = -g_0 \delta_{x_1' x_2'} \delta_{x_1' x_3'} \delta_{x_1' x_4'}$$

In Fourier space

$$\tilde{T}(p_1 p_2 p_3 p_4) \equiv \sum_{x_1 x_2 x_3 x_4} e^{-ip_1 x_1 - ip_2 x_2 - ip_3 x_3 - ip_4 x_4} (-g_0) \delta_{x_1 x_2} \delta_{x_1 x_3} \delta_{x_1 x_4}$$

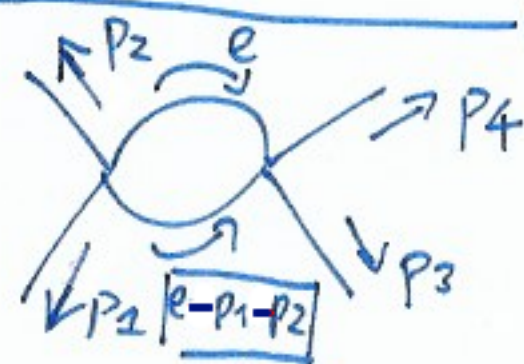
$$\equiv (-g_0) \sum_{x_1} e^{-i(p_1 + p_2 + p_3 + p_4)x_1} = (-g_0) \Omega \delta_{p_1 + \dots + p_4, 0} \rightarrow (-g_0) \delta(p_1 + \dots + p_4)$$

at higher orders:

$$\tilde{T}(p_1 p_2 p_3 p_4) \equiv T(p_1 p_2 p_3 p_4) \underbrace{\bar{S}(p_1 + p_2 + p_3 + p_4)}$$

but this will no longer be indep. of the momenta

there is still overall momentum conservation, since the elementary/tree level vertex has momentum conservation



momentum dependence in this propagator

wavefn. renorm?

$$T_{x_1' x_2' x_3' x_4'} \underbrace{G_{x_1' x_1}}_{\sim Z} \underbrace{G_{x_2' x_2}}_{\sim Z} \underbrace{G_{x_3' x_3}}_{\sim Z} \underbrace{G_{x_4' x_4}}_{\sim Z} = \underbrace{G_{x_1 x_2 x_3 x_4}^{\text{conn}}}_{\sim Z^2 \text{ (twice as many legs w. external momenta)}}$$

$\Downarrow \sim Z^{-2}$

$G^{(n)} = Z^{\frac{n}{2}} G_{\text{ren}}^{(n)}$

(twice as many fields)
 $\phi = \sqrt{Z} \phi_R$

renormalized coupling:

$T(p_1 = p_2 = p_3 = p_4 = 0) \equiv \frac{-g_R}{Z^2}$

THIS IS A DEFINITION

How to get it from a simulation?

kurtosis of the magnetization

$$\frac{1}{\Omega} \sum_{x_1 x_2 x_3 x_4} G^{\text{conn}}(\underbrace{x_1 x_2 x_3 x_4}_3) = \frac{1}{\Omega} (\langle M^4 \rangle - 3 \langle M^2 \rangle^2)$$

$$= \left(\frac{Z}{m_R^2} \right)^4 \frac{-g_R}{Z^2}$$

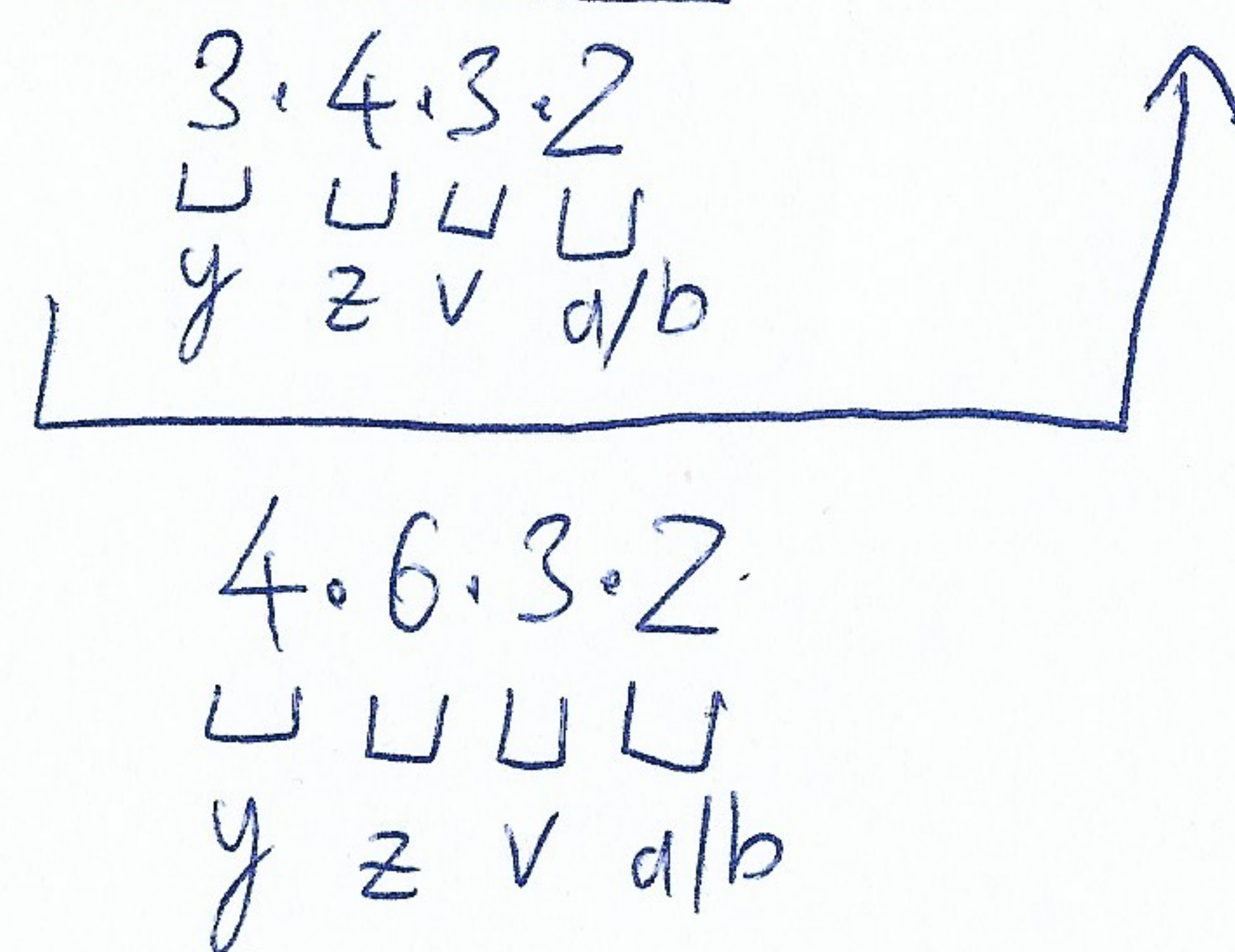
Like m_R before it, g_R also diverges if we take $a \rightarrow 0$ while keeping m_0 and g_0 fixed BUT: this should be not taken how the cont. lim.

1-loop correction to renormalized coupling (we can set $Z=1$ at this level)

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$$\frac{-m_0^8}{\Omega} \sum_{xyzv} \text{diagram} = \frac{-m_0^8}{\Omega} \left(\frac{-g_0}{4!} \right)^2 \sum_{xyzv} \sum_{ab} \langle \phi_x \phi_y \phi_z \phi_v \phi_a \phi_a \phi_a \phi_a \phi_b \phi_b \phi_b \phi_b \rangle$$

$$\frac{8 \cdot (3 \cdot 4 \cdot 3 \cdot 2 + 4 \cdot 6 \cdot 3 \cdot 2)}{(4!)^2} = \frac{3}{2}$$



$$\frac{-m_0^8}{\Omega} \sum_{xyzv} \sum_{ab} G_{xa}^{(0)} G_{ya}^{(0)} G_{zb}^{(0)} G_{vb}^{(0)} G_{ab}^{(0)} G_{ab}^{(0)} = -\frac{1}{\Omega} \sum_{ab} G_{ab}^{(0)} G_{ab}^{(0)} = -\sum_b G_{ob}^{(0)} G_{ob}^{(0)}$$

$$\sum_b G_{ob}^{(0)} G_{ob}^{(0)} = \int_{\vec{p}} \int_{\vec{q}} \sum_b e^{i(p+q)b} \frac{1}{m^2 + \vec{p}^2} \frac{1}{m^2 + \vec{q}^2}$$

$$\sum_b \rightarrow \Omega \delta_{p+q,0} \Rightarrow \exp = 1 \text{ and } \int_{\vec{p}} \text{ sets } \vec{q} = -\vec{p} \text{ and cancels } \Omega$$

$$\Rightarrow \left[-\frac{3}{2} g_0^2 \int_{\vec{q}} \frac{1}{(m^2 + \vec{q}^2)^2} \right]$$

$$g_R = g_0 - \frac{3}{2} g_0^2 \int_{\vec{q}} \frac{1}{(m^2 + \vec{q}^2)^2}$$