2016/12/07 seminar@Eötvös Loránd Univ., Budapest, Hungary

Recent progress on Yang-Baxter deformations of type IIB superstring on $AdS_5 \times S^5$

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Introduction

The AdS/CFT correspondence

type IIB string on $AdS_5 \times S^5$ $\Longleftrightarrow$ 4D $\mathcal{N}=4$ SU(N) SYM $(N \to \infty)$

Recent progress: the discovery of integrability

[For a big review, Beisert et al., 1012.3982]

Integrability is so powerful!

The integrability enables us to compute exactly physical quantities even at finite coupling, without relying on supersummetries.

EX anomalous dimensions, amplitudes etc.

Indeed, there are many directions of study with this integrability.

Here, among them, we are concerned with

the classical integrability on the string-theory side.

$\hookrightarrow$ The existence of Lax pair (kinematical integrability)

The classical integrability of the $AdS_5 \times S^5$ superstring

The coset structure of $AdS_5 \times S^5$ is closely related to the integrability.

$$\boxed{\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}}$$

: symmetric coset

Z_2-grading $\Rightarrow$ classical integrability

Including fermions

$$\boxed{\frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}}$$

: super coset

[Metsaev-Tseytlin, 1998]

Z_4-grading $\Rightarrow$ classical integrability

elucidated by
[Bena-Polchinski-Roiban, 2003]

This fact is the starting point of our later argument.

The next issue

Integrable deformations of the $AdS_5 \times S^5$ superstring

Integrable deformations → Deformed $AdS_5 \times S^5$ geometries

(as a 2D non-linear sigma model)

Questions

Do the integrable deformations lead to solutions of type IIB SUGRA?

If not, does a new theory appear instead?

The main subject of my talk is to answer these questions.

Along this direction, we will follow a systematic way to study integrable deformations.

→ Yang-Baxter deformations

Yang-Baxter deformations

What is Yang-Baxter deformation?

Recent progress on this issue

[7 slides]

Yang-Baxter deformations

[Klimcik, 2002, 2008]

Integrable deformation!

An example

G-principal chiral model

Yang-Baxter sigma model

$$S = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}(J_\mu J_\nu) \quad \longrightarrow \quad S^{(\eta)} = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}\left(J_\mu \frac{1}{1-\eta R} J_\nu\right)$$

$$J_\mu = g^{-1}\partial_\mu g\,, \quad g \in G$$

η : a const. parameter

What is *R*?

$$R \; : \; \mathfrak{g} \longrightarrow \mathfrak{g}$$

a linear op.

a classical r-matrix satisfying

the modified classical Yang-Baxter eq. (mCYBE)

An integrable deformation can be specified by a classical r-matrix.

Strong advantage

Given a classical r-matrix, a Lax pair follows automatically.

No need to construct it in an intuitive manner case by case

Relation between R-operator and classical r-matrix

A linear R-operator ⟷ A skew-symmetric classical r-matrix

$R \ : \quad \mathfrak{g} \longrightarrow \mathfrak{g}$ $\qquad\qquad$ $r \in \mathfrak{g} \otimes \mathfrak{g}$

$$R(X) \equiv \langle r_{12}, 1 \otimes X \rangle = \sum_i a_i \langle b_i, X \rangle \qquad \text{for} \quad X \in \mathfrak{g}$$

$$r_{12} = \sum_i a_i \otimes b_i \qquad \text{with} \qquad a_i, b_i \in \mathfrak{g}$$

Two sources of classical r-matrices

1) modified classical Yang-Baxter eq. (mCYBE) ⟵ the original work by Klimcik

$$[R(X), R(Y)] - R\left([R(X), Y] + [X, R(Y)]\right) = \underline{-c^2 [X, Y]} \qquad (c \in \mathbb{C})$$

2) classical Yang-Baxter eq. (CYBE) $(c = 0)$ ⟵ a possible generalization

The list of generalizations of Yang-Baxter deformations (2 classes)

(i) modified CYBE (trigonometric class)

a) Principal chiral model [Klimcik, hep-th/0210095, 0802.3518]

b) Symmetric coset sigma model [Delduc-Magro-Vicedo, 1308.3581]

1) c) The $AdS_5 \times S^5$ superstring [Delduc-Magro-Vicedo, 1309.5850]

(ii) CYBE (rational class)

a) Principal chiral model [Matsumoto-KY, 1501.03665]

b) Symmetric coset sigma model [Matsumoto-KY, 1501.03665]

2) c) The $AdS_5 \times S^5$ superstring [Kawaguchi-Matsumoto-KY, 1401.4855]

NOTE bi-Yang-Baxter deformation [Klimcik, 0802.3518, 1402.2105]

(applicable only for principal chiral models)

YB deformation of the $AdS_5 \times S^5$ superstring

[Delduc-Magro-Vicedo, 1309.5850]

[Kawaguchi-Matsumoto-KY, 1401.4855]

$$S = -\frac{1}{2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma\; P_-^{\alpha\beta}\,\mathrm{Str}\left[A_\alpha d \circ \boxed{\frac{1}{1-\eta\,[R]_g \circ d}}(A_\beta)\right]$$

R satisfies (m)CYBE.

The undeformed limit: $\eta \to 0$ $\Rightarrow$ the Metsaev-Tseytlin action

[Metsaev-Tseytlin, hep-th/9805028]

- Lax pair is constructed : classical integrability
- Kappa invariance : a consistency as string theory at classical level

NOTE

The difference between mCYBE and CYBE is reflected in some coefficients of some quantities such as Lax pair and kappa-transformation.

A group element: $g = g_{\rm b}\, g_{\rm f} \quad \in SU(2,2|4)$

$$g_{\rm b} = {g_{\rm b}}^{\rm AdS_5}\, {g_{\rm b}}^{\rm S^5}\,,$$

[For a big review, Arutyunov-Frolov, 0901.4937]

$$g_{\rm f} = \exp(\mathbf{Q}^I \theta_I)\,, \quad \mathbf{Q}^I \theta_I \equiv (\mathbf{Q}^{\check{\alpha}\hat{\alpha}})^I\, (\theta_{\check{\alpha}\hat{\alpha}})_I \quad (I = 1,2;\ \check{\alpha}, \hat{\alpha} = 1,\dots,4)$$

When we take a parametrization like

$$\begin{aligned}
{g_{\rm b}}^{\rm AdS_5} &= \exp\Big[x^0\, P_0 + x^1\, P_1 + x^2\, P_2 + x^3\, P_3\Big]\, \exp\Big[(\log\, z)\, D\Big]\,, \\
{g_{\rm b}}^{\rm S^5} &= \exp\Big[\frac{i}{2}(\phi_1\, h_1 + \phi_2\, h_2 + \phi_3\, h_3)\Big]\, \exp\Big[\xi\, \mathbf{J}_{68}\Big]\, \exp\Big[-i\, r\, \mathbf{P}_6\Big]\,,
\end{aligned}$$

the metric of AdS_5 x S^5 is given by

$$\begin{aligned}
ds^2 &= ds^2_{\rm AdS_5} + ds^2_{\rm S^5}\,, \\
ds^2_{\rm AdS_5} &= \frac{-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{z^2} + \frac{dz^2}{z^2}\,, \\
ds^2_{\rm S^5} &= dr^2 + \sin^2 r\, d\xi^2 + \cos^2\xi\, \sin^2 r\, d\phi_1^2 + \sin^2 r\, \sin^2\xi\, d\phi_2^2 + \cos^2 r\, d\phi_3^2
\end{aligned}$$

An outline of supercoset construction

[Arutyunov-Borsato-Frolov, 1507.04239]
[Kyono-KY, 1605.02519]

The deformed action can be rewritten into the canonical form:

$$S = -\frac{\sqrt{\lambda_c}}{4}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \left[\gamma^{ab}\boxed{G_{MN}}\partial_a X^M \partial_b X^N - \epsilon^{ab}\boxed{B_{MN}}\partial_a X^M \partial_b X^N\right]$$
$$-\frac{\sqrt{\lambda_c}}{2}\, i\bar{\Theta}_I(\gamma^{ab}\delta^{IJ} - \epsilon^{ab}\sigma_3^{IJ})\, e_a^m \Gamma_m \boxed{D_b^{JK}}\Theta_K + \mathcal{O}(\theta^4)$$

This action is expanded w.r.t the fermions.

In general, the covariant derivative D is given by

[Cvetic-Lu-Pope-Stelle, hep-th/9907202]

$$D_a^{IJ} \equiv \delta^{IJ}\left(\partial_a - \frac{1}{4}\omega_a^{mn}\Gamma_{mn}\right) + \frac{1}{8}\sigma_3^{IJ} e_a^m H_{mnp}\Gamma^{np}$$
$$-\frac{1}{8}\boxed{e^{\Phi}}\left[\epsilon^{IJ}\Gamma^p \boxed{F_p} + \frac{1}{3!}\sigma_1^{IJ}\Gamma^{pqr}\boxed{F_{pqr}} + \frac{1}{2\cdot 5!}\epsilon^{IJ}\Gamma^{pqrst}\boxed{F_{pqrst}}\right] e_a^m \Gamma_m$$

From this expression, one can read off all of the fields of type IIB SUGRA.

Summary of the resulting backgrounds

1) The mCYBE case [Delduc-Magro-Vicedo, 1309.5850]

η-deformation or standard *q*-deformation [Arutyunov-Borsato-Frolov, 1312.3542]

The background is not a solution of the usual type IIB SUGRA,
but satisfies the generalized type IIB SUGRA. [Arutyunov-Borsato-Frolov, 1507.04239]
[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

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2) The CYBE case [Kawaguchi-Matsumoto-KY, 1401.4855]

- A certain class of classical r-matrices satisfying

The unimodularity condition [Borsato-Wulff, 1608.03570]

$$r^{ij}[b_i, b_j] = 0 \quad \text{for a classical r-matrix} \quad r = r^{ij} b_i \wedge b_j$$

⟹ Solutions of the standard type IIB SUGRA

- The other ones lead to solutions of the ``generalized'' type IIB SUGRA

In the following, we will concentrate on

2) The CYBE case

Contents

1. What is the generalized type IIB SUGRA?

The generalized eqns of type IIB SUGRA

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

$$R_{MN} - \frac{1}{4} H_{MKL} H_N{}^{KL} - T_{MN} + D_M \boxed{X_N} + D_N \boxed{X_M} = 0\,,$$

$$\frac{1}{2} D^K H_{KMN} + \frac{1}{2} F^K F_{KMN} + \frac{1}{12} F_{MNKLP} F^{KLP} = \boxed{X^K} H_{KMN} + D_M \boxed{X_N} - D_N \boxed{X_M}$$

$$R - \frac{1}{12} H^2 + 4 D_M \boxed{X^M} - \boxed{4 X_M X^M} = 0\,,$$

$$D^M \mathcal{F}_M - \boxed{Z^M} \mathcal{F}_M - \frac{1}{6} H^{MNK} \mathcal{F}_{MNK} = 0\,, \qquad \boxed{I^M} \mathcal{F}_M = 0\,, \qquad \mathcal{F}_{n_1 n_2 \ldots} = \mathrm{e}^{\Phi} F_{n_1 n_2 \ldots}$$

$$D^K \mathcal{F}_{KMN} - \boxed{Z^K} \mathcal{F}_{KMN} - \frac{1}{6} H^{KPQ} \mathcal{F}_{KPQMN} - (\boxed{I} \wedge \mathcal{F}_1)_{MN} = 0\,,$$

$$D^K \mathcal{F}_{KMNPQ} - \boxed{Z^K} \mathcal{F}_{KMNPQ} + \frac{1}{36} \epsilon_{MNPQRSTUVW} H^{RST} \mathcal{F}^{UVW} - (\boxed{I} \wedge \mathcal{F}_3)_{MNPQ} = 0$$

$$T_{MN} \equiv \frac{1}{2} \mathcal{F}_M \mathcal{F}_N + \frac{1}{4} \mathcal{F}_{MKL} \mathcal{F}_N{}^{KL} + \frac{1}{4 \times 4!} \mathcal{F}_{MPQRS} \mathcal{F}_N{}^{PQRS} - \frac{1}{4} G_{MN} (\mathcal{F}_K \mathcal{F}^K + \frac{1}{6} \mathcal{F}_{PQR} \mathcal{F}^{PQR})$$

Modified Bianchi identities

$$(d\mathcal{F}_1 - \boxed{Z} \wedge \mathcal{F}_1)_{MN} - \boxed{I^K} \mathcal{F}_{MNK} = 0\,,$$

$$(d\mathcal{F}_3 - \boxed{Z} \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{MNPQ} - \boxed{I^K} \mathcal{F}_{MNPQK} = 0\,,$$

$$(d\mathcal{F}_5 - \boxed{Z} \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{MNPQRS} + \frac{1}{6} \epsilon_{MNPQRSTUVW} \boxed{I^T} \mathcal{F}^{UVW} = 0$$

New ingredients: $X, \quad I, \quad Z$ 3 vector fields

But $X_M \equiv I_M + Z_M$, so two of them are independent.

Then I & Z satisfy the following relations:

$$D_M I_N + D_N I_M = 0\,, \quad D_M Z_N - D_N Z_M + I^K H_{KMN} = 0\,, \quad I^M Z_M = 0$$

Assuming that I is chosen such that the Lie derivative

$$(\mathcal{L}_I B)_{MN} = I^K \partial_K B_{MN} + B_{KN} \partial_M I^K - B_{KM} \partial_N I^K$$

vanishes, the 2nd equation above can be solved by

$$Z_M = \partial_M \Phi - B_{MN} I^N \quad .$$

Thus only I is independent after all.

Note When $I = 0$, the usual type IIB SUGRA is reproduced.

A great progress on the Green-Schwarz string

Relation between kappa-symmetry and SUGRA

Old result: the on-shell condition of the standard type IIB SUGRA

$\Rightarrow$ kappa-invariant GS string theory [Grisaru-Howe-Mezincescu-Nilsson-Townsend, 1985]

The inverse was conjectured.

New result: kappa-invariant GS string theory

$\Rightarrow$ the generalized type IIB SUGRA [Tseytlin-Wulf, 1605.04884]

This issue has been resolved after more than 30 years from the old work.

Pathology? [Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

The string world-sheet is not Weyl invariant but scale invariant,
if the background is not a solution of type IIB SUGRA.

2. Examples of classical r-matrices

- 2 examples of unimodular classical r-matrices
  - i) gamma-deformations of S5
  - ii) Gravity duals for SYM on non-commutative space

Supercoset construction for them [Kyono-KY, 1605.02519]

- 1 example of non-unimodular classical r-matrices
  - iii) A non-unimodular classical r-matrix

i) gamma-deformations of S^5

c.f. Leigh-Strassler deformation

[Matsumoto-KY, 1404.1838]

Abelian classical r-matrix: $r = \frac{1}{8}(\mu_3\, h_1 \wedge h_2 + \mu_1\, h_2 \wedge h_3 + \mu_2\, h_3 \wedge h_1)$

where μ_i and h_i $(i = 1, 2, 3)$ are deformation parameters and the Cartan generators of $\mathfrak{su}(4)$.

Metric: $$ds^2 = ds^2_{\mathrm{AdS}_5} + \sum_{i=1}^{3}(d\rho_i^2 + G\rho_i^2 d\phi_i^2) + \eta^2 G\rho_1^2\rho_2^2\rho_3^2\left(\sum_{i=1}^{3}\mu_i\, d\phi_i\right)^2 ,$$

B-field: $$B_2 = \eta\, G\left(\mu_3\, \rho_1^2\rho_2^2\, d\phi_1 \wedge d\phi_2 + \mu_1\, \rho_2^2\rho_3^2\, d\phi_2 \wedge d\phi_3 + \mu_2\, \rho_3^2\rho_1^2\, d\phi_3 \wedge d\phi_1\right) ,$$

dilaton: $$\Phi = \frac{1}{2}\log G\,, \quad G^{-1} \equiv 1 + \eta^2(1 + \mu_3^2\rho_1^2\rho_2^2 + \mu_1^2\rho_2^2\rho_3^2 + \mu_2^2\rho_3^2\rho_1^2)\,, \qquad \textstyle\sum_{i=1}^{3}\rho_i^2 = 1$$

R-R: $$F_3 = -4\eta\sin^3\alpha\,\cos\alpha\,\sin\theta\,\cos\theta\left(\sum_{i=1}^{3}\mu_i\, d\phi_i\right)\wedge d\alpha \wedge d\theta\,,$$

$$F_5 = 4\left[\omega_{\mathrm{AdS}_5} + G\,\omega_{\mathrm{S}^5}\right] .$$

$$\rho_1 = \sin\alpha\cos\theta\,,\quad \rho_2 = \sin\alpha\sin\theta\,,\quad \rho_3 = \cos\alpha\,.$$

[Lunin-Maldacena, Frolov, 2005]

ii) Gravity duals for SYM on non-commutative space

c.f. Seiberg-Witten, 1999

Abelian Jordanian r-matrix: $r = \frac{1}{2}\, p_2 \wedge p_3$

[Matsumoto-KY, 1404.3657]

where $p_\mu \equiv \frac{1}{2}\gamma_\mu - m_{\mu 5}\,,\quad m_{\mu 5} = \frac{1}{4}\,[\gamma_\mu, \gamma_5]\,,\quad \gamma_\mu\text{: a basis of } \mathfrak{su}(2,2)$

Metric: $$ds^2 = \frac{1}{z^2}(-dx_0^2 + dx_1^2) + \frac{z^2}{z^4+\eta^2}(dx_2^2 + dx_3^2) + \frac{dz^2}{z^2} + d\Omega_5^2$$

B-field: $$B_2 = \frac{\eta}{z^4+\eta^2}\, dx^2 \wedge dx^3\,,$$

dilaton: $$\Phi = \frac{1}{2}\log\left(\frac{z^4}{z^4+\eta^2}\right)$$

R-R : $$F_3 = \frac{4\eta}{z^5}\, dx^0 \wedge dx^1 \wedge dz\,, \qquad F_5 = 4\left[\mathrm{e}^{2\Phi}\omega_{AdS_5} + \omega_{\mathrm{S}^5}\right]\,.$$

[Hashimoto-Itzhaki, Maldacena-Russo, 1999]

Note This solution can also be reproduced as a special limit of η-deformed AdS_5 .

[Arutyunov-Borsaro-Frolov, 1507.04239] [Kameyama-Kyono-Sakamoto-KY, 1509.00173]

iii) A non-unimodular classical r-matrix

$$\begin{aligned} r &= E_{24} \wedge (c_1 E_{22} - c_2 E_{44}) \\ &= (p_0 - p_3) \wedge \left[a_1 \left(\frac{1}{2}\gamma_5 - n_{03} \right) - a_2 \left(n_{12} - \frac{i}{2}\mathbf{1}_4 \right) \right] \end{aligned}$$

$$a_1 \equiv \frac{c_1 + c_2}{2} = \mathrm{Re}(c_1)\,, \qquad a_2 \equiv \frac{c_1 - c_2}{2\,i} = \mathrm{Im}(c_1)$$

The resulting background: [Kyono-KY, 1605.02519]

$$\begin{aligned} ds^2 &= \frac{-2dx^+dx^- + d\rho^2 + \rho^2 d\phi^2 + dz^2}{z^2} - 4\eta^2 \left[(a_1^2 + a_2^2)\frac{\rho^2}{z^6} + \frac{a_1^2}{z^4} \right] (dx^+)^2 + ds^2_{\mathrm{S}^5}\,, \\ B_2 &= 8\eta \left[\frac{a_1 x^1 + a_2 x^2}{z^4} dx^+ \wedge dx^1 + \frac{a_1 x^2 - a_2 x^1}{z^4} dx^+ \wedge dx^2 + a_1 \frac{1}{z^3} dx^+ \wedge dz \right]\,, \\ F_3 &= 8\eta \left[\frac{a_2 x^1 - a_1 x^2}{z^5} dx^+ \wedge dx^1 \wedge dz + \frac{a_1 x^1 + a_2 x^2}{z^5} dx^+ \wedge dx^2 \wedge dz + \frac{a_1}{z^4} dx^+ \wedge dx^1 \wedge dx^2 \right]\,, \\ F_5 &= \text{undeformed}\,, \qquad \Phi = \text{const} \end{aligned}$$

$$\boxed{dF_3 = 16\eta\,\frac{a_1}{z^5}\,dx^+ \wedge dx^1 \wedge dx^2 \wedge dz \neq 0}$$

The Bianchi identity is broken.

The e.o.m. of B_2 is not satisfied as well.

Some comments

- Special case

The a1=0 case is special. The classical r-matrix becomes unimodular.

The background is a solution of type IIB SUGRA. [Hubeney-Rangamani-Ross, hep-th/0504034]

- General case

The resulting background is not a solution of type IIB SUGRA,

but still satisfies the generalized equations with

$$\boxed{I = -\frac{2\eta a_1}{z^2}\, dx^+ \,, \quad Z = 0}$$

It is more interesting to perform ``generalized T-dualities'' for this solution

(i.e., a generalized Buscher rule) $\Rightarrow$ a solution of the usual type IIB SUGRA.

Furthermore, this ``T-dualized'' background is locally equivalent to

the undeformed $AdS_5 \times S^5$! [Orlando-Reffert-Sakamoto-KY, 1607.00795]

What is the physical interpretation of this result?

3. Generalized T-duality & modified DFT

Generalized T-duality

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

The generalized Buscher rule has been determined in a heuristic way.

There is no derivation

There are 2 classes

i) generalized T-duality along an isometric direction

Generalized solutions are mapped to generalized solutions (invertible)

ii) generalized T-duality along a non-isometric direction (due to liner dilaton)

Generalized solutions are mapped to solutions of the standard SUGRA

NOTE: the class ii) is just one way ! (non-invertible)

To get the deeper understanding of the generalized T-dualities,
it would be interesting to construct a Double Field Theory (DFT) picture.

Generalized gravity from modified DFT

[Sakatani-Uehara-KY, 1611.05856]

DFT = Double Field Theory (NS-NS sector)

c.f., EFT = Exceptional Field Theory (NS-NS + R-R sector)

This is a manifestly T-duality covariant formulation

section conditions

Type IIB SUGRA ← DFT, EFT

Type IIA SUGRA ← DFT, EFT

[Siegel, Hull, Zwiebach, Park, , , ,]

section conditions

generalized Type IIB SUGRA ← modified DFT, modified EFT (?)

generalized Type IIA SUGRA (?) ← R-R (?) modified DFT, modified EFT (?)

[Sakatani-Uehara-KY, 1611.05856]
[Sakatani-Uehara-KY, work in progress]

c.f., no generalized 11D SUGRA [Bergshoeff-Sezgin-Townsend, 1987]

Advantages of the modified DFT

1) Before taking a section condition, the two classes of the T-dualities are treated on the equal footing.

$\Rightarrow$ The inverse process of the class ii) is possible as well.

It may be possible to construct generalized sols. from standard sols.

e.g., generalized D-brane backgrounds, generalized black holes etc.

The extra vector fields may contribute to the black hole entropy.

2) It might enable us to construct the classical action of the generalized supergravities.

Does it reveal the relation to 11D supergravity?

Summary and Discussion

Summary

I have given a review of recent progress on

Yang-Baxter deformations of the $AdS_5 \times S^5$ superstring

- The mCYBE case

  An η-deformation of $AdS_5 \times S^5$ is not a solution of type IIB SUGRA, but satisfies the generalized type IIB SUGRA.

  There would be no solutions of the usual type IIB SUGRA.

- The CYBE case

  1) The unimodular classical r-matrices lead to sols. of type IIB SUGRA.

     EX Lunin-Maldacena-Frolov, Maldacena-Russo, Schrödinger spacetimes

  2) The other non-unimodular ones lead to sols. of the generalized SUGRA.

Discussion

Non-abelian T-duality conjecture [Hoare-Tseytlin, 1609.02550]

The Yang-Baxter deformations with CYBE

$\Updownarrow$ conjecture

Non-abelian T-dualities

For proof in the bosonic case, [Borsato-Wulff, 1609.09834]

For a non-symmetric case, [J. Sakamoto-KY, 1612.nnnnn]

This equivalence would shed light on the following questions:

What is the origin or mechanism of the broken Weyl invariance?

What happens to the gauge-theory side due to the deformations?

Is it related to the modified DFT picture? Only at stringy level?

It may be interesting to generalize this conjecture to the Minkowski spacetime.

For Yang-Baxter deformations of 4D Minkowski spacetime, see
[Matsumoto-Orlando-Reffert-Sakamoto-KY, 1505.04553][Borowiec-Kyono-Lukierski-Sakamoto-KY, 1510.03083]

Thank you!

Back up

Further generalizations

1) YB deformations of non-integrable backgrounds:

EX $AdS_5 \times T^{1,1}$: non-integrability & chaos

[Basu-Pando Zayas, 1103.4107]
[Asano-Kawai-Kyono-KY, 1505.07583]

YB deformations can capture TsT trans. of $T^{1,1}$ as well. [Crichigno-Matsumoto-KY, 1406.2249]

2) YB deformations of 4D Minkowski space: [Matsumoto-Orlando-Reffert-Sakamoto-KY, 1505.04553]

Melvin, pp-wave, T-duals of (A)dS,
Hashimoto-Sethi, Spradlin-Takayanagi-Volovich, etc.

[Borowiec-Kyono-Lukierski-Sakamoto-KY, 1510.03083]
[Kyono-Sakamoto-KY, 1512.00208]

[Pachol-van Tongeren, 1510.02389]

A scaling limit of η–deformed AdS_5xS^5

Yang-Baxter deformations of 4D Minkowski spacetime

r-matrix	Type of Twist	Background
$p_i \wedge p_j \ (i,j = 1,2,3)$	Melvin Shift Twist	Seiberg-Witten
$p_0 \wedge p_i$	Melvin Shift Twist	NCOS
$(p_0 + p_i) \wedge p_j \ (i \neq j)$	Null Melvin Shift Twist	light-like NC
$\frac{1}{2} p_3 \wedge n_{12}$	Melvin Twist	T–dual Melvin
$\frac{1}{2\sqrt{2}} p_2 \wedge (n_{01} + n_{13})$	Melvin Null Twist	Hashimoto–Sethi
$\frac{1}{2} n_{12} \wedge n_{03}$	R Melvin R Twist	Spradlin–Takayanagi–Volovich
$\frac{1}{2} p_1 \wedge n_{03}$	Melvin Boost Twist	T–dual of Grant space
$\frac{1}{2\sqrt{2}} (p_0 - p_3) \wedge n_{12}$	Null Melvin Twist	pp-wave
$\frac{1}{2\sqrt{2}} (\hat{d} - n_{03}) \wedge (p_0 - p_3)$	Non-Twist	pp-wave
$\frac{1}{2} \hat{d} \wedge p_0$	Non-Twist	T–dual of dS_4
$\frac{1}{2} \hat{d} \wedge p_1$	Non-Twist	T–dual of AdS_4
DJ-type (mCYBE)	Non-Twist	q-deformation (?)

Definitions of the quantities

Maurer-Cartan 1-form

$$A_\alpha \equiv g^{-1}\partial_\alpha g\,, \quad g \in SU(2,2|4)$$

,

Projection on the group manifold

$$d \equiv P_1 + 2P_2 - P_3$$

Projection on the world-sheet

$$P_\pm^{\alpha\beta} \equiv \frac{1}{2}(\gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$$

$$\left[\begin{array}{l} \gamma^{\alpha\beta} = \mathrm{diag}(-1,1) \\ \epsilon^{\alpha\beta} : \ \text{anti-symm. tensor} \end{array}\right.$$

A chain of operations

$$R_g(X) \equiv g^{-1}R(gXg^{-1})g\,, \qquad \forall X \in \mathfrak{su}(2,2|4)$$

The current status of two kinds of YB deformations

1) *η*-deformation ← mCYBE

- A *q*-deformed classical action with classical r-matrix of Drinfeld-Jimbo type [Delduc-Magro-Vicedo, 1309.5850]

  The metric and B-field [Arutyunov-Borsaro-Frolov, 1312.3542]

- Supercoset construction [Arutyunov-Borsaro-Frolov, 1507.04239]

  → NOT satisfy the e.o.m. of type IIB SUGRA c.f., [Hoare-Tseytlin, 1508.01150]

- Solution of generalized SUGRA eqns.

- The world-sheet theory is not Weyl invariant, but scale invariant.

[Arutyunov-Frolov-Hoare -Roiban-Tseytlin, 1511.05795]

+ many other works

minimal surfaces, quark-antiquark potential [Kameyama-KY, 1410.5544, 1602.06786]

The current status of two kinds of YB deformations

2) Jordanian deformations ← CYBE [Kawaguchi-Matsumoto-KY, 1401.4855]

A lot of classical r-matrices

i) Partial deformations are possible (i.e., only AdS_5 or only S^5)

This is an advantage of Jordanian deformations

ii) Candidates of gravity solutions have been argued for many classical r-matrices

[Matsumoto-KY, 1404.1838, 1404.3657, 1412.3658, 1502.00740] [S.J. van Tongeren, 1506.01023]

iii) Supercoset construction has been done for some examples. [Kyono-KY, 1605.02519]

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

In the following, I will introduce

(a) The deformed classical action & the outline of supercoset construction

(b) Some classical r-matrices and the resulting backgrounds

iii) Schrödinger spacetimes

c.f. [Son, 0804.3972], [Balasubramanian-McGreevy, 0804.4053]

Mixed r-matrix: $r = -\frac{i}{4}p_- \wedge (h_4 + h_5 + h_6)$

[Matsumoto-KY, 1502.00740]

Metric: $$ds^2 = \frac{-2dx^+dx^- + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} - \eta^2\frac{(dx^+)^2}{z^4} + ds^2_{S^5}$$

B-field: $$B_2 = \frac{\eta}{z^2}dx^+ \wedge (d\chi + \omega)\,,$$

dilaton: $$\Phi = \text{const.}$$

The R-R sector is the same as $AdS_5 \times S^5$.

[Herzog-Rangamani-Ross, 0807.1099]
[Maldacena-Martelli-Tachikawa, 0807.1100]
[Adams-Balasubramanian-McGreevy, 0807.1111]

S^5-coordinates: $$ds^2_{S^5} = (d\chi + \omega)^2 + ds^2_{\mathbb{CP}^2}\,,$$
$$ds^2_{\mathbb{CP}^2} = d\mu^2 + \sin^2\mu\left(\Sigma_1^2 + \Sigma_2^2 + \cos^2\mu\,\Sigma_3^2\right)$$

NOTE the dilaton and R-R sector have not been deformed.

In the middle of computation, the fermionic sector becomes really messy and quite complicated. So the cancellation of the deformation effect seems miraculous.

Let's return to the present example.

By taking

$$\boxed{I = -\frac{2\eta a_1}{z^2}\,dx^+\,, \quad Z_M = 0}$$

the background satisfies the generalized eqs. [Orlando-Reffert-Sakamoto-KY, 1607.00795]

Then by performing ``T-dualities'' along x^+, x^-, ϕ_1, ϕ_2 , one can obtain

$$\begin{aligned}
ds^2 &= -2z^2\,dx^+dx^- + \frac{(d\rho - \eta a_1 \rho\, dx^-)^2 + \rho^2(d\theta + \eta a_2\, dx^-)^2 + (dz - \eta a_1 z\, dx^-)^2}{z^2} \\
&\quad + dr^2 + \sin^2 r\, d\xi^2 + \frac{d\phi_1^2}{\cos^2\xi \sin^2 r} + \frac{d\phi_2^2}{\sin^2 r \sin^2\xi} + \cos^2 r\, d\phi_3^2\,, \\
\mathcal{F}_5 &= \frac{4i\rho}{z^3 \sin\xi \cos\xi \sin^2 r}(d\rho - \eta a_1 z\, dx^-)\wedge(d\theta + \eta a_2\, dx^-)\wedge(dz - \eta a_1 z\, dx^-)\wedge d\phi_1 \wedge d\phi_2 \\
&\quad + 4iz^2 \sin r \cos r\, dx^+ \wedge dx^- \wedge dr \wedge d\xi \wedge d\phi_3\,, \\
\Phi &= -2\eta a_1 x^- + \log\left[\frac{z^2}{\sin^2 r \sin\xi \cos\xi}\right]
\end{aligned}$$

The other components are zero.

This is a solution of the standard type IIB SUGRA. [Orlando-Reffert-Sakamoto-KY, 1607.00795]

By performing the following coordinate transformations,

$$\rho = \tilde{\rho}\, \mathrm{e}^{\eta a_1 x^-}\,, \quad z = \tilde{z}\, \mathrm{e}^{\eta a_1 x^-}\,, \quad \theta = \tilde{\theta} - \eta a_2 x^-\,, \quad \boxed{x^- = \frac{1}{2\eta a_1}\log(2\eta a_1\, \tilde{x}^-)},$$

the ``T-dualized'' background can be rewritten as

$$\begin{aligned}
ds^2 &= -2\tilde{z}^2\, dx^+ d\tilde{x}^- + \frac{d\tilde{\rho}^2 + \tilde{\rho}^2 d\tilde{\theta}^2 + d\tilde{z}^2}{\tilde{z}^2} \\
&\quad + dr^2 + \sin^2 r\, d\xi^2 + \frac{d\phi_1^2}{\cos^2\xi \sin^2 r} + \frac{d\phi_2^2}{\sin^2 r \sin^2\xi} + \cos^2 r\, d\phi_3^2\,, \\
\mathcal{F}_5 &= \frac{4i\tilde{\rho}}{\tilde{z}^3 \sin\xi \cos\xi \sin^2 r}\, d\tilde{\rho}\wedge d\tilde{\theta}\wedge d\tilde{z}\wedge d\phi_1\wedge d\phi_2 \\
&\qquad + 4i\tilde{z}^2 \sin r \cos r\, dx^+\wedge d\tilde{x}^-\wedge dr\wedge d\xi\wedge d\phi_3\,, \\
\Phi &= \log\left[\frac{\tilde{z}^2}{\sin^2 r \sin\xi \cos\xi}\right]
\end{aligned}$$

After doing T-dualities along $x^0,\ x^3,\ \phi_1,\ \phi_3$, the undeformed AdS_5 x S^5 is reproduced.

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

Can one interpret this result as a twisted b.c. + alpha ?

c.f., Frolov, Alday-Arutyunov-Frolov

Undoing a Drinfeld twist?

NOTES We have studied other examples of non-abelian classical r-matrices.

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

All of them satisfy the generalized equations,

but other properties are different depending on the cases.

- For some examples, ``T-dualized'' backgrounds are not mapped to $AdS_5 \times S^5$.
- For others, it seems that even ``T-dualized'' background would not exist.

→ It would be significant to classify non-abelian classical r-matrices.

EX What kinds of classical r-matrices are mapped to $AdS_5 \times S^5$?

c.f., an argument based on scaling limits of the η-model

[Hoare-van Tongeren, 1605.03554]

Double covering by an AdS$_5$

For simplicity, let us take that η is positive and $a_1 = 1, \; a_2 = 0$.

One can realize that the ``T-dualized'' background is doubly covered by a single AdS$_5$ via the coordinate transformations:

$$x^- = \frac{1}{2\eta}\log(2\eta\tilde{x}^-) \qquad (\tilde{x}^- > 0)$$

$$x^- = \frac{1}{2\eta}\log(-2\eta\tilde{x}^-) \qquad (\tilde{x}^- < 0)$$

It would be interesting to consider an affine symmetry algebra associated with the ``T-dualized'' background. A folded Yangian?

Question

How prevalent is integrability in various kinds AdS/CFTs?

NOTE: Integrable subsectors are ubiquitous. EX integrable subsectors in large N QCD in 4 D

So we will concentrate on the full integrability below.

There are various kinds of AdS/CFT

Real (or complex) beta-deformations [Lunin-Maldacena, Frolov]

Gravity duals for NC gauge theories [Hashimoto-Itzhaki, Maldacena-Russo]

$AdS_5 \times T^{1,1}$ [Klebanov-Witten] $AdS_5 \times Y^{p,q}$ [Gauntlett-Martelli-Sparks-Waldram]

Klebanov-Strassler, Maldacena-Nunez

AdS BH [Horowitz-Strominger] AdS solitons [Witten, Horowitz-Myers]

AdS/NRCFT [Son,Balasubramanian-McGreevy, Kachru-Liu-Mulligan]

q-deformation of AdS_5xS^5 [Delduc-Magro-Vicedo, Arutyunov-Borsato-Frolov]

A classification list

(would not be complete)

Integrable backgrounds

Real beta-deformations [Frolov, hep-th/0503201]

Gravity duals for NC gauge theories [Matsumoto-KY, 1403.2703]

q-deformation of $AdS_5 \times S^5$ [Delduc-Magro-Vicedo, 1309.5850, Arutyunov-Borsato-Frolov, 1312.3542]

TsT transformatios of AdS_5 [Hubeny-Rangamani-Ross , hep-th/0504034] [Dhokarh-Haque-Hashimoto , 0801.3812] [Kawaguchi-Matsumoto-KY, 1401.4855]

Non-integrable backgrounds

Complex beta-deformations [Giataganas-Pando Zayas-Zoubos, 1311.3241]

$AdS_5 \times T^{1,1}$ [Basu-Pando Zayas, 1103.4107]

$AdS_5 \times Y^{p,q}$ [Basu-Pando Zayas, 1105.2540]

AdS BH [Pando Zayas-Terrero Escalante, 1007.0277]

AdS solitons [Basu-Das-Ghosh, 1103.4101]

Klebanov-Strassler, Maldacena-Nunez [Basu-Das-Ghosh-Pando Zayas, 1201.5634]

Schrödinger spacetime with $z = 4, 5, 6$ [Giataganas-Sfetsos, 1403.2703]

Lifshitz space (with hyper-scaling violation) [Giataganas-Sfetsos, 1403.2703] [Bai-Chen-Lee-Moon, 1406.5816]

p-brane backgrounds [Stepanchuk-Tseytlin, 1211.3727] [Chervonyi-Lunin, 1311.1521]