Probing the Standard Model with flavor physics: an exclusive determination of $|V_{cb}|$ from the $B \to D^* \ell \nu$ semileptonic decay at non-zero recoil

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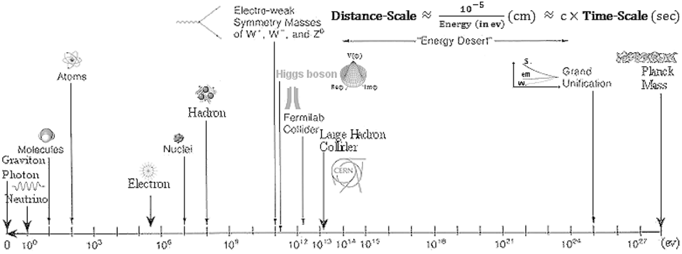
The Standard Model (SM)

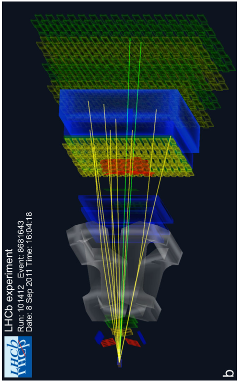
- The Standard Model is (arguably) the most successful theory describing nature we have ever had
- The theory is not completely satisfactory
  - Situation similar to that at the end of the XIX century
- The SM can explain phenomena in a large range of scales



- Yet there is a region where we expect the SM to fail
- The SM is regarded as an effective theory at low energies (low means $E \lesssim v_{EW} \approx 0.1 - 1$ TeV)

Where to look for new physics?

Energy frontier



Intensity frontier

Cosmology frontier

The V_{cb} matrix element: Tensions

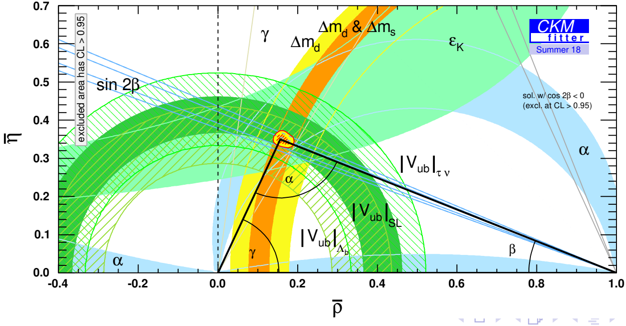
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\lvert V_{cb}\rvert\,(\cdot 10^{-3})$	PDG 2016	PDG 2018	PDG 2020
Exclusive	39.2 ± 0.7	41.9 ± 2.0	39.5 ± 0.9
Inclusive	42.2 ± 0.8	42.2 ± 0.8	42.2 ± 0.8

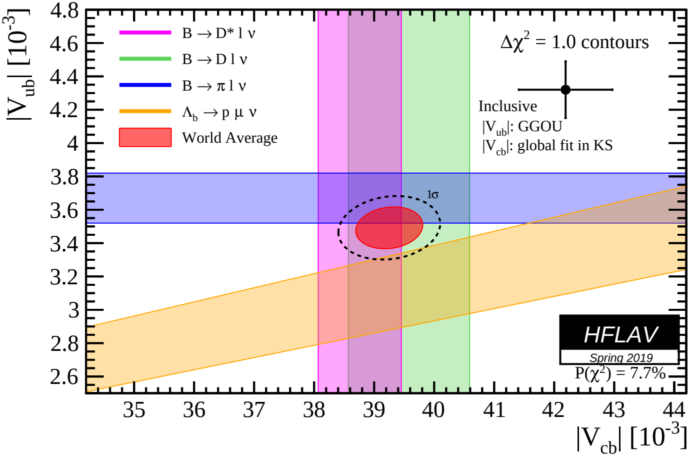
- Matrix must be unitary (preserve the norm)
- Inclusive 2021 $\lvert V_{cb}\rvert = 42.16(59) \times 10^{-3}$

Bordone, Capdevila, Gambino; arXiv:2107.00604

- **Current tensions (2021) stand at** $\approx 3\sigma$



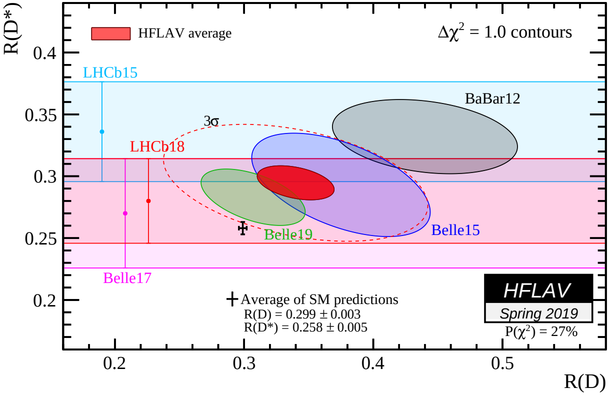
Break: Reminder of $|V_{ub}|$ vs $|V_{cb}|$



Current status of $|V_{ub}|$ vs $|V_{cb}|$ (HFLAV 2019)

The V_{cb} matrix element: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma$ tension with the SM

The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}}P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}}\underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}}\left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q\to\infty}\mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2\left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: Calculating $R(D^*)$

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} + \underbrace{K_2(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}_2(w)|^2}_{\text{Theory}}\right] \times |V_{cb}|^2$$

- The amplitudes $\mathcal{F}$,$\mathcal{F}_2$ must be calculated in the theory
- Since $K_2(w,0)=0$, $\mathcal{F}_2$ only contributes significantly with the τ
- Knowing these amplitudes, one can extract $|V_{cb}|$ from experiment
  - It is possible to extract $R(D^*)$ without experimental data!

$$R(D^*) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw\,\left[K_1(w,m_\tau)\,|\mathcal{F}(w)|^2 + K_2(w,m_\tau)\,|\mathcal{F}_2(w)|^2\right] \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\mathrm{Max}}} dw\,\left[K_1(w,0)\,|\mathcal{F}(w)|^2\right] \times \cancel{|V_{cb}|^2}}$$

- $|V_{cb}|$ cancels out

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion in the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

- Boyd-Grinstein-Lebed (BGL)

  Phys. Rev. Lett. 74 (1995) 4603-4606
  Phys.Rev. D56 (1997) 6895-6911
  Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

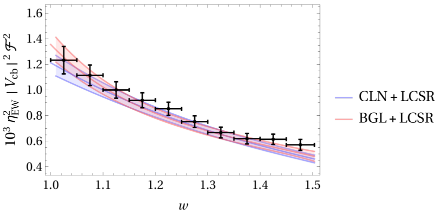
- Caprini-Lellouch-Neubert (CLN)

  Nucl. Phys. B530 (1998) 153-181

$$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

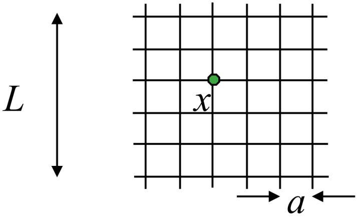
$$|V_{cb}| = 41.7 \pm 2.0 (\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
  - From Babar's paper PRL **123**, 091801 (2019) **BGL is compatible with CLN and far from the inclusive value**
  - Belle's paper PRD **100**, 052007 (2019) finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

Break: Introduction to Lattice QCD

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f \left(\gamma^\mu D_\mu + m_f\right) \psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu} F^{\mu\nu}$$



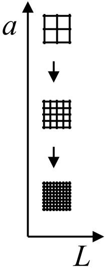
- Discretize space-time in a computer
  - Finite lattice spacing a
  - Finite spatial volume L
  - Finite time extent T

- Perform simulations in an unphysical setup and approach the physical limit
  - Enlarge the volume and reduce a
  - Quark masses $\implies$ Pion masses (hadrons are matched)
  - Number of sea quarks $n_f = 2+1, \quad 2+1+1, \quad 1+1+1+1\ldots$

Break: Introduction to Lattice QCD

The systematic error analysis is based on **EFT** descriptions of QCD
The EFT description:

- provides functional form for different extrapolations (or interpolations)
- can be used to construct improved actions
- can estimate the size of the systematic errors



In order to keep the systematic errors under control we must repeat the calculation for several lattice spacings, volumes, light quark masses... and use the EFT to extrapolate to the physical theory

Break: Heavy quarks in Lattice QCD

Heavy quark treatment in Lattice QCD

- For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
- For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
  - In this work $am_c \sim 0.15 - 0.6$, but $am_b > 1$

Need special actions and ETs to describe the bottom quark

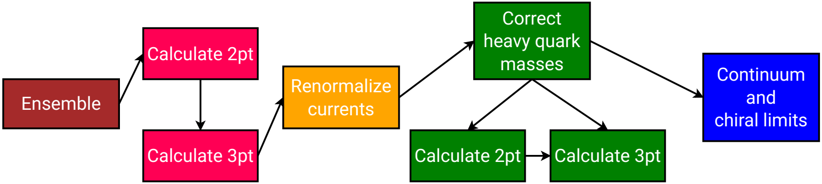
- Relativistic HQ actions (this work $\rightarrow$ FermiLab)
- Non-Relativistic QCD (NRQCD)

If the action is improved enough, one can treat the bottom as a light quark

- Highly improved action AND small lattice spacing
- Use unphysical values for m_b and extrapolate

The discretization errors needn't disappear **as long as we keep them under control**

Break: Lattice workflow



Calculating $|V_{cb}|$ on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\,(1+w)\,\boldsymbol{h_{A_1}}(w) - v_B^\nu\left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating $|V_{cb}|$ on the lattice: Formalism

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

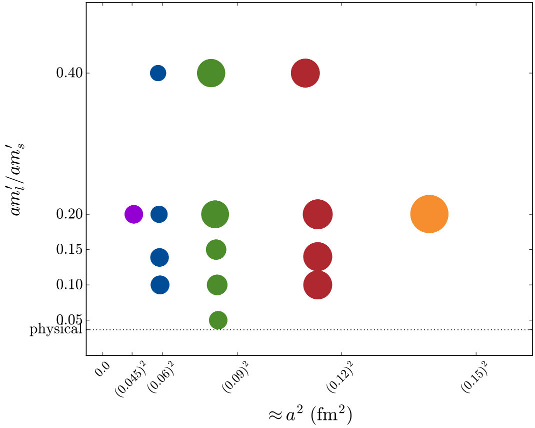
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

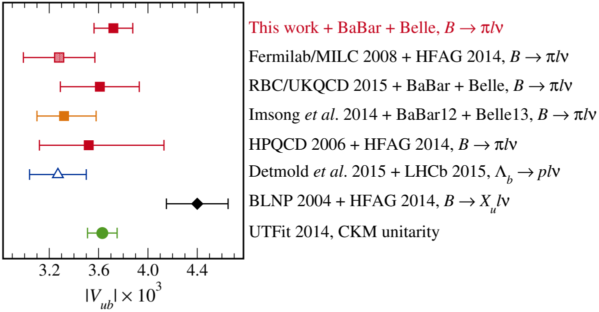
Introduction: Available data and simulations

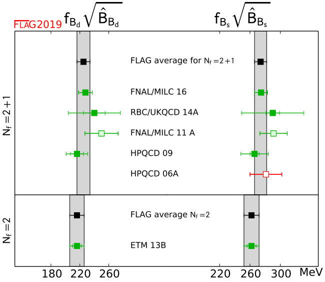
- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action



Introduction: The asqtad ensembles

- The asqtad data is being superseded by newer data with improved actions
  - 2^{nd} generation $N_f = 2 + 1$ HISQ and Fermilab charm/bottom quarks
  - 3^{rd} generation $N_f = 2 + 1 + 1$ HISQ and a HISQ bottom quark
- Some results from the asqtad ensembles are still competitive today





PRD**93**, (2016) 113016, arXiv:1602.03560

PRD**92**, (2015) 014024, arXiv:1503.07839

This is the last analysis done with asqtad data

Analysis: Extracting the form factors

Calculated ratios

$$\frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} \to x_f, \qquad w = \frac{1+x_f^2}{1-x_f^2}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle\left\langle \bar{B}(0)\right|\mathbf{A}\left|D^*(p_\perp,\varepsilon_\parallel)\right\rangle^*}{\langle D^*(0)|\,V_4\,|D^*(0)\rangle\left\langle \bar{B}(0)\right|V_4\left|\bar{B}(0)\right\rangle} \to R_{A_1}^2, \qquad h_{A_1} = \left(1-x_f^2\right)R_{A_1}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\perp)\right|\mathbf{V}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to X_V, \qquad h_V = \frac{2}{\sqrt{w^2-1}}R_{A_1}X_V$$

$$\frac{\left\langle D^*(p_\parallel,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_1, \qquad h_{A_3} = \frac{2}{w^2-1}R_{A_1}\left(w-R_1\right)$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|A_4\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_0,$$

$$h_{A_2} = \frac{2}{w^2-1}R_{A_1}\left(wR_1 - \sqrt{w^2-1}R_0 - 1\right)$$

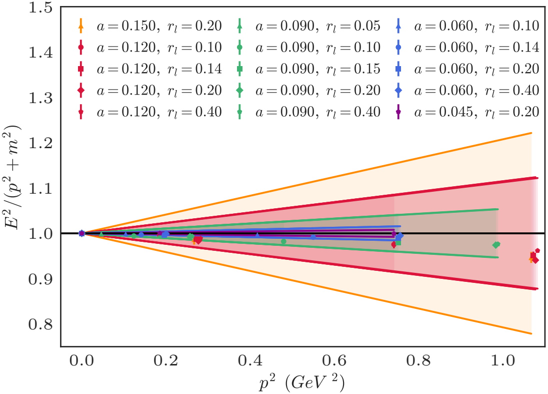
* Phys.Rev. **D**66, 01503 (2002)

Analysis: Systematics in the two-point function fits

- Heavy quark discretization effects break the dispersion relation
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha\, m)$

$$a^2E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- Deviations from the continuum expression measure the size of the discretization errors
- As long as the discretization errors are within expected bounds, this is all right
- Data for B meson only at rest $\longrightarrow$ Ok in the past



Analysis: Current renormalization

- In the coefficients of the terms of our effective theory a dependence arises with the scale (i.e. a)
- The renormalization tries to account for the right dependence
- The scheme we employ is called *Mostly non-perturbative renormalization* of results

$$Z_{V^{1,4},A^{1,4}} = \underbrace{\rho_{V^{1,4},A^{1,4}}}_{\text{Perturbative factor}} \times \underbrace{\sqrt{Z_{V_{bb}} Z_{V_{cc}}}}_{\text{Non-perturbative piece}}$$

- The (relatively large) non-perturbative piece cancels in our ratios
- The (close to one) perturbative piece (matching factor ρ) is calculated at one-loop level for $w = 1$ and $m_c = 0$
- The errors for $w \neq 1$ and $m_c \neq 0$ are estimated and added to the factor
- We calculate ρ_{A_1} and ratios of ρ_X/ρ_{A_1} for the other form factors
- ρ_{A_1} is **blinded** during analysis, hence all the form factors are multiplied by the same blinding factor
- The results shown here are **unblinded**

Analysis: Chiral-continuum fits

- Our data represents the form factors at non-zero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

$$h_{A_1}(w) = \underbrace{\left[1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*D\pi}}{48\pi^2 f_\pi^2 r_1^2}\text{logs}_{\text{SU3}}(a, m_l, m_s, \Lambda_{QCD})\right.}_{\text{NLO }\chi\text{PT + HQET}}$$

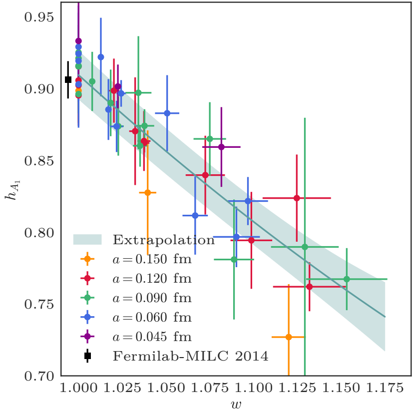
$$\left.\underbrace{+c_1 x_l + c_{a1} x_{a^2}}_{\text{NLO }\chi\text{PT}} \underbrace{-\rho^2_{A_1}(w-1) + k_{A_1}(w-1)^2}_{w\text{ dependence}} \underbrace{+ c_2 x_l^2 + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}}_{\text{NNLO }\chi\text{PT}}\right] \times$$

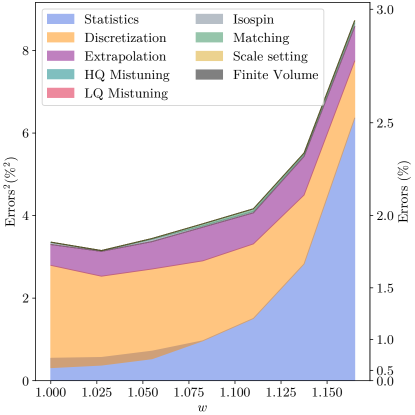
$$\underbrace{\left(1 + \beta_{11}^{A_1}\alpha_s a \Lambda_{\text{QCD}} + \cancel{\beta_{02}^{A_1} a^2 \Lambda^2_{\text{QCD}}} + \beta_{03}^{A_1} a^3 \Lambda^3_{\text{QCD}}\right)}_{\text{HQ discretization errors}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

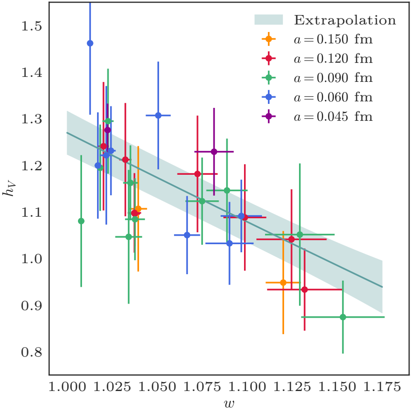
Analysis: Chiral-continuum fits

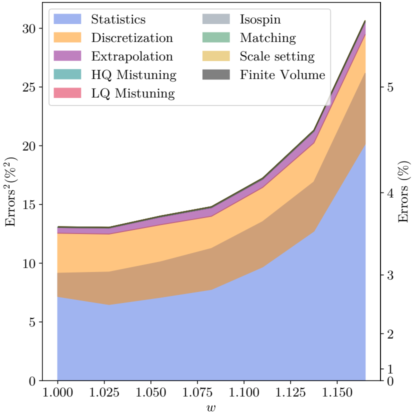




- Combined fit $p - \text{value} = 0.96$
- $h_{A_1}(1) = 0.909(17)$

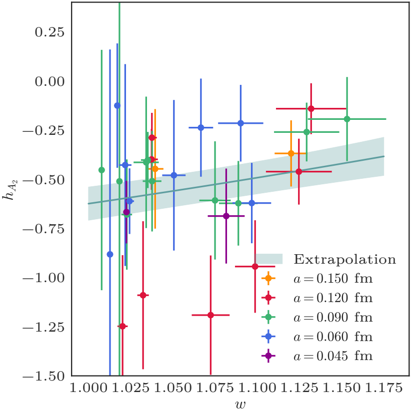
Analysis: Chiral-continuum fits

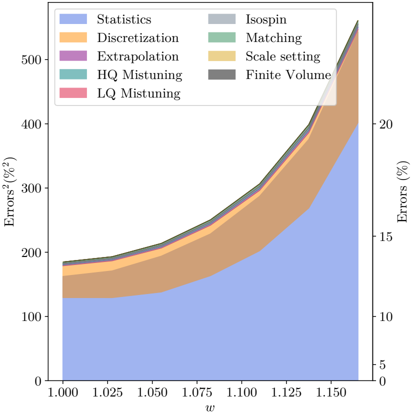




- Combined fit $p - \text{value} = 0.96$
- $h_V(1) = 1.270(46)$

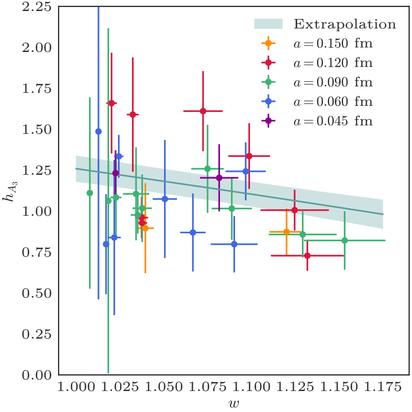
Analysis: Chiral-continuum fits

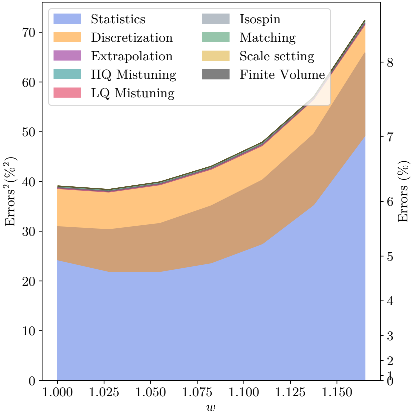




- Combined fit $p - \text{value} = 0.96$
- $h_{A_2}(1) = -0.624(85)$

Analysis: Chiral-continuum fits





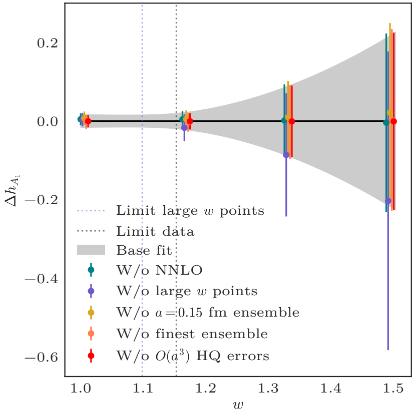
- Combined fit $p - \text{value} = 0.96$
- $h_{A_3}(1) = 1.259(79)$

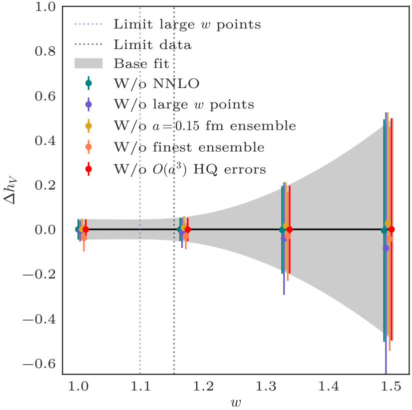
Analysis: Error budget

Source	$h_V(\%)$	$h_{A_1}(\%)$	$h_{A_2}(\%)$	$h_{A_3}(\%)$
Chiral-continuum fit error	4.2	2.0	17.4	6.9
(Statistics)	(3.7)	(1.2)	(16.9)	(6.3)
(Chiral-continuum extrapolation)	(0.8)	(0.9)	(1.7)	(0.5)
(LQ and HQ discretization)	(2.6)	(1.3)	(9.7)	(4.4)
(Matching)	(0.3)	(0.2)	(1.7)	(0.5)
(HQ mistuning)	(0.0)	(0.0)	(1.7)	(0.0)
LQ mistuning	0.0	0.0	0.1	0.0
Scale settings	0.0	0.0	0.3	0.1
Isospin effects	0.1	0.2	1.2	0.5
Finite volume	-	-	-	-
Total error	4.2	2.0	17.4	6.9

Errors at $w = 1.11$

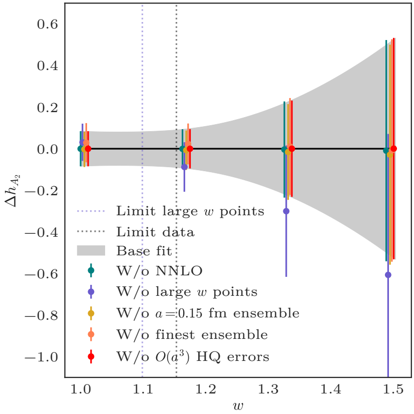
- **The discretization errors are one of the most important contributions to the final error**

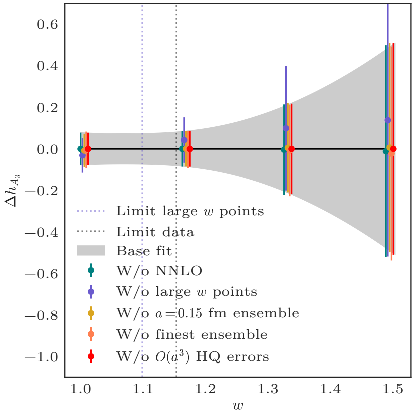




	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	**85.5/110**	86.1/111	71.5/93	79.7/101
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.9/101	85.6/111	

Results: Stability of chiral-continuum fits





	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	**85.5/110**	86.1/111	71.5/93	79.7/101
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.9/101	85.6/111	

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z)B_g(z)}\sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w)h_{A_1}(w) = \frac{1}{\phi_f(z)B_f(z)}\sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z)B_{\mathcal{F}_1}(z)}\sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z)B_{\mathcal{F}_2}(z)}\sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*})f(z=0)$
- Constraint $(1+w)m_B^2(1-r)\mathcal{F}_1(z=z_{\mathrm{Max}}) = (1+r)\mathcal{F}_2(z=z_{\mathrm{Max}})$
- BGL (weak) unitarity constraints

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

Analysis: z expansion fit procedure

- Several different datasets
  - Our lattice data
  - BaBar BGL fit — arXiv:1903.10002; Phys.Rev.Lett. **123**, 091801 (2019)
    - Generate synthetic data and include the data points to our joint fit
    - Limited by the order of BaBar BGL fit (222) $\rightarrow$ Truncation errors?
    - Fit dominated by Belle data anyway
  - Belle untagged dataset — arXiv:1809.03290; Phys.Rev. D**100**, 052007 (2019)
    - Data binned in four variables: $w, \cos\theta_v, \cos\theta_l$ and χ
    - Same normalization per binning $\sum \mathrm{Bins}(\alpha) = N, \quad \alpha = w, \cos\theta_v, \cos\theta_l, \chi$
    - Correlation matrices should reflect the normalization constraints $\rightarrow$ they don't
    - We use the data as it is published anyway (in Phys.Rev. D, the arXiv correlation matrices are wrong, even on v3!!)
    - $f_{00} = 0.486(6)$ in our analysis right from the start

See Belle's erratum Phys.Rev. D**103** Phys.Rev.D 103, 079901 (2021)

All the experimental and theoretical **correlations are included** in all fits

Analysis: Constraints and number of coefficients

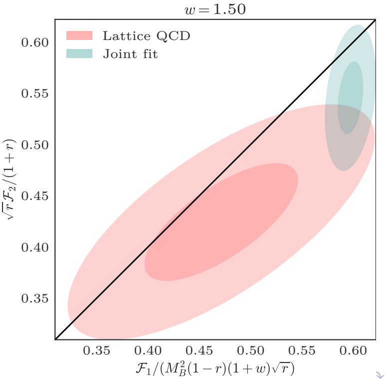
Constraints

- The constraint at zero recoil is used to remove a coefficient of the BGL expansion
- Neither the constraint at maximum recoil nor the unitarity constraints are imposed

How many coefficients in the BGL z-expansion?

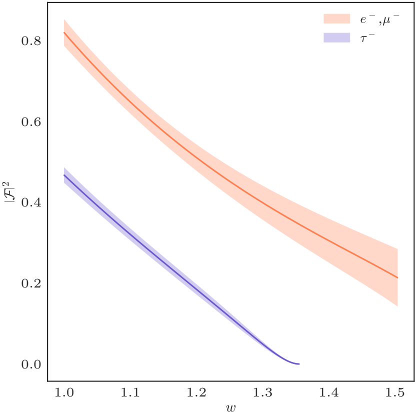
Phys.Rev. **D100 (2019), 013005**

- Add coefficients until
  - We exhaust the degrees of freedom
  - The error is saturated
- Compared linear/quadratic/cubic fits
  - Agreement in the low order coefficients
  - Quadratic saturates error, cubic no new information

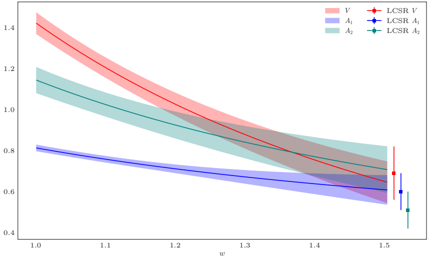


Results: Decay amplitude and form factors

Lattice prediction for the decay amplitude



Comparison with LCSR



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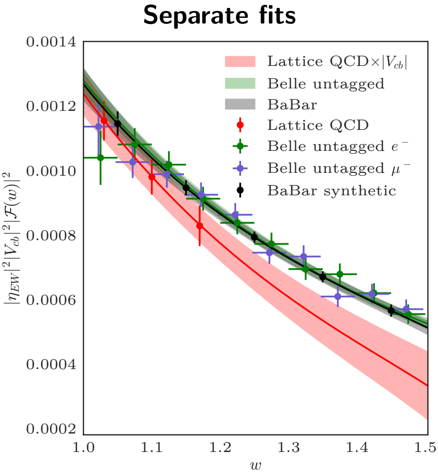
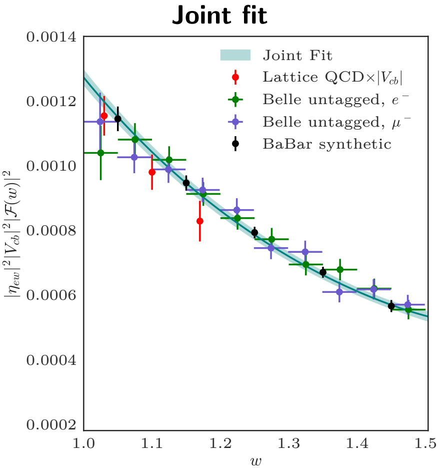
- Combined fit $p - \text{value} = 0.88$
- Good agreement for A_1, V
- Reasonable agreement for A_2

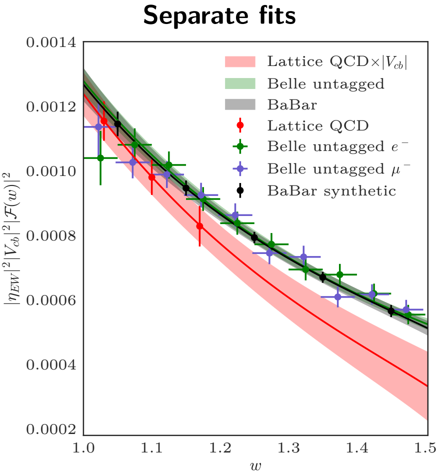
Results: Separate fits and joint fit

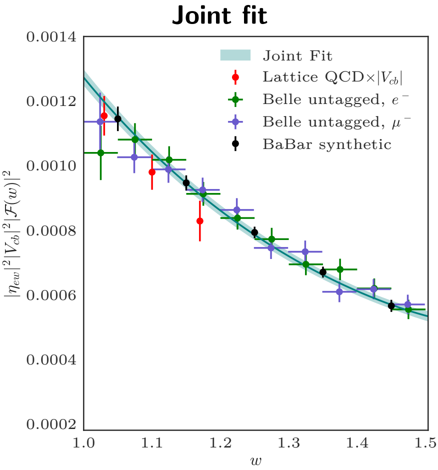




Fit	Lattice	Exp	Lat + Belle	Lat + BaBar	Lat + Exp
p-Value	0.88	0.037	0.015	0.088	0.002

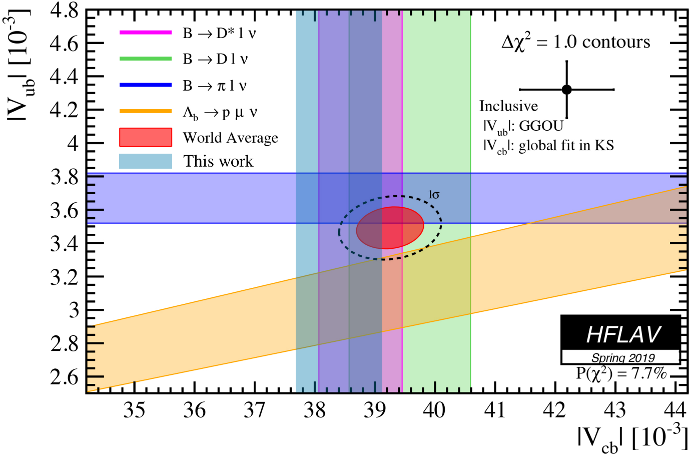
Results: Separate fits and joint fit





Unblinded, final result $|V_{cb}| = 38.40(74) \times 10^{-3}$

Results: Update of $|V_{ub}|$ vs $|V_{cb}|$

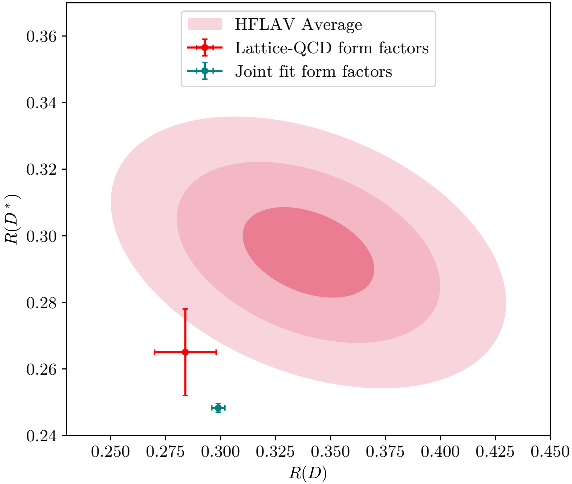


The $|V_{cb}|$ puzzle remains

Results: $R(D^*)$ in context

No constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.265(13)$ $R(D^*)_{\mathbf{Lat+Exp}} = 0.2483(13)$
W/ constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.274(10)$ $R(D^*)_{\mathbf{Lat+Exp}} = 0.2492(12)$

Phys.Rev.D**92** (2015), 034506; Phys.Rev.D**100** (2019), 052007; Phys.Rev.D**103** (2021), 079901; Phys.Rev.Lett. **123** (2019), 091801



Conclusions

- This is the **first**, unquenched, completed $B \to D^* \ell \nu$ calculation at non-zero recoil on the lattice
- The **main new information of this analysis** comes from the behavior at small recoil of the form factors
- Main sources of errors of our form factors are
  - Statistics
  - Light- and heavy-quark discretization errors
- We have a short-term plan to reduce the discretization errors by improving the light-quark regularization

Conclusions

- The value of $|V_{cb}|$ from this analysis agrees with the one obtained from $B \to D\ell\nu$ analysis at non-zero recoil
  - Our newer value decreases the errors
- The inclusive-exclusive tension in the determination of $|V_{cb}|$ remains unsolved
- Results show $R(D^*)$ very close to the **theoretical prediction**
- The tension with the experimental average is reduced
  - Newest experimental determinations show values closer to the theoretical determination
- Further lattice analysis and refinements of our analysis can potentially settle the $R(D^*)$ issue
  - Pending JLQCD calculation on $B \to D^*\ell\nu$ form factor on the lattice
  - Next FNAL/MILC calculation of $B \to D^{(*)}\ell\nu$ is in the queue
- Our next calculation will allow us to confirm this results and have a better handle on the systematic errors
  - HISQ 2+1 + Fermilab HQ, analyze simultaneously $B \to D\ell\nu$ and $B \to D^*\ell\nu$

Thank you for your attention

BACKUP SLIDES

Analysis: Heavy quark mistuning corrections

- The simulations are run at approximate physical values of m_c, m_b
- After the runs the differences between the calculated and the physical masses is corrected non-perturbatively
  - The Fermilab action uses the kinetic mass m_2 to compute these corrections
  - $m_1 \to m_2$ as $a \to 0$

Correction process

1. For a particular ensemble correlators are computed at different m_c, m_b
2. All the ratios are calculated for the new values of the heavy quark masses, and the form factors are extracted
3. The derivative of combinations of the form factors with respect to the heavy quark masses is fitted to a suitable function
4. All the form factors are corrected using these results

Shifts are small in most cases, but add a small correlation among all data points

Analysis: Comparison with an improved CLN

- CLN is much more constraining than BGL, using only 4 fit parameters
- We can relax the constraints by allowing errors in the coefficients
  - We take into account the full correlation between ρ^2, c_{A_1} and d_{A_1}
- Update HQET relations between the form factors JHEP 11 (2017) 061

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + \left(64c_{A_1} - 16\rho^2\right)z^2 + \left(512d_{A_1} + 256c_{A_1} - 6\rho^2\right)z^3\right]$$

$$R_0^{\mathrm{CLN}}(w) = 1.25(35) - 0.183(77)\,(w-1) + 0.063(23)\,(w-1)^2$$

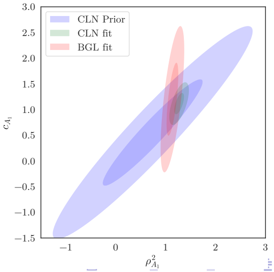
$$R_1^{\mathrm{CLN}}(w) = 1.28(36) - 0.101(51)\,(w-1) + 0.066(24)\,(w-1)^2$$

$$R_2^{\mathrm{CLN}}(w) = 0.744(44) + 0.128(38)\,(w-1) - 0.079(19)\,(w-1)^2$$

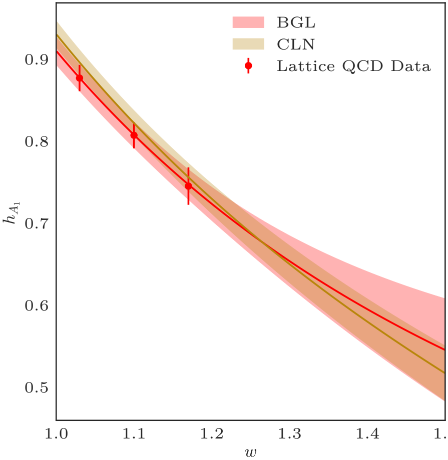
$$R_0^{\mathrm{CLN}}(w) = \frac{\sqrt{r}\mathcal{F}_2(w)}{(1+r)h_{A_1}(w)}$$

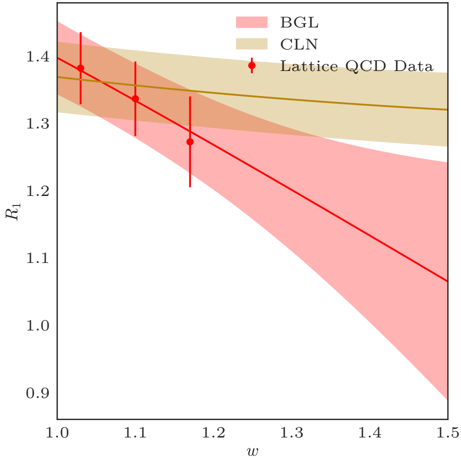
$$R_1^{\mathrm{CLN}}(w) = \frac{h_V(w)}{h_{A_1}(w)}$$

$$R_2^{\mathrm{CLN}}(w) = \frac{\frac{m_{D^*}}{m_B}h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$



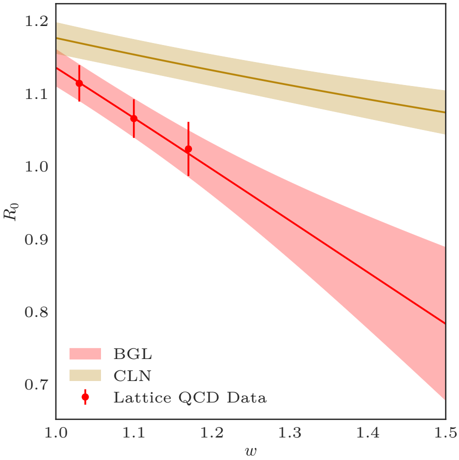
Analysis: Comparison with an improved CLN

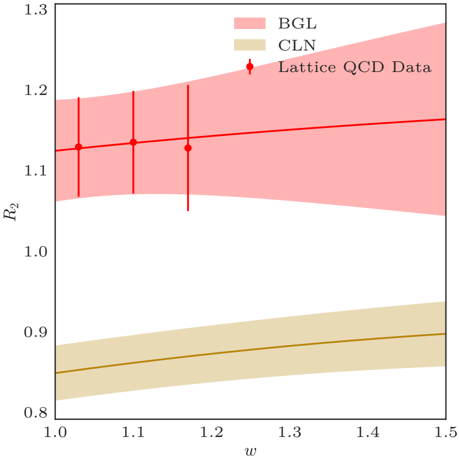




- Lattice only $p-\text{value} =\sim O(10^{-5})$
- Predictions for h_{A_1} and R_1^{CLN} look fine

Analysis: Comparison with an improved CLN





- Lattice only $p-\text{value} =\sim O(10^{-5})$
- Predictions for $R_0^{\rm CLN}$ and $R_2^{\rm CLN}$ show tensions

Analysis: The recoil parameter w

- The recoil parameter is measured dynamically
- In the lab frame (B meson at rest)

$$w^2 = 1 + v_{D^*}^2$$

- Ratio of three point functions

$$X_f(p) = \frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} = \frac{\mathbf{v}_{D^*}}{w+1}$$

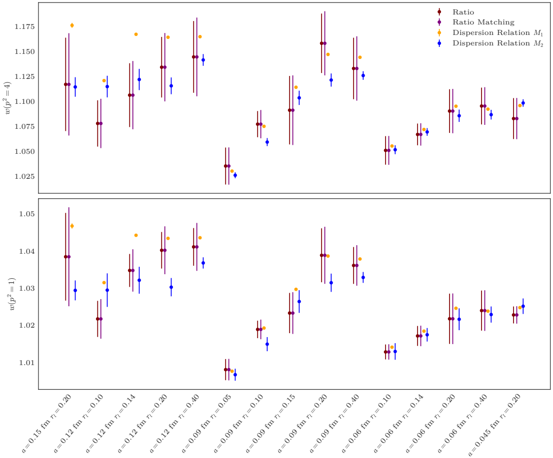
- From here

$$w(p) = \frac{1+\mathbf{x}_f^2}{1-\mathbf{x}_f^2}$$

- Alternatively one can use the dispersion relation

Analysis: The recoil parameter w

- Different methods to calculate the recoil parameter
  - In this analysis, we choose the ratio (more conservative)
  - The difference in the final result for the form factors, $|V_{cb}|$ and $R(D^*)$ is not significative



Analysis: Systematic errors

- Error contributions considered:
  - Correlator fits and excited states
    - Use the same fit ranges for all the correlators
    - Make sure the fits are stable under small variations
    - Add extra excited states

      We assume no extra errors
  - Light- and heavy-quark discretization errors
    - These errors are already taken into account in the chiral-continuum extrapolation
    - The NLO terms of tour ansatz include $O(a^2)$ corrections to describe the light-quark discretization errors
    - We employ generic discretization terms $\beta_X \alpha_s^p a^q$ for the heavy-quark
    - Try the chiral-continuum extrapolation with and without these terms to estimate the size of the correction
  - Chiral extrapolation
    - The errors are already taken into account in the chiral-continuum extrapolation
    - Try the chiral-continuum extrapolation with and without the NNLO and NNNLO terms to estimate their size
  - Matching
    - The errors are already taken into account in the chiral-continuum extrapolation
    - Try the chiral-continuum extrapolation with and without the matching errors to estimate their size

Analysis: Systematic errors

- Error contributions considered:
  - Heavy quark mistuning
    - The errors are already taken into account in the chiral-continuum extrapolation
    - Try the chiral-continuum extrapolation with and without mistuning corrections to estimate their size
  - Light quark mistuning
    - Try the chiral-continuum extrapolation with $m_{ud} \pm \sigma$ and compare
      The difference is negligible
  - Lattice scale dependence
    - Redo the chiral-continuum extrapolation with $r_1 \pm \sigma$ and compare
      The difference is negligible
  - Isospin effects
    - Try the chiral-continuum extrapolation with $m_{ud} = m_{u,d}$ and compare
    - The difference is added as an extra error to the final result
  - Finite volume errors
    - Following Arndt and Lin we estimate the size of the finite volume errors

      Phys. Rev. D70, 014503 (2004)

      The errors are negligible