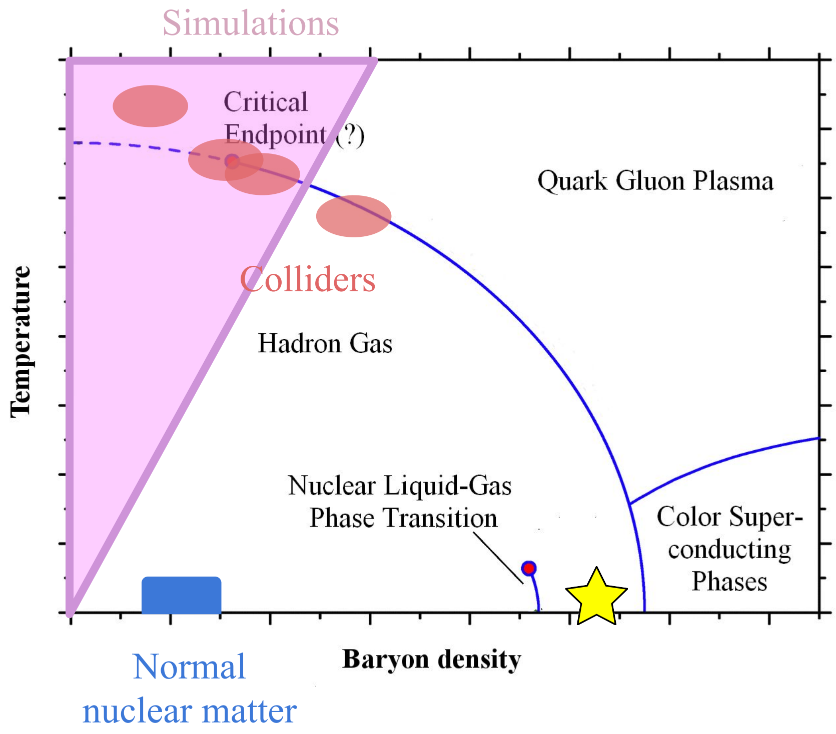
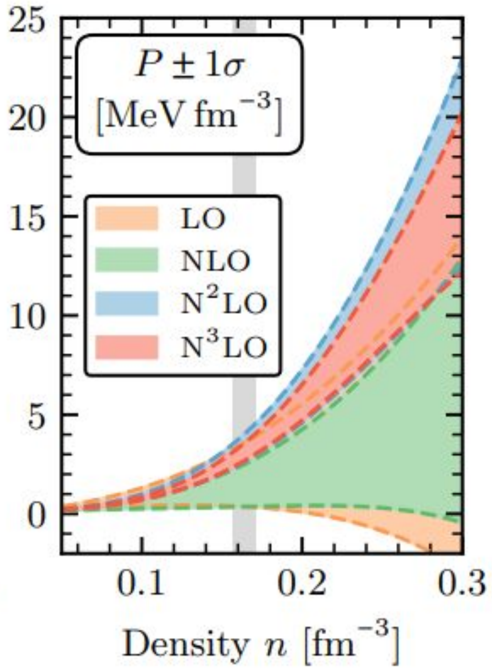
What neutron stars tell about the hadron-quark phase transition

János Takátsy

ELFT seminar, 2023.05.09.

Why study neutron stars?





Basic properties of neutron stars

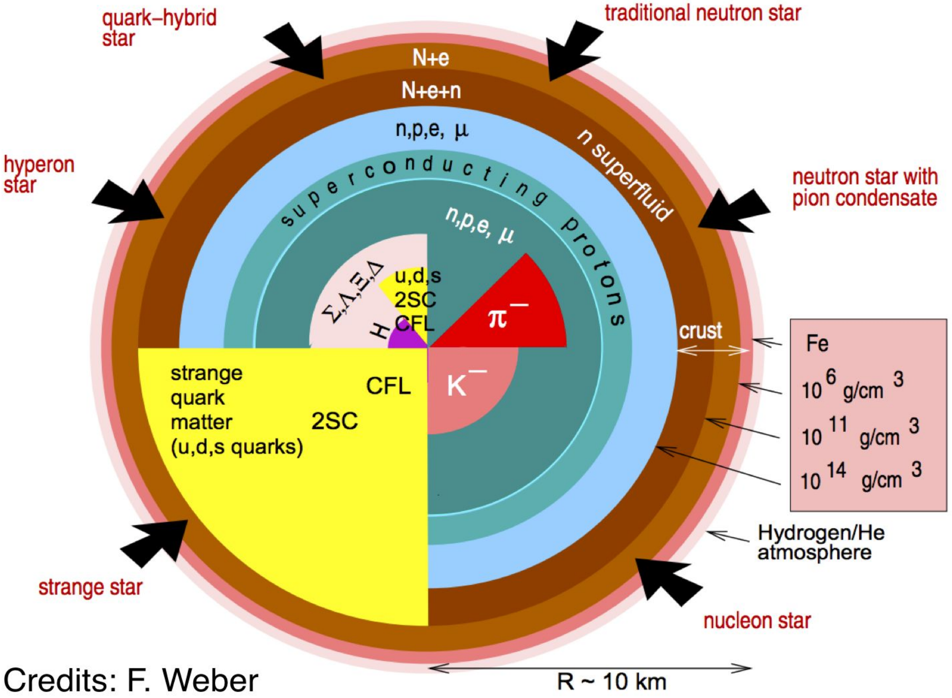
Size: $R \sim 10$ km

Mass: $M = 1.2\, M_\odot - 2.3\, M_\odot$

$\longrightarrow \rho \cong 5 \cdot 10^{17}\ \mathrm{kg/m^3}$

Strong magnetic field: $10^4 - 10^{11}$ T (16 T in laboratory)

Fast rotation (rotational period can be as low as several ms)



Credits: F. Weber

Tolmann-Oppenheimer-Volkoff equation

Spherically symmetric metric: $\mathrm{d}s^2 = e^{2\nu(r)}\mathrm{d}t^2 - e^{2\lambda(r)}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$

Einstein's equations (ideal fluid): $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$

$$T_{\mu\nu} = (p+\varepsilon)u_\mu u_\nu - p g_{\mu\nu}$$

After fiddling with the equations one can get:

$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r) + p(r)]\,\frac{M(r)+4\pi r^3 p(r)}{r^2 - 2M(r)r}}$$

$$e^{-2\lambda} = 1 + \frac{2M(r)}{r} \equiv 1 + \frac{2}{r}\int_0^r 4\pi r^2 \varepsilon(r)\mathrm{d}r$$

The mass-radius relation

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r) + p(r)]\,\frac{M(r)+4\pi r^3 p(r)}{r^2-2M(r)r}$$

How to get a mass radius relation:

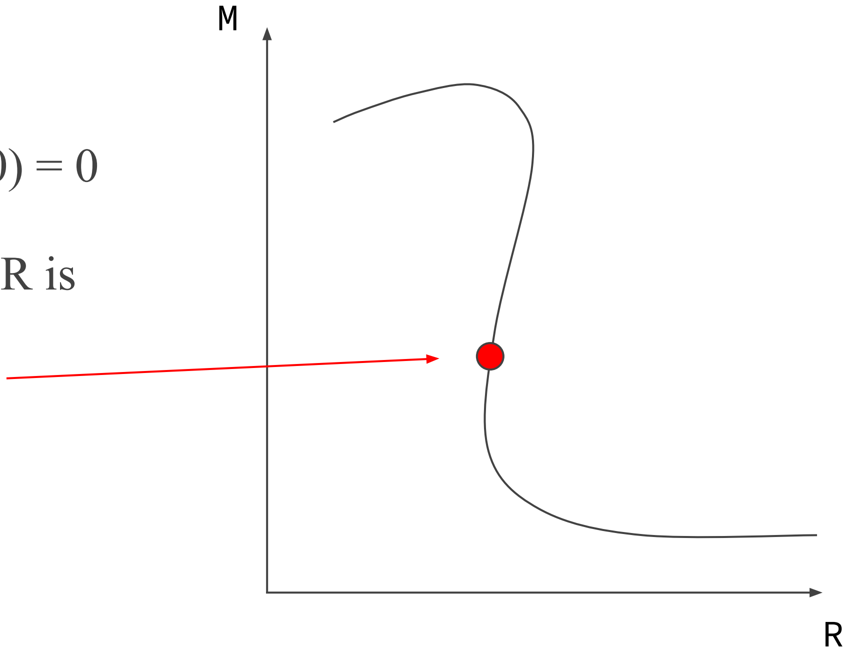
→ get an equation of state p(ε)

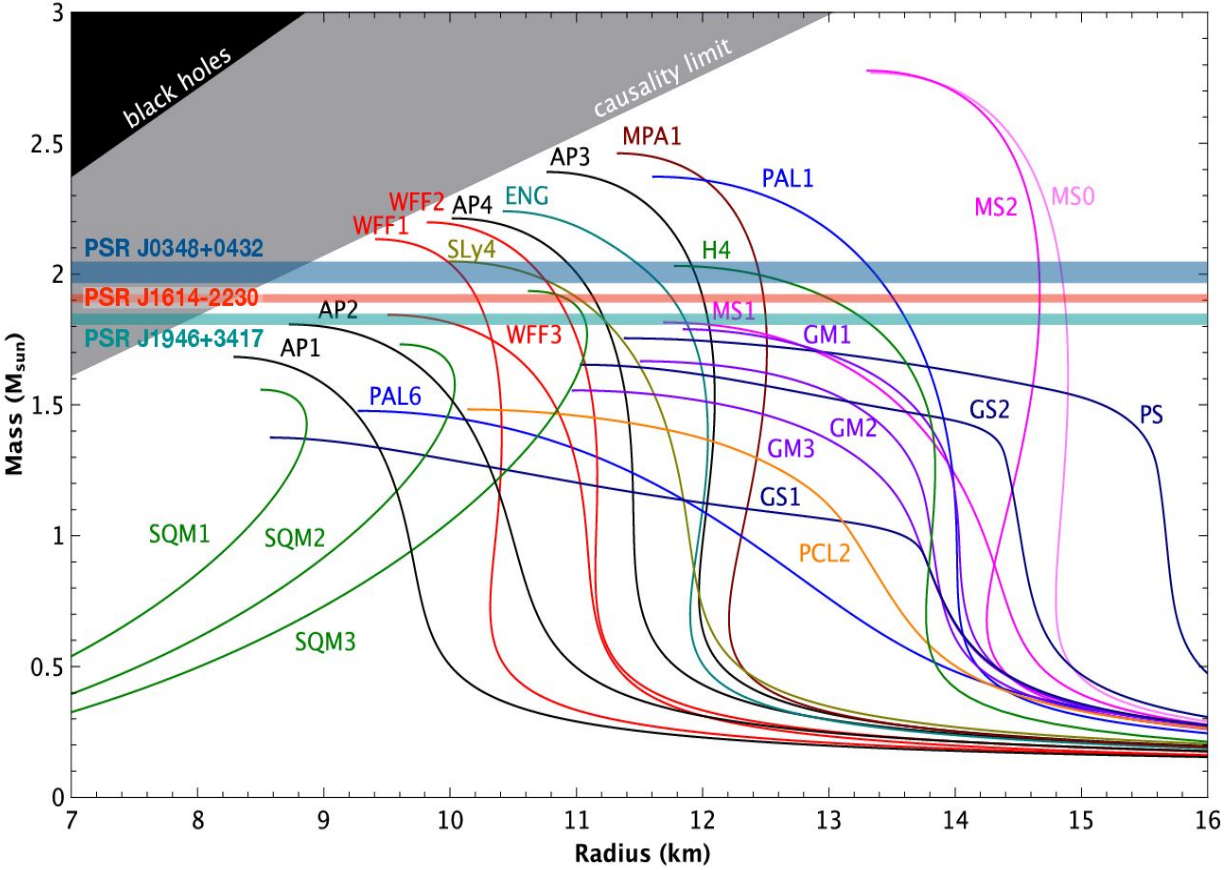
→ start with a specific central density: ε_c, p_c, M(0) = 0

→ integrate the TOV equations until p(R) = 0 → R is the radius of the NS

→ M(R) is the mass of the NS

→ change ε_c and repeat → M-R relation

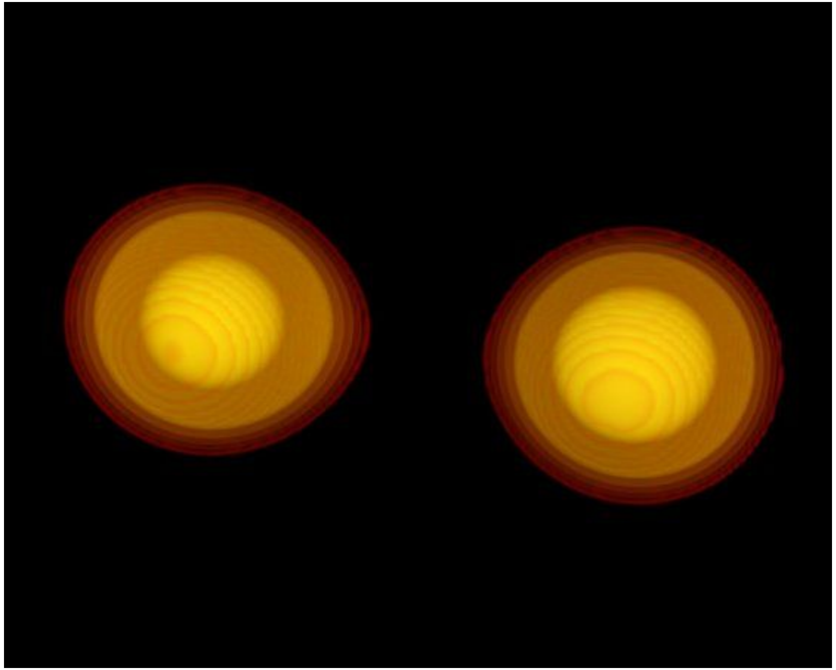




black holes
causality limit
PSR J0348+0432
PSR J1614-2230
PSR J1946+3417
MPA1
AP3
PAL1
MS2
MS0
WFF2
AP4
ENG
WFF1
SLy4
H4
MS1
AP2
WFF3
GM1
AP1
PAL6
GS2
PS
GM2
GM3
GS1
SQM1
SQM2
PCL2
SQM3
Mass (M_{sun})
Radius (km)

Tidal deformations

- Neutron stars have a finite size
  → they can be deformed by external forces
- The deformation depends on the mass and radius of the NS and on the EoS
- The tidal deformability can be a useful tool to determine these



Analogy: dielectric sphere in constant external electric field

Laplace's equation: $\Delta\Phi_{\text{ex}} = \Delta\Phi_{\text{int}} = 0$

Boundary conditions: $\Phi_{\text{ex}}(R) = \Phi_{\text{int}}(R)$

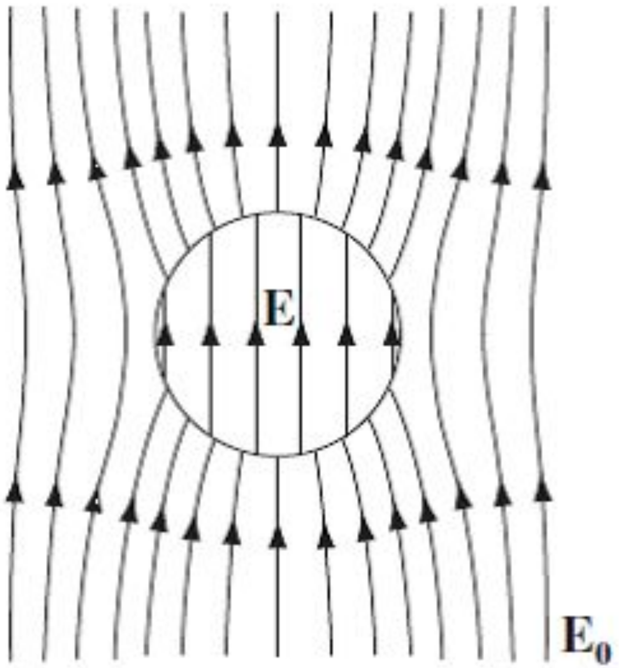
$$\varepsilon_{\text{ex}} \partial\Phi_{\text{ex}}/\partial r|_{r=R} = \varepsilon_{\text{in}} \partial\Phi_{\text{in}}/\partial r|_{r=R}$$

$$\Phi_{\text{ex}}(r \to \infty) = -\mathbf{E}_0\mathbf{r} = -E_0 r \cos\vartheta$$

General solution:

$$\Phi_{\text{in}} = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\vartheta)$$

$$\Phi_{\text{ex}} = \sum_{l=0}^{\infty} (C_l r^l + D_l / r^{l+1}) P_l(\cos\vartheta)$$



Analogy: dielectric sphere in constant external electric field

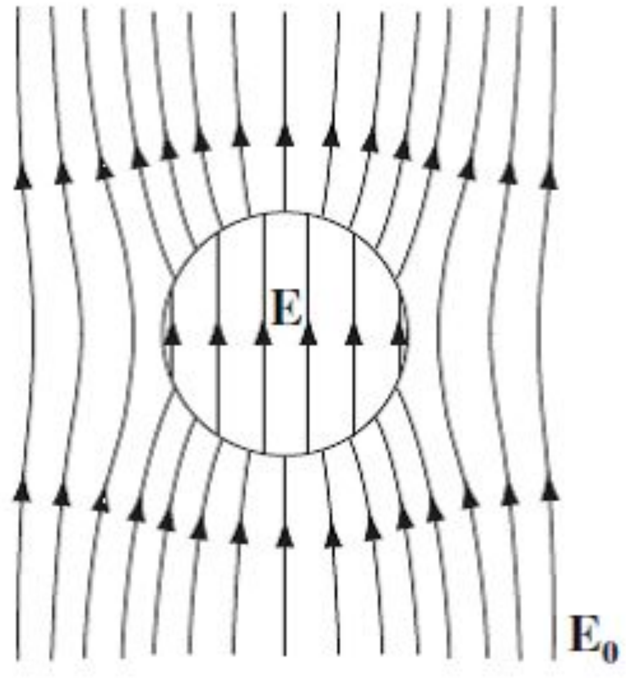
Coefficients: $C_1 = -E_0$

$$\rightarrow A_1 = \frac{-3\varepsilon_{\rm ex}}{\varepsilon_{\rm in}+2\varepsilon_{\rm ex}} E_0 \qquad D_1 = \frac{\varepsilon_{\rm in}-\varepsilon_{\rm ex}}{\varepsilon_{\rm in}+2\varepsilon_{\rm ex}} R^3 E_0$$

External solution: $\Phi_{\rm ex} = -\mathbf{E}_0\mathbf{r} + \frac{1}{4\pi\varepsilon_{\rm ex}}\frac{\mathbf{pr}}{r^3}$

Polarizability: $\mathbf{p} = \varepsilon_{\rm ex}\gamma\mathbf{E}_0$

$$\boxed{\gamma = 4\pi R^3 \frac{\varepsilon_{\rm r}-1}{\varepsilon_{\rm r}+2}}$$



Deformation of neutron stars

External and induced gravitational potential:

$$\Phi_{\text{ext}}(\mathbf{r}) = \Phi_{\text{ext}}(0) + \mathbf{r}_i \partial_i \Phi_{\text{ext}}(0) + \tfrac{1}{2}\mathbf{r}_i \mathbf{r}_j \partial_i \partial_j \Phi_{\text{ext}}(0) + \dots$$

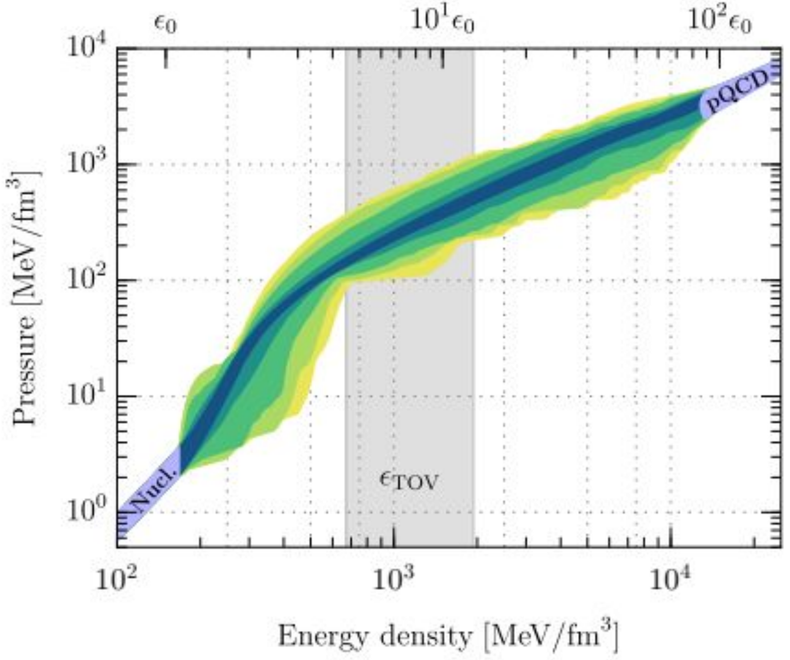
$$\Phi_{\text{ind}}(\mathbf{r}) = -G\left(\frac{M}{r} + \frac{\mathbf{p}_i \mathbf{r}_i}{r^3} + \frac{3}{2} Q_{ij} \frac{\mathbf{r}_i \mathbf{r}_j}{r^5} + \dots\right)$$

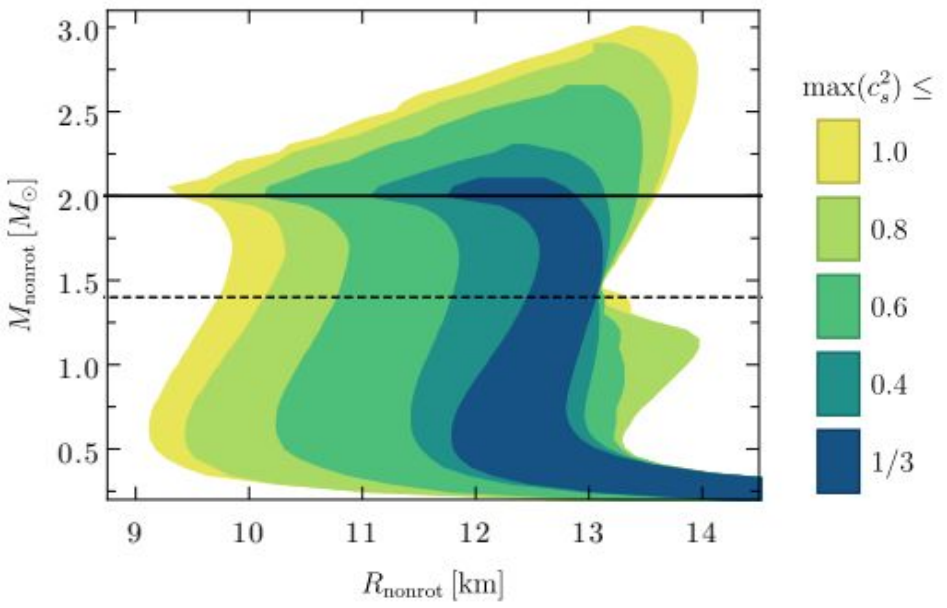
The external potential: $$\Phi(\mathbf{r}) = -G\left(\frac{M}{r} + \frac{3}{2} Q_{ij} \frac{\mathbf{r}_i \mathbf{r}_j}{r^5} + \dots\right) + \frac{1}{2}\mathcal{E}_{ij}\mathbf{r}_i \mathbf{r}_j + \dots$$

The tidal deformability and Love number:

$$Q_{ij} = -\lambda \mathcal{E}_{ij} \qquad\qquad k_2 = \frac{3}{2} G \lambda R^{-5}$$

Neutron star EoS constraints

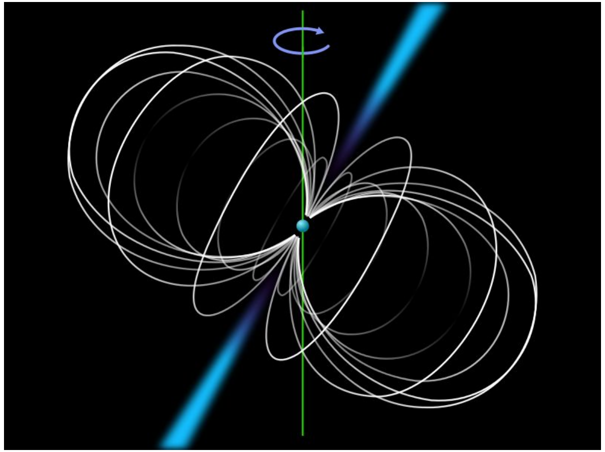


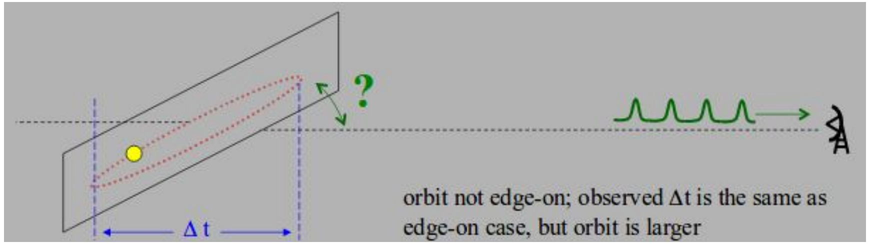


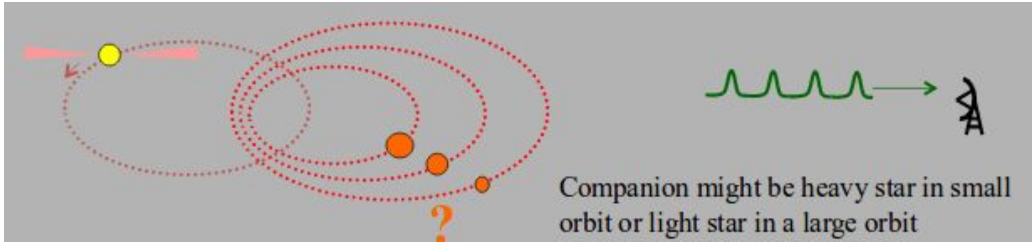
Mass measurement

→ Pulsars in binary systems + Doppler shift

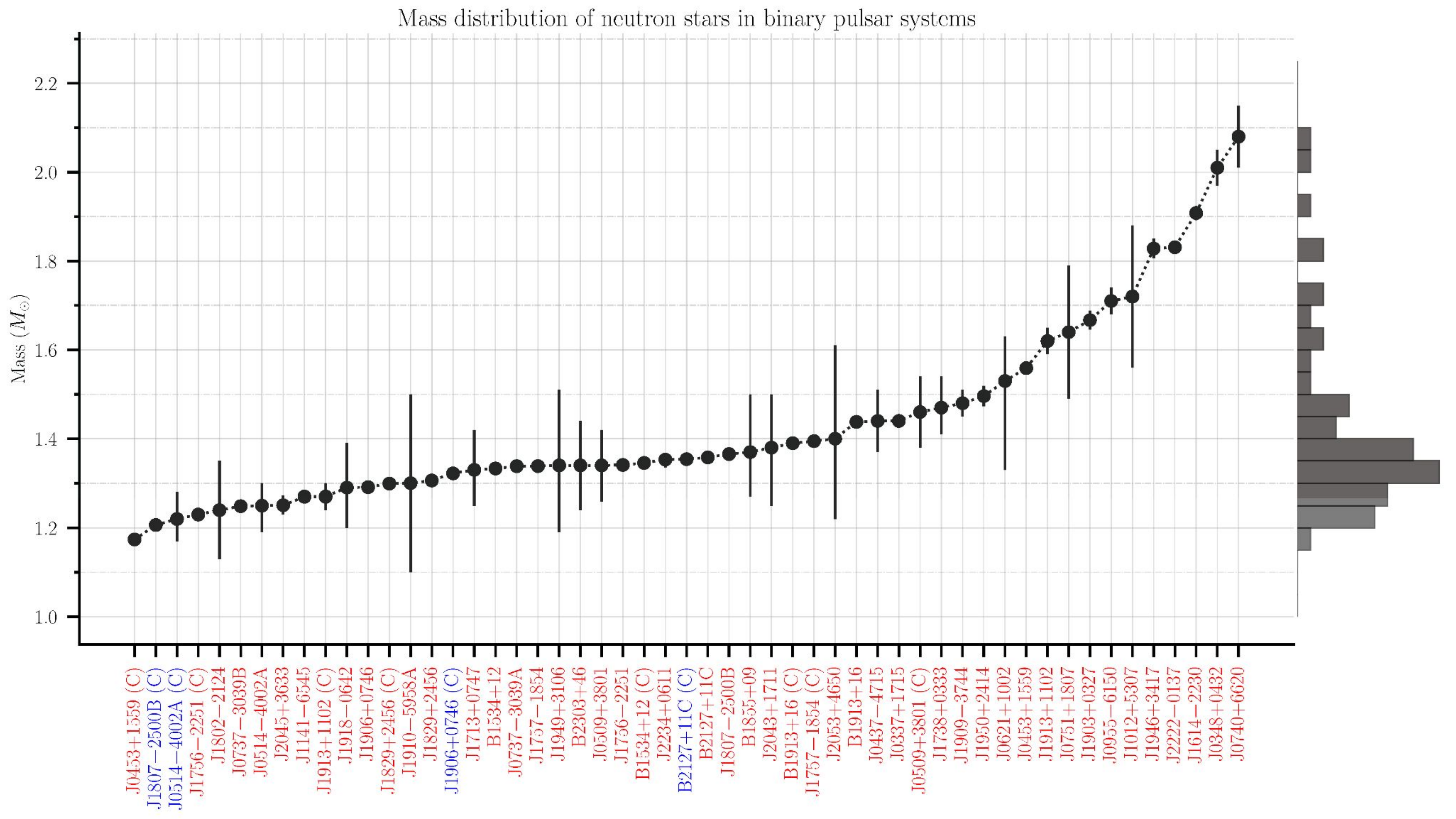
→ Degeneracies → only projected semi-major axis

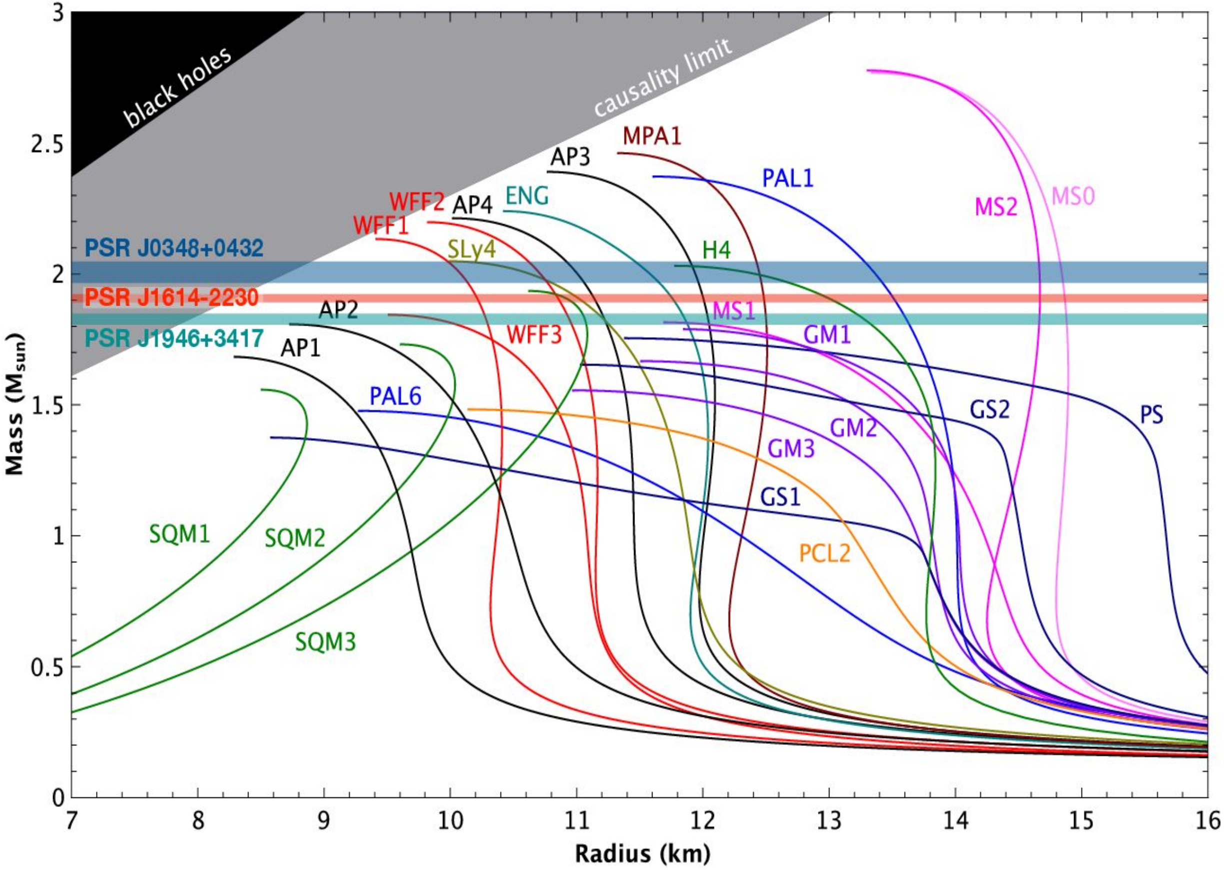






→ Observation of the companion object: another pulsar, white dwarf (+X-ray binaries)


Mass distribution of neutron stars in binary pulsar systems
Mass ($M_\odot$)
2.2
2.0
1.8
1.6
1.4
1.2
1.0
J0453+1559 (C)
J1807−2500B (C)
J0514−4002A (C)
J1756−2251 (C)
J1802−2124
J0737−3039B
J0514−4002A
J2045+3633
J1141−6545
J1913+1102 (C)
J1918−0642
J1906+0746
J1829+2456 (C)
J1910−5958A
J1829+2456
J1906+0746 (C)
J1713+0747
B1534+12
J0737−3039A
J1757−1854
J1949+3106
B2303+46
J0509+3801
J1756−2251
B1534+12 (C)
J2234+0611
B2127+11C (C)
B2127+11C
J1807−2500B
B1855+09
J2043+1711
B1913+16 (C)
J1757−1854 (C)
J2053+4650
B1913+16
J0437−4715
J0337+1715
J0509+3801 (C)
J1738+0333
J1909−3744
J1950+2414
J0621+1002
J0453+1559
J1913+1102
J0751+1807
J1903+0327
J0955−6150
J1012+5307
J1946+3417
J2222−0137
J1614−2230
J0348+0432
J0740+6620



black holes
causality limit
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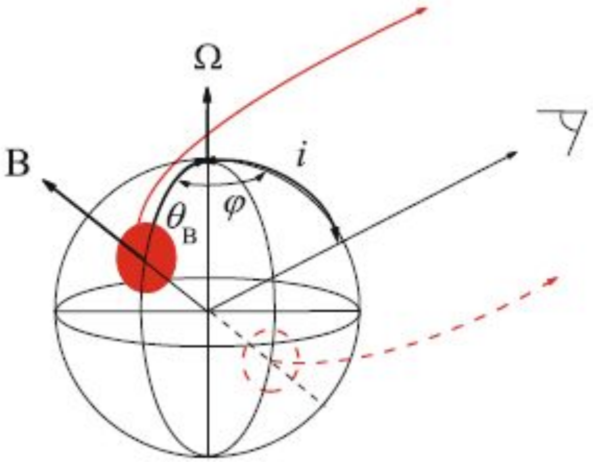
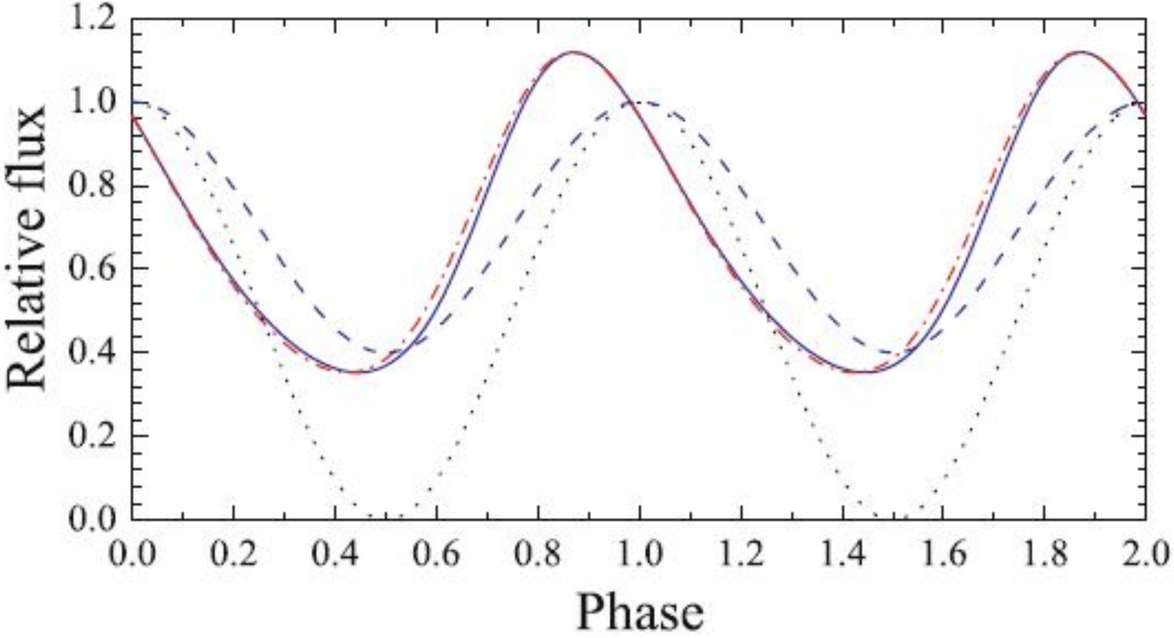
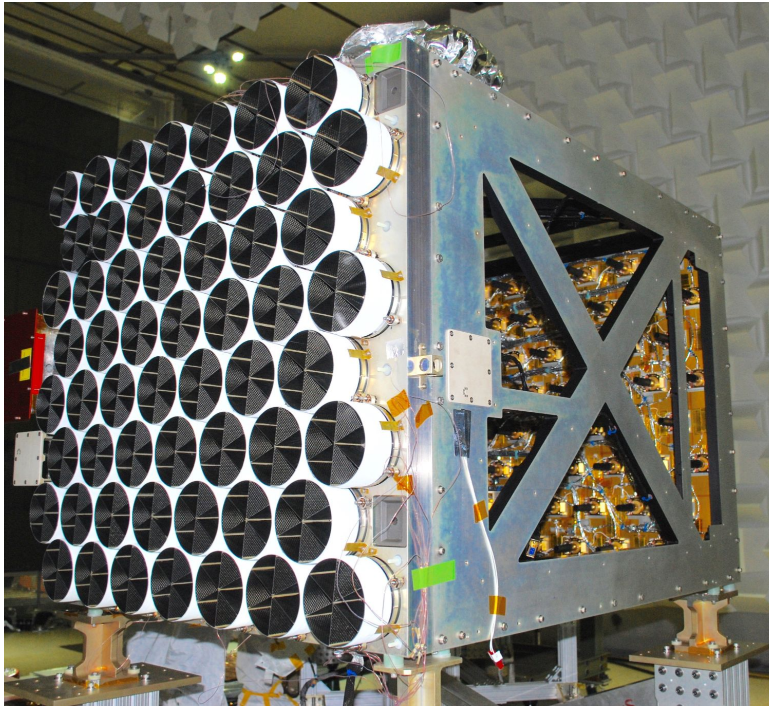
MS0
WFF2
AP4
ENG
WFF1
SLy4
H4
AP2
MS1
WFF3
GM1
AP1
PAL6
GS2
PS
GM2
GM3
GS1
SQM1
SQM2
PCL2
SQM3
Mass (M_sun)
Radius (km)

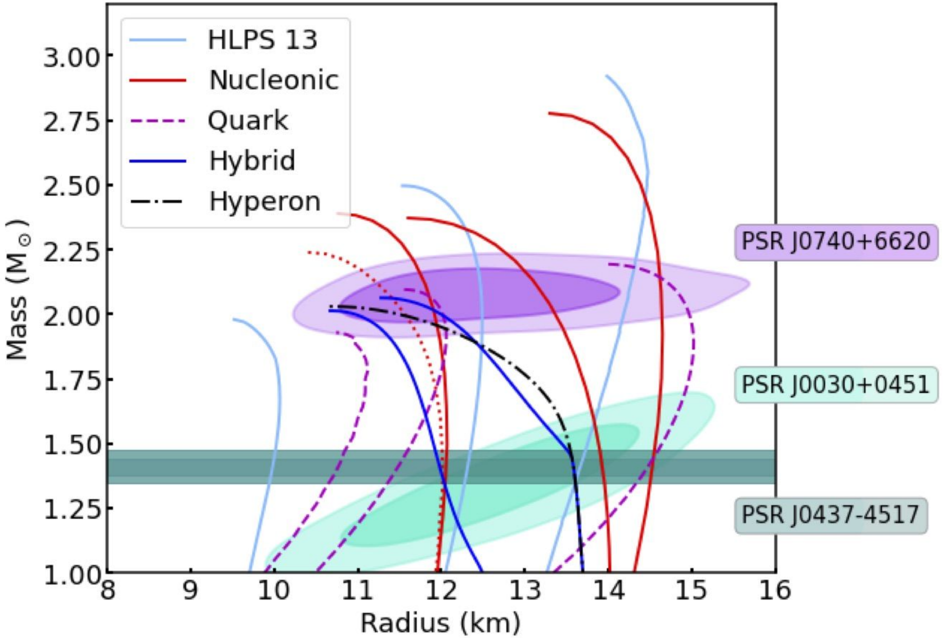
Radius measurement: pulse profile modeling





NICER measurements





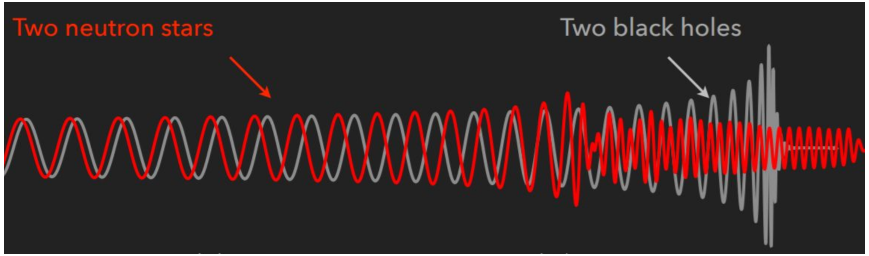
Measuring tidal deformability: gravitational waves

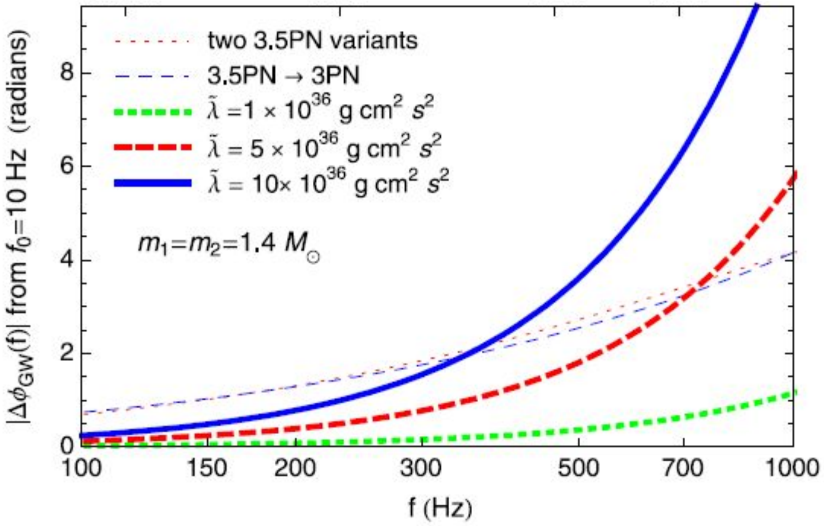
In the inspiral phase, far from the merger tidal effects cause a phase shift in the gravitational wave signal:

$$\delta\Psi \propto \tilde{\Lambda} f^{5/3}$$

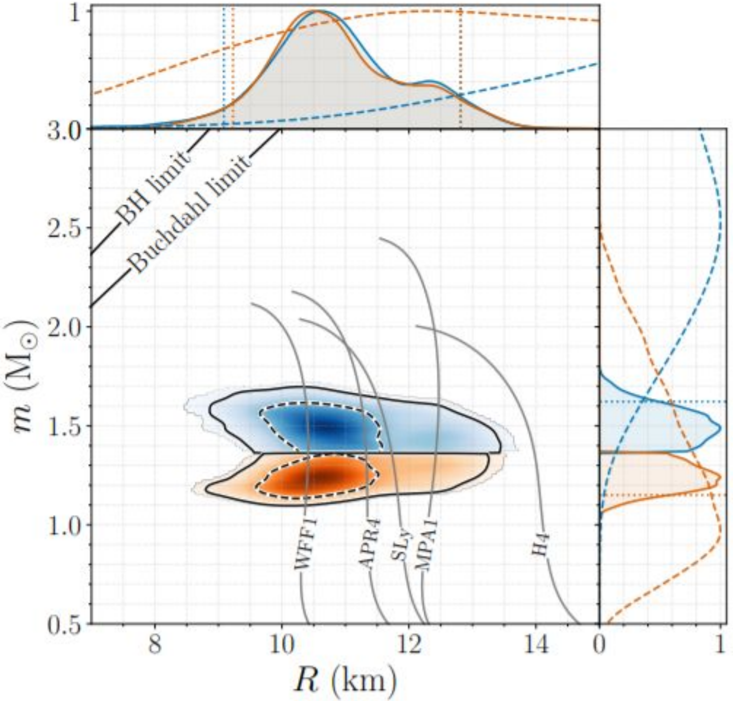
$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1+12m_2)m_1^4\Lambda_1+(m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda_i = \frac{\lambda_i}{m_i^5} = \frac{2}{3}k_{2,i}\frac{R_i^5}{m_i^5}$$

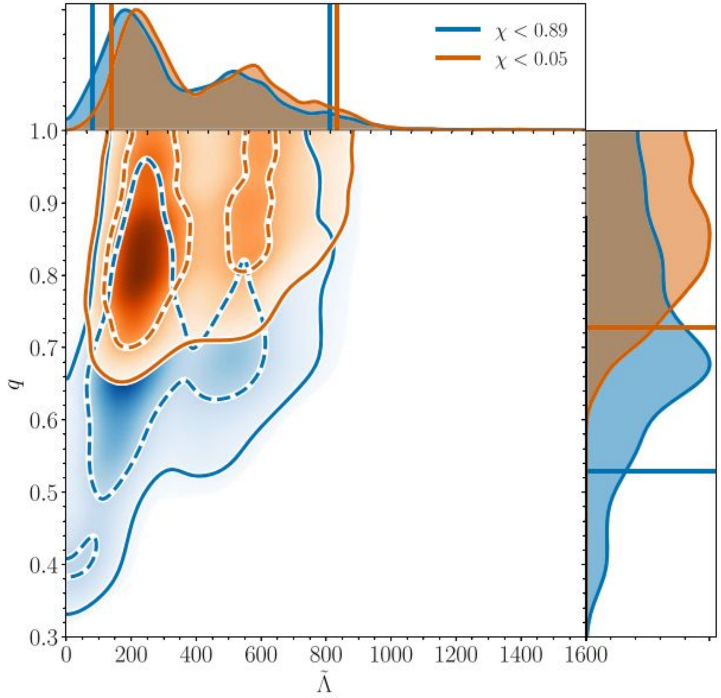




GW170817



BH limit
Buchdahl limit
WFF1
APR4
SLy
MPA1
H4
m ($M_\odot$)
R (km)



$\chi < 0.89$
$\chi < 0.05$
q
$\tilde{\Lambda}$

Using a constituent quark model

Our method: using a constituent quark model at intermediate densities

→ model is parametrized by meson phenomenology and finite temperature behaviour

→ at low densities we use various hadronic models and some general connection scheme

→ we have 4 free parameters (2 from quark model, other 2 from concatenation)

→ we use astrophysical measurements in a Bayesian framework to get posterior probabilities

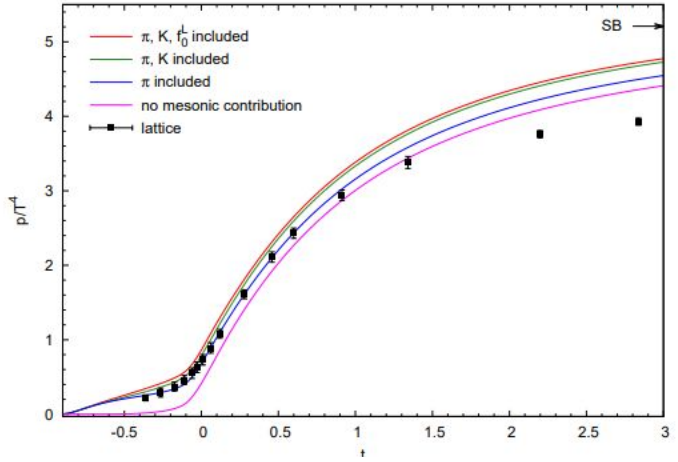
The constituent quark model

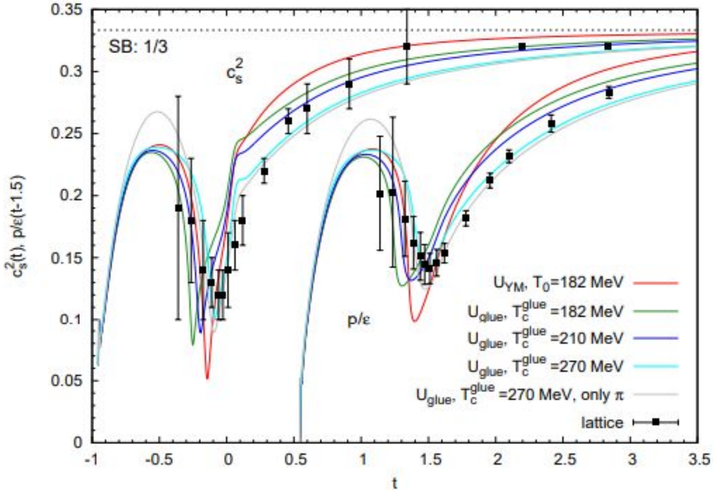
We use the (axial)vector meson extended linear sigma model

↳ SU(3) constituent quark-meson model with the complete (pseudo)scalar and (axial)vector meson nonets

↳ parameterized with meson vacuum masses and decay widths

↳ agrees well with lattice results at finite temperature





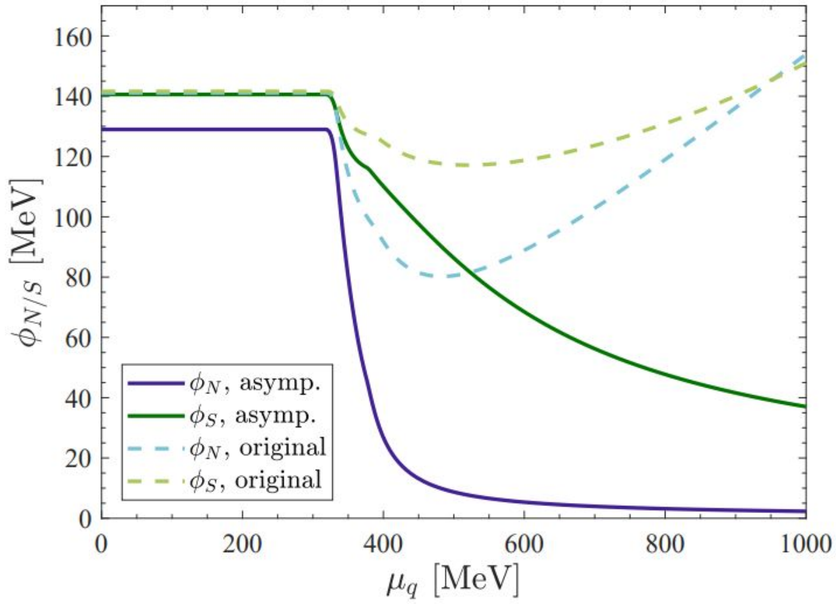
Vector condensates and asymptotic behaviour

For arbitrary parametrization chiral symmetry is not restored at high density.

↪ we need to include this extra requirement

↪ we get an extra constraint for the parameters

Increasing the vector coupling, the phase transition turns into a crossover (g_V>3.1)



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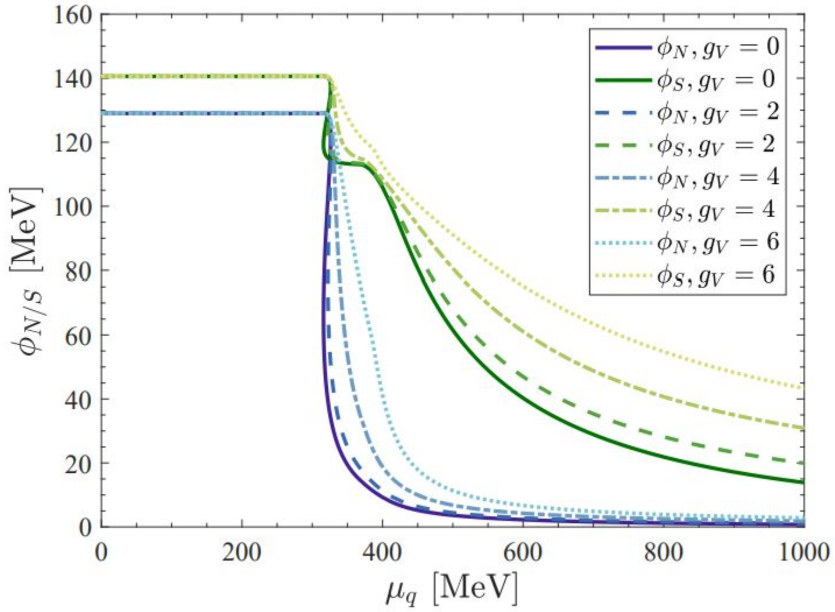
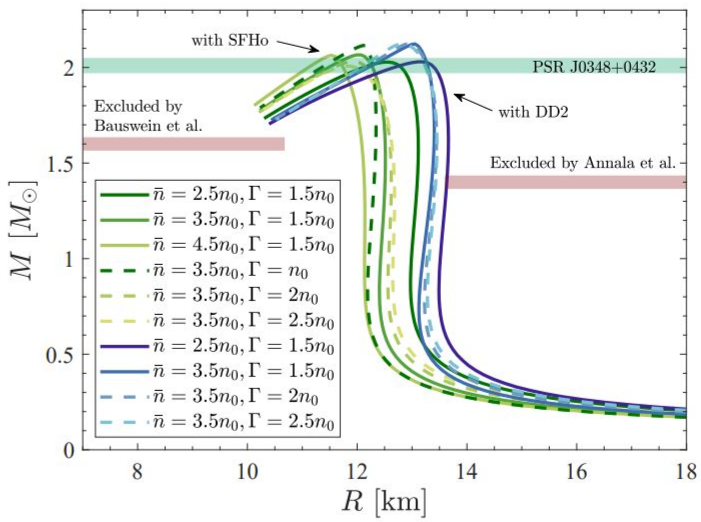


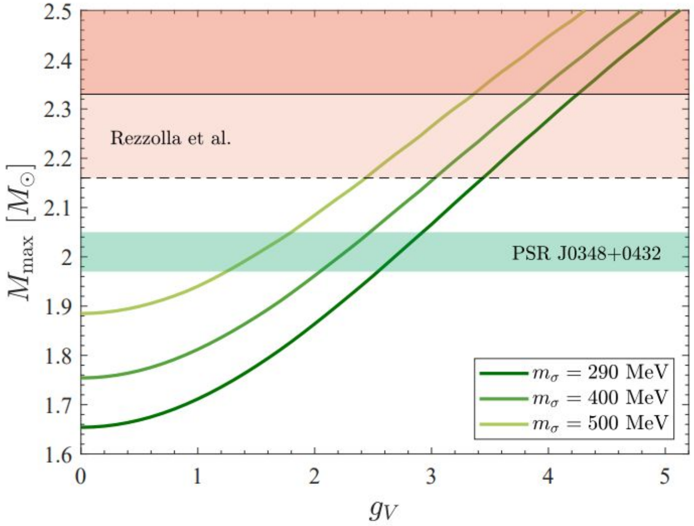
Hybrid equation of state

Hybrid stars also have a hadronic crust and outer core:

- at low densities we use hadronic EoSs:
  - ↪ the SFHo EoS to represent soft hadronic EoSs
  - ↪ the DD2 as a stiff EoS
- we apply a smooth connection between the two phases:
  - ↪ $\varepsilon(n_B)$ interpolation with polynomial
- we have 4 tunable parameters:
  - ↪ 2 from the constituent quark model: m_σ, g_V
  - ↪ 2 describing the concatenation: $\tilde{n}$, Γ

Constraint from maximum mass of neutron stars





- maximum mass mostly depend on quark model parameters
- with m_σ=290 MeV g_V is constrained to 2.5 < g_V < 4.3

Bayesian analysis

Bayes' theorem:

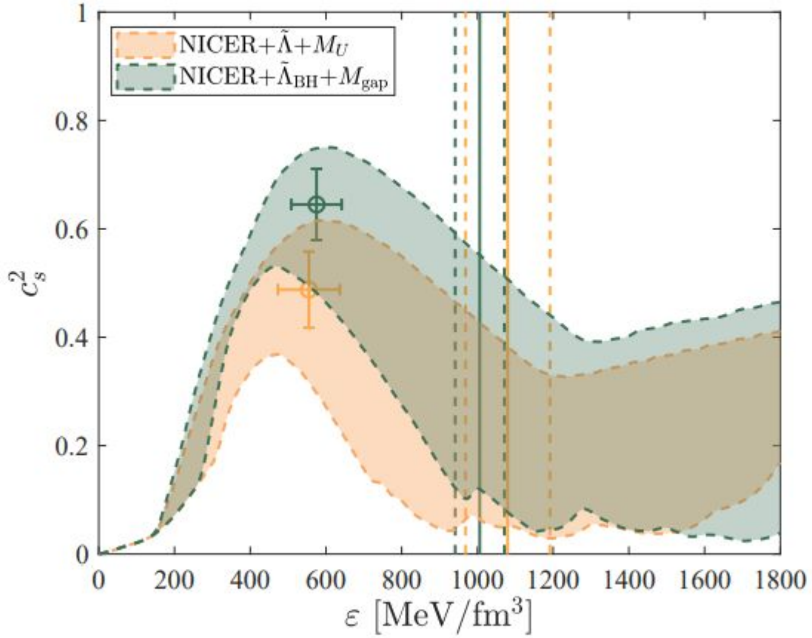
$$p(\vartheta|\mathrm{data}) = \frac{p(\mathrm{data}|\vartheta)p(\vartheta)}{p(\mathrm{data})}$$

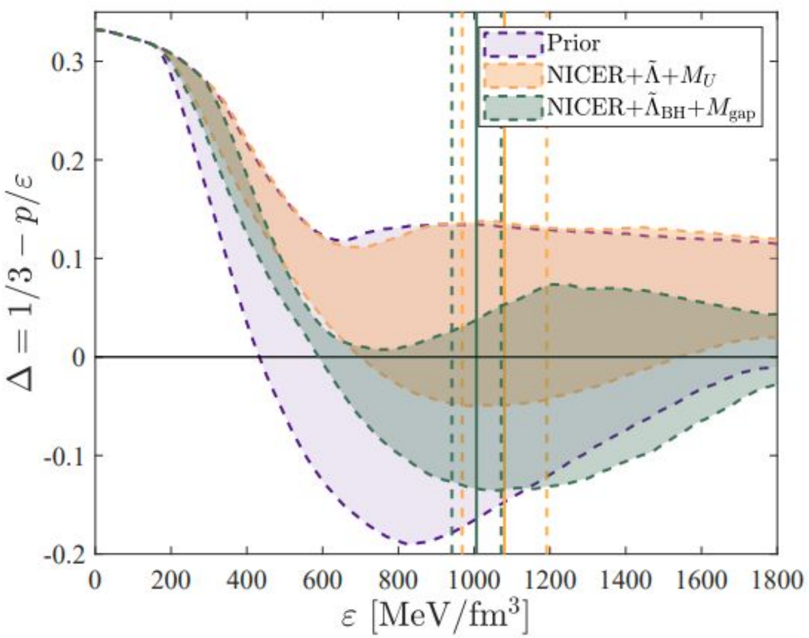
$$p(\mathrm{data}|\vartheta) = p(M_{\mathrm{max}}|\vartheta)p(\mathrm{NICER}|\vartheta)p(\tilde{\Lambda}|\vartheta)$$

- Lower limit on maximum mass from $2M_\odot$ NS observations
- Mass-radius probability densities from observations of PSR J0030+0451 and PSR J0740+6620 with NICER
- Tidal deformability data from LVC for GW170817 + constraint from no prompt collapse to BH
- Upper mass constraint from hypermassive NS hypothesis

Speed of sound peak

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\varepsilon} \qquad \Delta = \frac{1}{3} - \frac{p}{\varepsilon}$$





Thank you for your attention!

Some references

[1] T. Hinderer, The Astrophysical Journal 677, 1216 (2008)

[2] S. Postnikov, M. Prakash, Phys. Rev. D 82, 024016 (2010)

[3] K. Yagi, N. Yunes, Physics Reports 681, 1 (2017)

[4] T. Hinderer, et al., Phys. Rev. D 81, 123016 (2010)

[5] B. P. Abbott, et al., Phys. Rev. Lett. 121, 161101 (2018)

[6] B. P. Abbott, et al., Astrophysical Journal Letters 892, L3 (2020)

Backup slides

Mass measurement

$$f_{ns} = \left(\frac{2\pi}{P_{orb}}\right)^2 \frac{(a_{ns}\sin i)^3}{G} = \frac{(M_c \sin i)^3}{M_T^2}$$

$$q = \frac{M}{M_c} = \frac{(a_c \sin i)}{(a_{ns} \sin i)}$$

<ins>Two other measurements</ins>:

1) Projected semi-major axis of the companion star → mass ratio

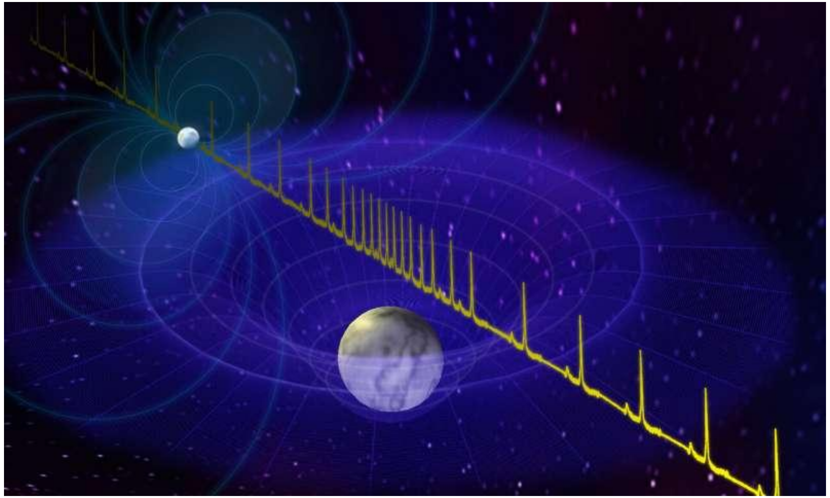
a) White dwarf → optical measurements

b) Another pulsar

2.1) Eclipses → observed edge-on

2.2) Independent mass measurement of the companion: Shapiro-delay

2.3) Relativistic effects: precession, gravitational radiation



Tolmann-Oppenheimer-Volkoff equation

Spherically symmetric metric: $\mathrm{d}s^2 = e^{2\nu(r)}\mathrm{d}t^2 - e^{2\lambda(r)}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$

Thus the non-zero components of the Ricci tensor:

$$R_{00} = \left(-\nu'' + \lambda'\nu' - \nu'^2 - \tfrac{2\nu'}{r}\right)e^{2(\nu-\lambda)} \qquad R_{11} = \nu'' - \lambda'\nu' + \nu'^2 - \frac{2\lambda'}{r}$$

$$R_{22} = (1 + r\nu' - r\lambda')e^{-2\lambda} - 1 \qquad R_{33} = R_{22}\sin^2\theta$$

The Ricci scalar: $R = e^{-2\lambda}\left(-2\nu'' + 2\lambda'\nu' - 2\nu'^2 - \frac{2}{r^2} + 4\frac{\lambda'}{r} - 4\frac{\nu'}{r}\right) + \frac{2}{r^2}$

TOV-equation

Einstein's equations (G = c = 1): $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$

$$T_{\mu\nu} = (p + \varepsilon) u_\mu u_\nu - p g_{\mu\nu}$$

Thus:

$$G_0^{\ 0} = e^{-2\lambda} \left(\frac{1}{r^2} - \frac{2\lambda'}{r} \right) - \frac{1}{r^2} = -8\pi\varepsilon(r) \qquad G_1^{\ 1} = e^{-2\lambda} \left(\frac{1}{r^2} + \frac{2\nu'}{r} \right) - \frac{1}{r^2} = 8\pi p(r)$$

$$G_2^{\ 2} = G_3^{\ 3} = e^{-2\lambda} \left(\nu'' + \nu'^2 - \lambda'\nu' + \frac{\nu' - \lambda'}{r} \right) = 8\pi p(r)$$

From $G_0^{\ 0}$: $\quad e^{-2\lambda} = 1 + \frac{2M(r)}{r} \equiv 1 + \frac{2}{r} \int\limits_0^r 4\pi r^2 \varepsilon(r) \mathrm{d}r$

TOV-equation

$$G_0^{\,0} = e^{-2\lambda}\left(\frac{1}{r^2} - \frac{2\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi\varepsilon(r)$$

$$G_1^{\,1} = e^{-2\lambda}\left(\frac{1}{r^2} + \frac{2\nu'}{r}\right) - \frac{1}{r^2} = 8\pi p(r)$$

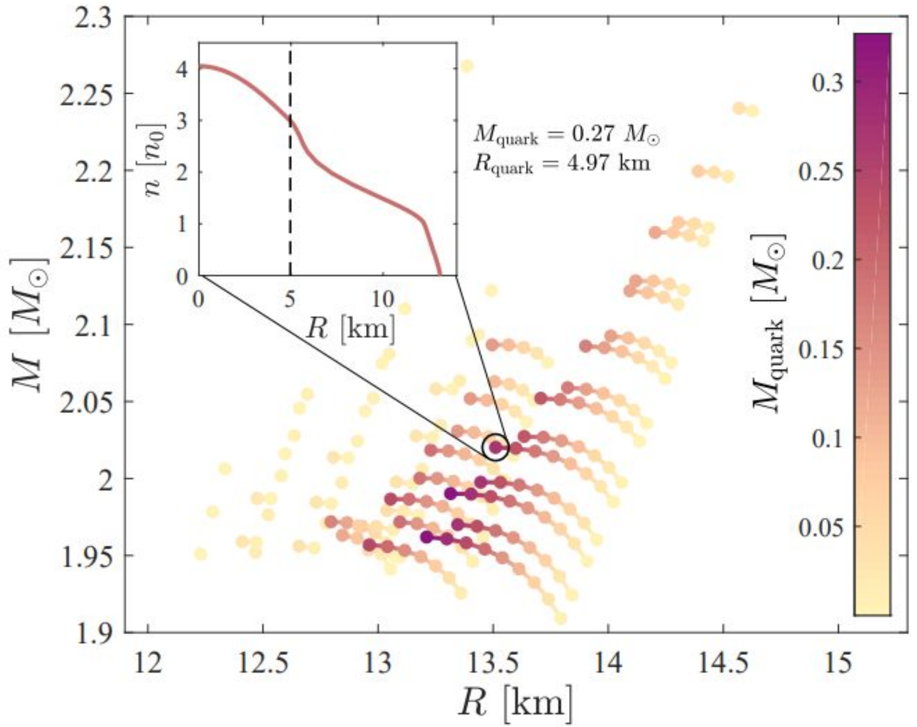
$$G_2^{\,2} = G_3^{\,3} = e^{-2\lambda}\left(\nu'' + \nu'^2 - \lambda'\nu' + \frac{\nu' - \lambda'}{r}\right) = 8\pi p(r)$$

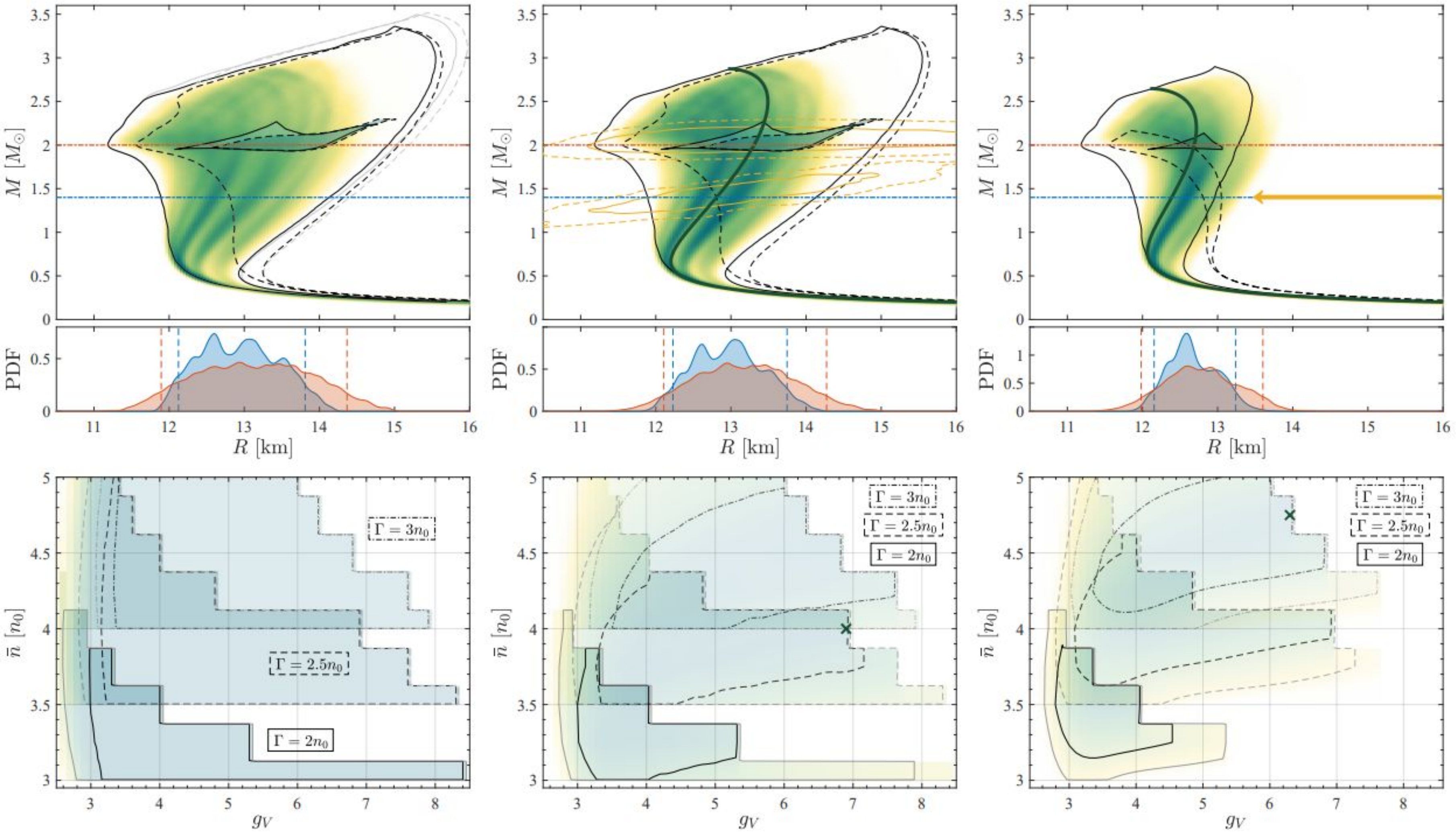
From $G_0^{\,0}$ and $G_1^{\,1}$: $-2r\lambda' = (1 - 8\pi r^2\varepsilon)e^{2\lambda} - 1$ $\quad 2r\nu' = (1 + 8\pi r^2 p)e^{2\lambda} - 1$

Expressing λ', ν' and ν''from these equations and substituting them to the remaining ones:

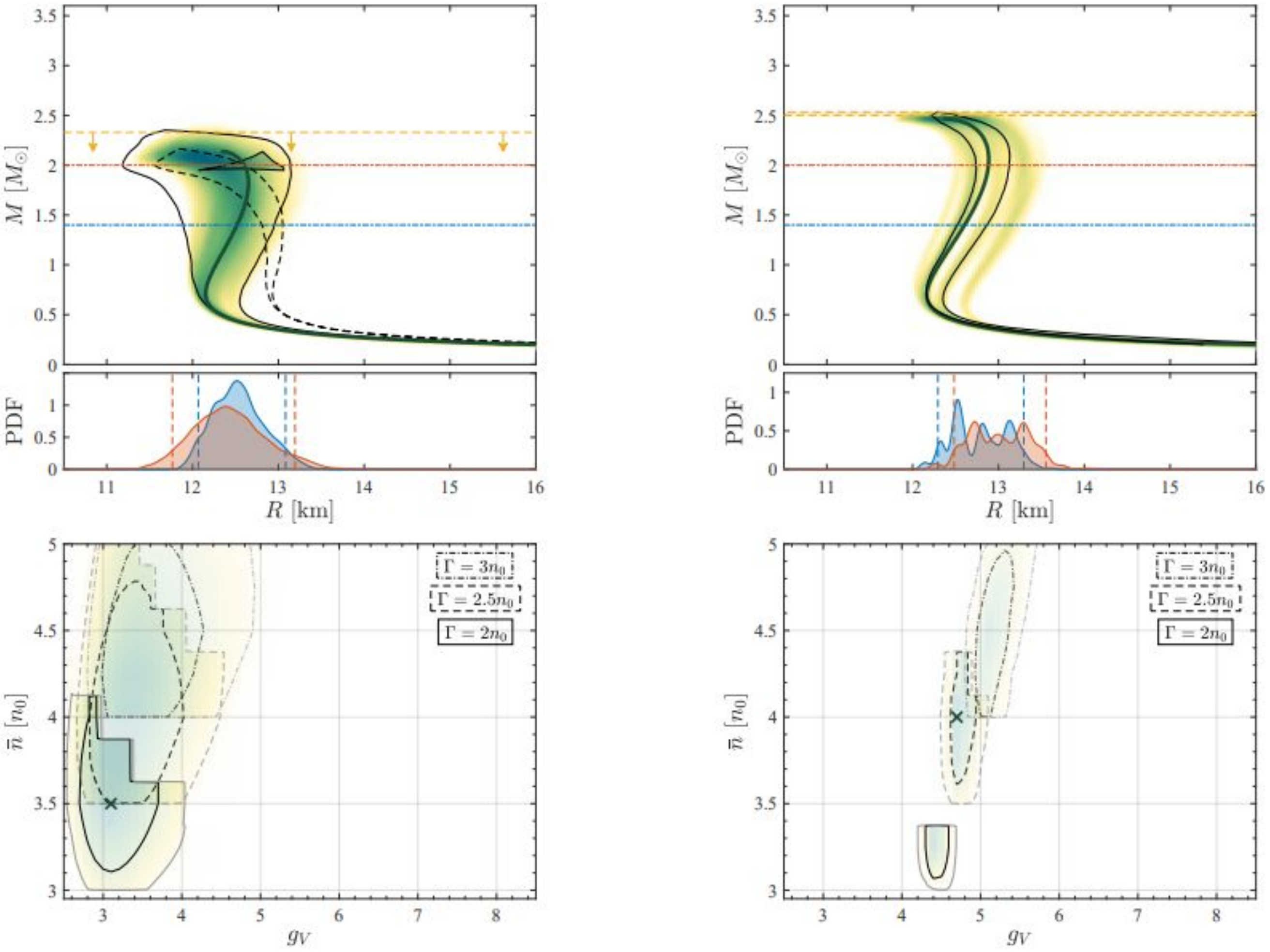
$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r) + p(r)]\frac{M(r) + 4\pi r^3 p(r)}{r^2 - 2M(r)r}}$$

Existence of a pure quark core





M [$M_\odot$]
PDF
R [km]
$\bar{n}$ [n_0]
g_V
$\Gamma = 3n_0$
$\Gamma = 2.5n_0$
$\Gamma = 2n_0$



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Constraints on concatenation parameters