Collapse instability and staccato decay of oscillons in various dimensions

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AZ NKFI ALAPBÓL MEGVALÓSULÓ PROJEKT

In memoriam

Gy. Fodor, P. Forgács, Z. Horváth, and Á. Lukács:
Small amplitude quasi-breathers and oscillons,
Phys. Rev. D78: 025003, 2008.

Gy. Fodor, P. Forgács, Z. Horváth, and M. Mezei:
Computation of the radiation amplitude of oscillons,
Phys. Rev. D79: 065002, 2009.

Gy. Fodor, P. Forgács, Z. Horváth, and M. Mezei:
Radiation of scalar oscillons in 2 and 3 dimensions,
Phys. Lett. B674: 319-324, 2009.

Gy. Fodor, P. Forgács, Z. Horváth, and M. Mezei:
Oscillons in dilaton-scalar theories,
JHEP 2009(8): 106, 2009.

Zalán Horváth
(1943-2011)

Outline of talk

1. Introduction
2. Oscillons and quasi-breathers
3. Decay mechanisms of oscillons
4. Collapse instability in dimension D
5. Staccato decay in dimension D
6. Conclusions and outlook

Introduction

Oscillons: metastable scalar field lumps with extraordinarily long life-time

Nonlinear scalar field theory in $D+1$ dimensions

$$\mathcal{L} = \frac{1}{2}\left(\partial_t \phi\right)^2 - \frac{1}{2}\left(\nabla \phi\right)^2 - V(\phi)$$

Assume: $V(\phi)$ has a minimum in $\phi = 0$ Mass scale: $m^2 = V''(0)$

Energy:

$$\mathcal{E} = \int d^D x \left\{ \frac{1}{2}\left(\partial_t \phi\right)^2 + \frac{1}{2}\left(\nabla \phi\right)^2 + V(\phi) \right\}$$

Equation of motion:

$$\left(\partial_t^2 - \nabla^2\right)\phi = -V'\left(\phi\right)$$

Assuming spherical symmetry:

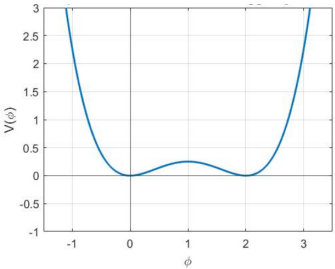
$$\left(\partial_t^2 - \partial_r^2 - \frac{D-1}{r}\partial_r\right)\phi = -V'\left(\phi\right)$$

$$\mathcal{E} = \frac{2\pi^{D/2}}{\Gamma\left(D/2\right)} \int dr r^{D-1} \left\{ \frac{1}{2}\left(\partial_t \phi\right)^2 + \frac{1}{2}\left(\partial_r \phi\right)^2 + V(\phi) \right\}$$

Example: ϕ^4 theory with $D = 3$

Potential:

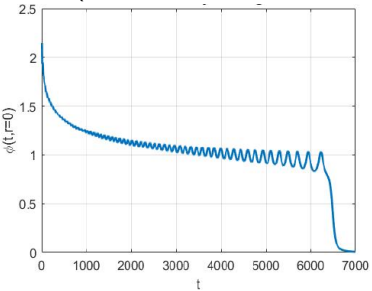
$$V(\phi) = \frac{1}{4}\phi^2 (\phi - 2)^2 \Rightarrow m^2 = 2$$

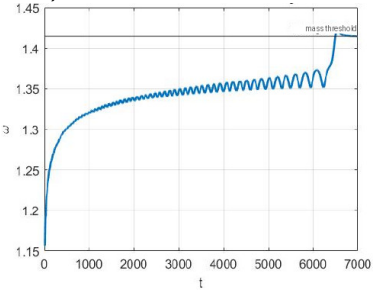


Initial condition:

$$\phi(r, 0) = C \exp\left(-\frac{r^2}{r_0^2}\right) \qquad \dot{\phi}(r, 0) = 0$$

Time evolution (our simulation with $C = 2$, $r_0 = 2.39$)





Numerical time evolution

Gy. Fodor: „A review on radiation of oscillons and oscillatons", arXiv: 1911.03340 [hep-th].

$$\partial_t^2 \phi = \left(\partial_r^2 + \frac{D-1}{r} \partial_r \right) \phi - V'(\phi)$$

$$\Downarrow \qquad r = \frac{2R}{\kappa \left(1 - R^2\right)} \qquad \kappa = 0.05 \quad (\text{ maps } r \in [0, \infty) \text{ to } R \in [0, 1) \,)$$

$$\partial_t^2 \phi = \frac{\kappa^2 \left(1 - R^2\right)^3}{2 \left(1 + R^2\right)} \left[\frac{\left(1 - R^2\right)}{2 \left(1 + R^2\right)} \partial_R^2 \phi - \frac{R \left(3 + R^2\right)}{\left(1 + R^2\right)} \partial_R \phi + \frac{D-1}{2R} \partial_R \phi \right] - V'(\phi)$$

Numerics: method of lines with Courant factor $\Delta t / \Delta R = 1$, with 4th order Runge-Kutta in t.
Interval extended to $R \in [-1, 1]$, $\partial_R \phi|_{R=0} = 0$ enforced by $\phi(R) = \phi(-R)$.
Choice of κ: tunes resolution of oscillon core vs. radiation tail.
Protection against short-wavelength instabilities: add dissipation term

$$\mathcal{D} = K \left(\partial_R^6 \Phi \right) (\Delta R)^5$$

Independence of results on choice of K must be always checked.

Quasi-breathers

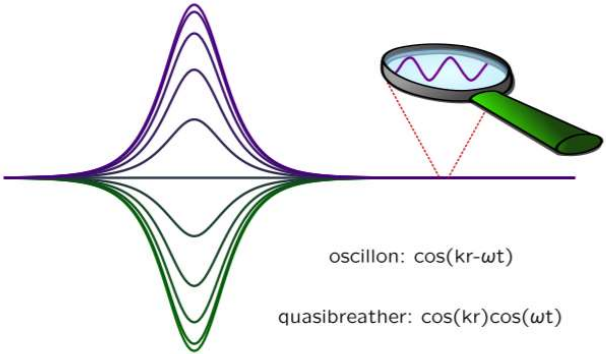
Oscillons slowly radiate and lose energy
Can be compensated by adding a small standing wave tail: **quasi-breathers**

For most of their life, the tail part is exponentially small. Typical radiation rate of small amplitude oscillons:

$$\frac{dE}{dt} \sim \frac{A}{\epsilon^{D-1}} \exp\left(-\frac{B}{\epsilon}\right)$$

where ϵ is the amplitude.

As a result, the oscillon core evolves essentially adiabatically via a succession of quasi-breather core configurations.



The core and the tail part of a quasi-breather.

Figure from Gy. Fodor: *A review on radiation of oscillons and oscillatons*, arXiv: 1911.03340 [hep-th].

Constructing quasi-breathers numerically

Quasi-breather: exactly periodic solution

Ansatz: $$\phi(t,r) = \sum_{m=1}^{N} \phi_m(r)\cos(m\omega t) \qquad N: \text{ mode truncation}$$

$$\left(\partial_t^2 - \partial_r^2 - \frac{D-1}{r}\partial_r\right)\phi = -V'(\phi)$$

$$\Downarrow$$

$$\left(n^2\omega^2 + \partial_r^2 + \frac{D-1}{r}\partial_r\right)\phi = \frac{\omega}{\pi}\int_0^{2\pi\omega} dt V'\left(\sum_{m=1}^{N}\phi_m(r)\cos(m\omega t)\right)\cos(n\omega t)$$

$\rightarrow$ coupled equations for the functions $\phi_n(r)$. Boundary conditions:

$$\partial_r\phi_n(r)|_{r=0} = 0$$

$$\phi_n(r\to\infty) \sim r^{(1-D)/2}\exp\left(-\sqrt{1-n^2\omega^2}\,r\right) \qquad n\omega < 1: \text{ localised modes}$$

$$\phi_n(r\to\infty) \sim r^{(1-D)/2}\sin\left(\sqrt{n^2\omega^2-1}\,r + \varphi_n\right) \qquad n\omega > 1: \text{ radiation modes}$$

Sine-Gordon theory in D=1: exact breathers

Sine-Gordon action

$$S = \int dtdx \left\{ \frac{1}{2} \left(\partial\phi\right)^2 + \mu \left(\cos\beta\phi - 1\right) \right\} \qquad \phi_0^{(n)} = \frac{2\pi}{\beta} n \qquad m = \sqrt{\mu}\beta$$

Breather solution: exactly periodic, no radiation!

$$\phi_B(t,x) = \frac{4}{\beta} \arctan\left(\frac{\epsilon \cos\left(m\omega t\right)}{\omega \cosh(m\epsilon x)} \right)$$

$$M_B(\epsilon) = \frac{16\sqrt{\mu}}{\beta}\epsilon \qquad 0 < \epsilon < 1$$

Frequency-amplitude relation

$$\omega = \sqrt{1-\epsilon^2}$$

For a general D and $V(\phi)$, it is always possible to define the amplitude parameter ϵ such that this relation is exact, and perform a small amplitude expansion of the oscillon and quasi-breather solutions:

Gy. Fodor, P. Forgács, Z. Horváth, and Á. Lukács: *Small amplitude quasi-breathers and oscillons*, Phys. Rev. D78: 025003, 2008.

Mechanism of oscillon longevity

$\omega < 1$: fundamental mode is localised
$\rightarrow$ only higher harmonics radiate,
but: they are suppressed!

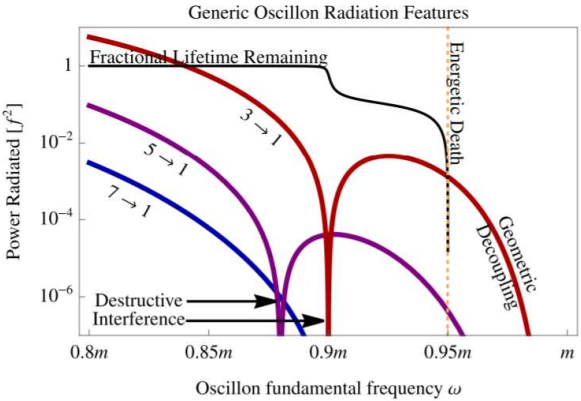
Oscillon with core size
$d\left(\sim \epsilon^{-1}\right) \gg \lambda_{\text{rad}}$: **geometric decoupling**

There are also windows of longevity due to destructive interference between outgoing radiation from higher harmonics.

Note: for even potentials

$$V(\phi) = V(-\phi)$$

only odd harmonics exist!



Axion theory (sine-Gordon in $D = 3$)

Figure from D. Cyncynates and T. Giurgica-Tiron, Phys. Rev. D103: 116011, 2021.

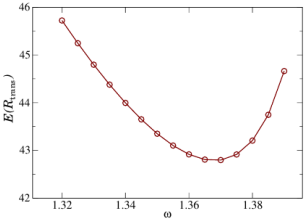
Sine-Gordon in $D = 1$: complete destructive interference of radiation - no decay!

Sudden collapse a.k.a. energetic death

Energy stored in quasi-breather core

$$E = \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^{R_{\text{core}}} dr r^{D-1} \left\{ \frac{1}{2} (\partial_t \phi)^2 + \frac{1}{2} (\partial_r \phi)^2 + V(\phi) \right\}$$

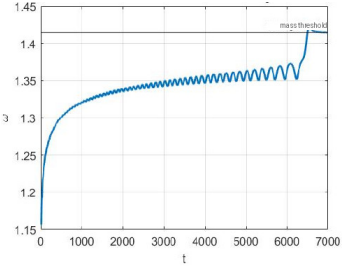
Radius of core R_{core}: dominant localised vs radiation mode amplitudes are equal.

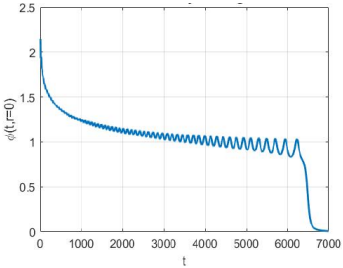


Core energy as a function of ω for $V(\phi) = \frac{1}{4}\phi^2 (\phi - 2)^2$, $D = 3$

Figure from Gy. Fodor: *A review on radiation of oscillons and oscillatons*, arXiv: 1911.03340

Sudden death: $\omega \approx 1.368 \approx 0.967m$





Staccato decay in $D = 1$

P. Dorey, T. Romańczukiewicz and Y. Shnir: *Staccato radiation from the decay of large amplitude oscillons*, Phys. Lett. B806: 135497, 2020.

Model:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_\lambda(\phi)$$

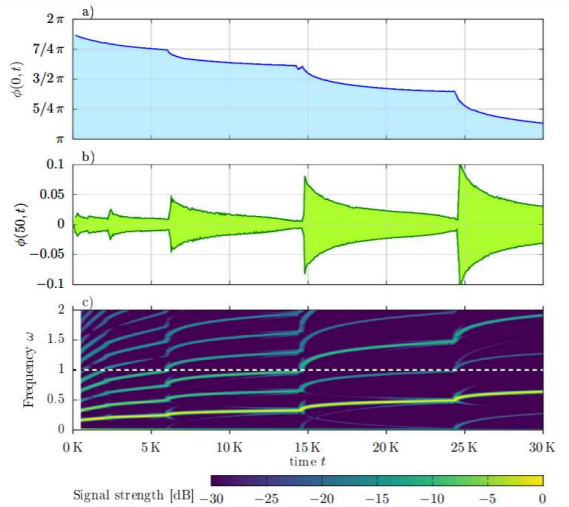
$$V_\lambda(\phi) = (1-\lambda)(1-\cos\phi) + \frac{\lambda}{8\pi^2}\phi^2(\phi - 2\pi)^2$$

$$\phi(0,x) = 4\arctan\left(\frac{\epsilon\cos(\omega t)}{\omega\cosh(\epsilon x)}\right)$$

$$\dot{\phi}(0,x) = 0$$

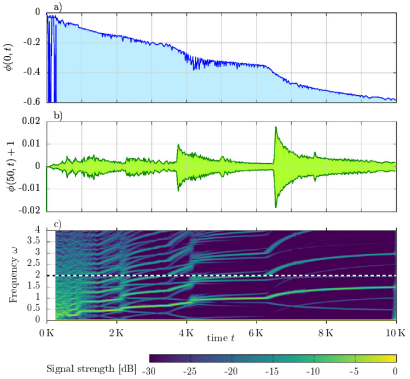
$$\epsilon = \sqrt{1-\omega^2}$$

Set coupling $\lambda = 0.0025$,
start evolution with $\omega = 0.1$

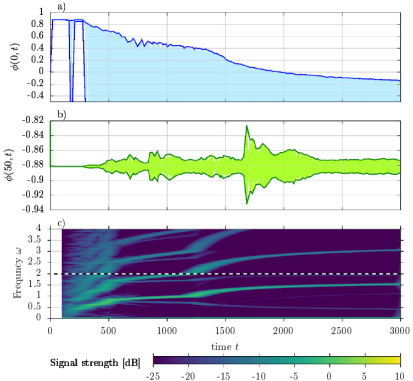


Staccato decay in $D = 1$

Staccato decay proves to be a robust feature in 1+1 D, e.g. in two-kink collisions



ϕ^6 model



hyperbolic ϕ^4 : $V(\phi) = \frac{1}{4}\left(\sinh^2\phi - 1\right)^2$

P. Dorey, T. Romańczukiewicz and Y. Shnir, Phys. Lett. B806: 135497, 2020.

Our research plan

- Construct quasi-breathers with given initial frequency in spatial dimension $D \in \mathbb{R}$

  This is necessary to

  - Control initial frequency
  - Optimize life-time of oscillon, minimize shape modes
- Determine energy-frequency diagram, sudden collapse as a function of D
- Study whether staccato decay persists for $D > 1$

Constructing quasi-breathers

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_\lambda(\phi)$$

$$V_\lambda(\phi) = (1-\lambda)(1-\cos\phi) + \frac{\lambda}{8\pi^2}\phi^2(\phi - 2\pi)^2$$

Ansatz: $$\phi(t,r) = \sum_{n=1}^{N} \phi_n(r)\cos(n\omega t)$$

Initialising solution:

$$D = 1: \quad \phi_n(0) \approx \phi_n^{sG}(0)$$

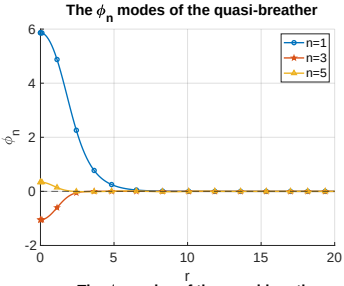
$$= \frac{\omega}{\pi}\int_0^{2\pi/\omega} 4\arctan\left(\frac{\epsilon\cos(\omega t)}{\omega}\right)\cos(n\omega t)$$

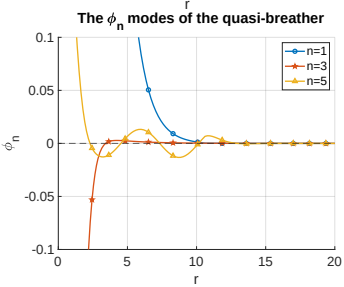
$$D > 1: \quad \phi_n^{D+\delta D}(0) \approx \phi_n^{D}(0)$$

Cutting off the tail of radiation modes:

$$\phi_n(r) \to \phi_n(r)\cdot e^{-\sqrt{n^2\omega^2-1}(r-r_0)} \qquad r_0 = 10/\sqrt{1-\omega^2}$$

B. C. Nagy and G. Takács, Phys. Rev. D 104: 056033, 2021.





$\omega = 0.3,\ D = 1,\ \lambda = 0,\ N = 5$

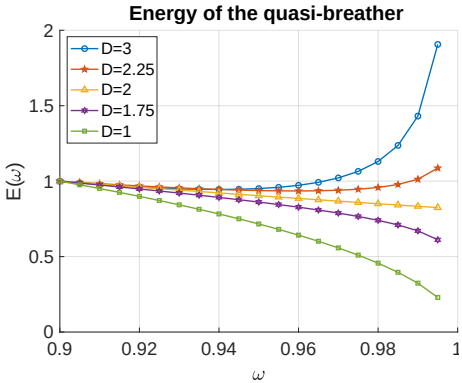
Sudden collapse in D spatial dimensions

B. C. Nagy and G. Takács, Phys. Rev. D 104: 056033, 2021.

$D > 2$: there is a minimum
$\Rightarrow$ critical frequency ω_c

D	2.25	2.50	2.75	3.00
ω_c	0.9591	0.9429	0.9381	0.9395

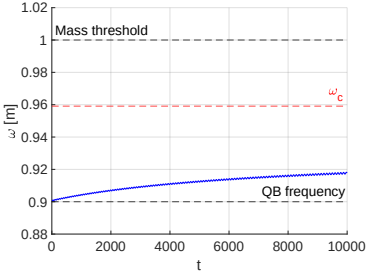
The disappearance of the minimum at $D = 2$ was previously derived in the small amplitude expansion by. Gy. Fodor et al in 2008.



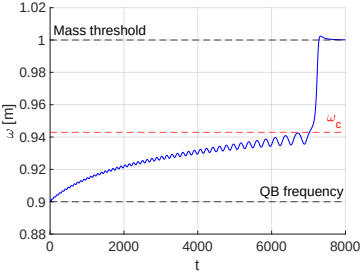
$\lambda = 0$ (sG/axion theory in D dim)
Quasibreather construction: $N = 5$

Sudden collapse: numerical time evolution

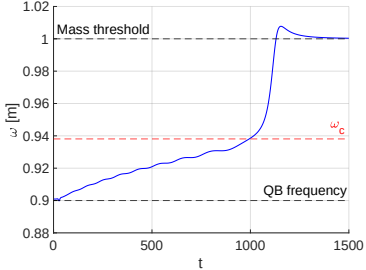
$D = 2.25$



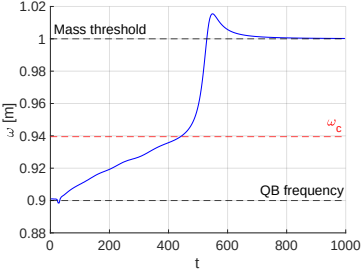
$D = 2.50$



$D = 2.75$

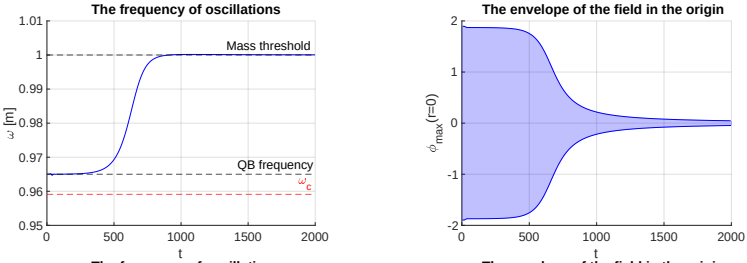


$D = 3.00$

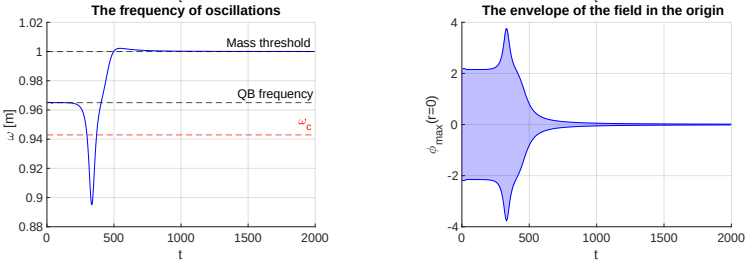


Starting with a supercritical frequency $\omega > \omega_c$

$D = 2.25$



$D = 2.50$

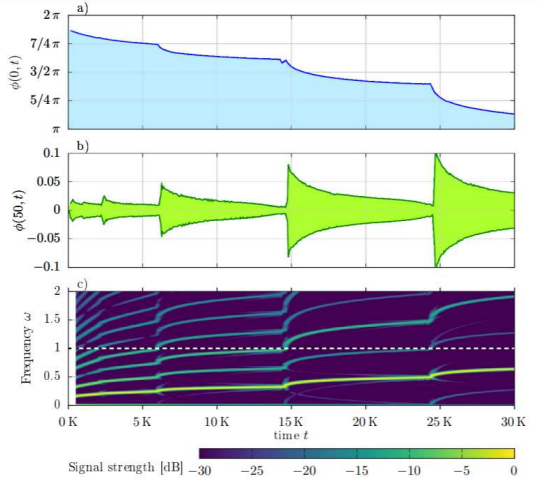


It is possible to switch between these behaviours by fine-tuning initial data

G. Fodor, P. Forgács, P. Grandclément, and I. Rácz, Phys. Rev. D 74 (2006) 124003.

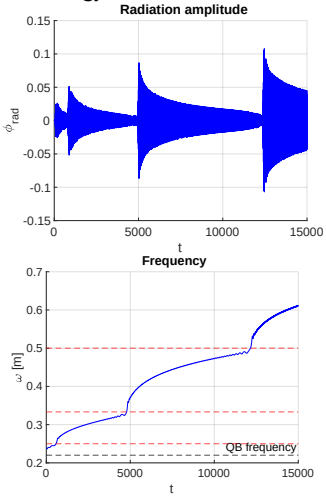
Reproducing staccato decay in $D = 1$ with our methods

P. Dorey, T. Romańczukiewicz and Y. Shnir, 2020.



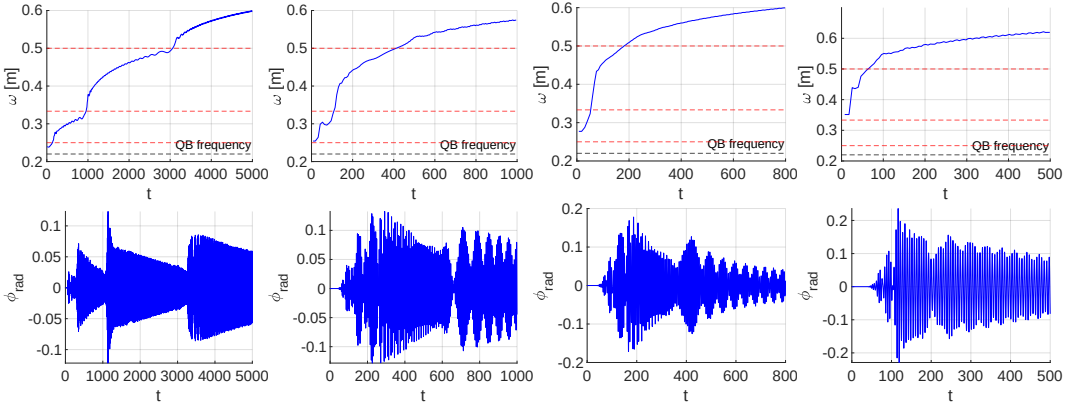
$\lambda = 0.0025$, $\omega_0 = 0.1$

B.C. Nagy and G. Takács , 2021.



$\lambda = 0.0025$, $\omega_0 = 0.22$

Staccato decay in $D > 1$?

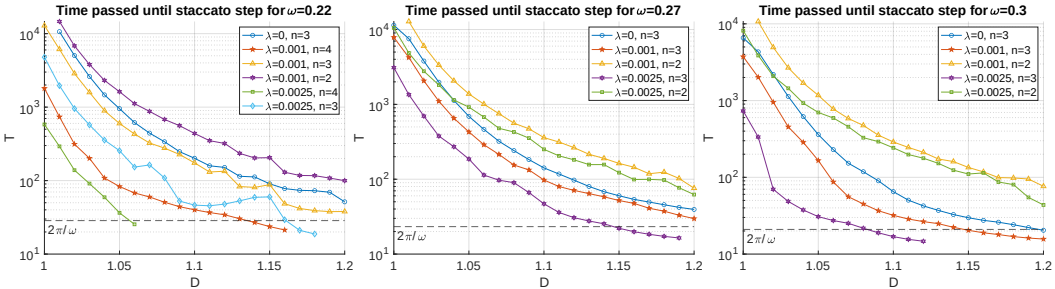


$D = 1.02$ $D = 1.08$ $D = 1.13$ $D = 1.20$

$(\lambda = 0.0025,\ \omega_0 = 0.22)$

- oscillon decay speeds up (note time scale!
- staccato steps become indistinct
  (although first burst at $\omega = m/2$ is still observable at $D = 1.20$)

Time passed until staccato steps in $D > 1$



- The time passed until staccato steps decreases with D
- When it is close to $2\pi/\omega$: staccato steps become unobservable
- Unobservability happens before reaching the next physical dimension $D = 2$, even for $\lambda = 0$ (sine-Gordon/axion theory)

Sadly, it looks likely that staccato decays are confined to $D = 1$.
Exceptional stability islands (destructive interference of radiation!) might help, but: known cases are not suitable - all have $\omega > m/2$.

Conclusions and outlook

1. Oscillons: long lived metastable configuration of interacting scalar field
   a) Small amplitude: analytic results for life time etc.
      Frequency: $\omega = m\sqrt{1-\epsilon^2}$ ϵ: amplitude parameter
   b) Large amplitude: interesting, novel dynamics

2. $D > 2$: critical frequency
   a) $\omega_0 < \omega_c$: slow radiative evolution followed by sudden decay
   b) $\omega_0 > \omega_c$: immediate decay, can be suppressed by fine-tuning

3. $D \leq 2$: no critical frequency, slow radiative evolution
   a) $D = 1$: staccato decay, very robust feature when $\omega_0 < m/2$
   b) $D > 1$: staccation decay disappears well below $D = 2$

4. Ongoing work
   a) Decay rate in quantum theory
      Small amplitudes: analytic results
      M.P. Hertzberg: *Quantum radiation of oscillons*, Phys. Rev. D82: 045022.
      Question: what happens for large amplitudes?
   b) Staccato decay at the quantum level?

Collaboration and references

Work was done in collaboration with Botond Nagy (MSc student)

Results appeared in:

B. C. Nagy: *Oszcillonok dinamikája*, TDK paper, Oct. 2020.

B. C. Nagy and G. Takács: *Collapse instability and staccato decay of oscillons in various dimensions*, Phys. Rev. D 104: 056033, 2021.

Thank you for your attention!