SU(2) Lattice Gauge Theory with a Topological Action

Lorinc Szikszai
in collaboration with Zoltan Varga
Supervisor
Daniel Nogradi

November 08, 2017

Outline

- Gauge Theory
- Lattice Gauge Theory
- Universality
- Topological action
- Our work

Pure SU(N) Gauge Theory

Action density

$$\mathcal{L} = -\frac{1}{2g^2} \mathrm{Tr} F_{\mu\nu} F_{\mu\nu}$$

Field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

A_μ is the gluon filed
g is the bare gauge coupling constant
Quantization with the Euclidean path integral formalism

$$\langle \mathcal{O} \rangle = \frac{\int \mathrm{D} A_\mu \mathcal{O}(A_\mu) e^{-S(A_\mu)}}{\int \mathrm{D} A_\mu e^{-S(A_\mu)}}.$$

Lattice Gauge Theory

Defining the theory on hypercubic space-time lattice

$$V = a^3 \cdot N_\sigma^3 \qquad T = \frac{1}{N_\tau a}$$

Non-perturbative gauge invariant UV regularization

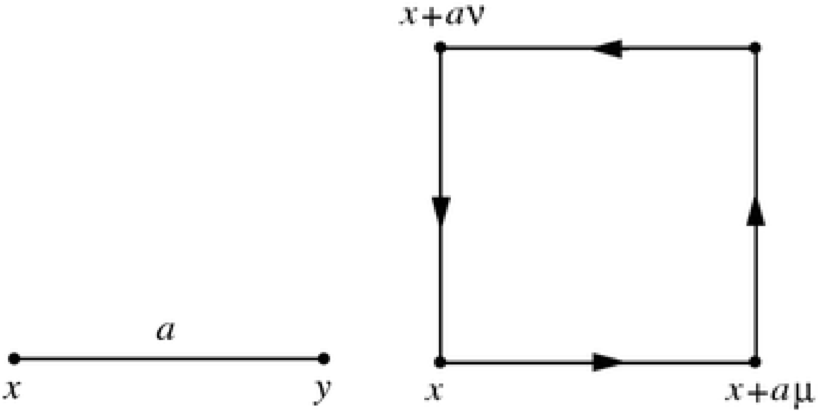
$$A_\mu(x) \to U_\mu(x) = \exp(a g_0 A_\mu(x))$$

$$\mathrm{D}A_\mu \to \mathrm{D}U_\mu = \prod_{x \in R, \mu = 1 \ldots 4} \mathrm{d}U_\mu(x)$$

Both high and low energy observables can be computed

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}U \mathcal{O}(U) e^{-S_{\mathrm{Lattice}}(U)}}{\int \prod \mathrm{d}U e^{-S_{\mathrm{Lattice}}(U)}}.$$

Finite integral, but what is the S_{Lattice}



(a) $U_\mu(x) = \exp(ag_0 A_\mu(x))$ (b) $U_P(x; \mu\nu)$

Figure: Ingredients of lattice gauge theory: link and plaquette

Universality

Locality is vital for the viability of universality
Universality classes are determined by

- the space-time dimension
- the symmetries

all action good, if its continuum

$$S = -\frac{\beta}{4N} \sum_x a^4 \mathrm{Tr} F_{\mu\nu}(x) F_{\mu\nu}(x) + \mathcal{O}(a^5)$$

$$\beta = \frac{2N}{g_0^2}$$

if we demand the reproduction of the classical action in the classical continuum we still have infinite number of choices for the action

Wilson plaquette Action

One good choice the Wilson plaquette action which is in the case of SU(2) gauge theory

$$S = \sum_{x,\mu<\nu} S_{\text{plaquette}}$$

where $S_{\text{plaquette}}$ is the contribution of squares with sides a

$$S_{\text{plaquette}} = \beta(1 - \frac{1}{2}\text{Tr}U_P(x;\mu\nu))$$

$U_P(x;\mu\nu)$ is the ordered product of $U_\mu(x)$ in the plaquette

$$U_P(x;\mu\nu) \equiv U_{(x+a\hat{\nu})(-\nu)}U_{(x+a\hat{\mu}+a\hat{\nu})(-\mu)}U_{(x+a\hat{\mu})\nu}U_{x\mu}$$

lot of other lattice action but all looks in the continuum

$$S = -\frac{\beta}{8}\sum_x a^4\text{Tr}F_{\mu\nu}(x)F_{\mu\nu}(x) + \mathcal{O}(a^5)$$

Topological Lattice Action

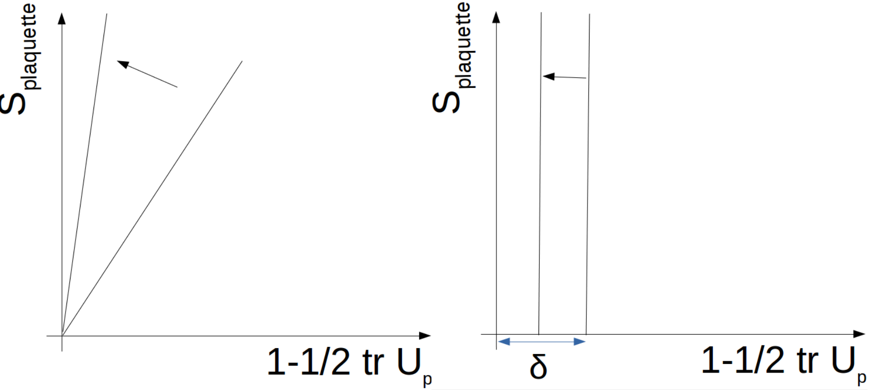
Is the classical continuum limit really important?
for the topological action as before

$$S = \sum_{x,\mu<\nu} S_{\text{plaquette}}$$

But, with

$$S_{\text{plaquette}} = \begin{cases} 0 \text{ if } (1 - \frac{1}{2}\text{Tr}U_P(x;\mu\nu)) < \delta \\ \infty \text{ if } (1 - \frac{1}{2}\text{Tr}U_P(x;\mu\nu)) > \delta \end{cases}$$

The topological action is invariant against small deformations of the fields



(a) Wilson (b) topological

Figure: Continuum limit

Topological Lattice Action

So

$$e^{-S} = \begin{cases} 1 \text{ for all alowed state} \\ 0 \text{ otherwise} \end{cases}$$

All allowed state has the same weight in the path integral
the action does not vary do not have classical continuum limit
Cannot be treated using perturbation theory
Our aim: to show they yield the correct quantum continuum limit which shows the robustness of universality

Former results with topological action

1d $O(2)$,$O(3)$ and 2d $O(2), O(3)$ model

- Bietenholz, Wolfgang, et al. *"Topological lattice actions."* Journal of High Energy Physics 2010.12 (2010): 20.
- Bietenholz, W., et al. *"Topological lattice actions for the 2d XY model."* Journal of High Energy Physics 2013.3 (2013): 141.

They show that this type of action has the correct continuum limit
4d U(1) model

- Akerlund, Oscar, and Philippe de Forcrand. *"Aspects of topological actions on the lattice."* (2015).

No continuum limit in the U(1)

Our aim

is decide whether the topological action works well so we determine dimensionless quantity with the standard Wilson action and the topological action, than compare them in the continuum We measure

- Zero temperature (topological susceptibility)
- High temperature (Deconfinement phase transition)
- Small volume (running coupling)

Topological susceptibility

The topological charge in the continuum

$$Q(x) = \int \mathrm{d}x^4 \frac{1}{32\pi^2} \epsilon_{\mu\nu\varrho\sigma} \mathrm{Tr} F_{\mu\nu} F_{\varrho\sigma}$$

On lattice we determine the $\langle Q^2 \rangle$ with Wilson flow at $t = t_0$
The topological susceptibility

$$\chi = \frac{\langle Q^2(t_0) \rangle}{V}$$

where V is the space-time volume of the lattice $\chi \sim a^{-4}$ so we use the $t_0 \sim a^2$ to make it dimensionless

The t_0 scale

Flow equation

$$\dot{A}_\mu = -\frac{\delta S}{\delta A_\mu} = D_\nu F_{\mu\nu}$$

The gauge filed at $t > 0$ is a renormalized filed
We can define a dimension full quantity (t_0)

$$t^2 \langle E \rangle |_{t_0} = 0.3$$

E is the action density

$$E = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$t^2\langle E \rangle$ is a dimensionless observable
applications of t_0: determines the relative lattice spacings

Dimensionless quantity

We can measure dimensionless quantity with our simulations
So we get from out simulation

$$\chi \cdot a^4$$

and

$$\frac{t_0}{a^2}$$

Our dimensionless observable is

$$\chi \cdot t_0^2$$

we can extrapolate to the continuum with

$$\frac{a^2}{t_0}$$

The continuum extrapolation of χt_0^2

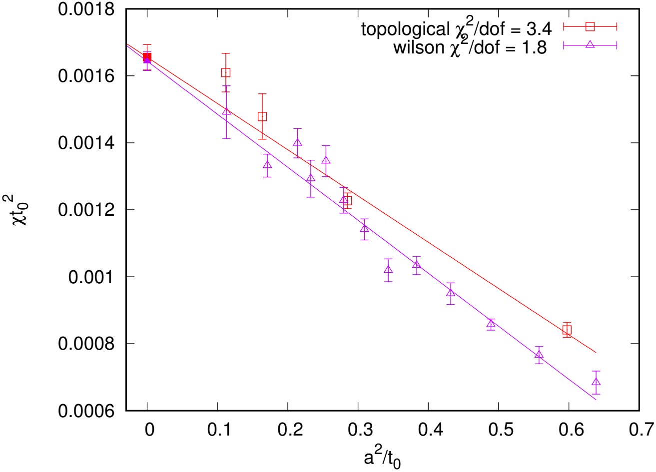


Figure: χt_0^2 continuum

Deconfinement phase transition

Second order phase transition
Same University class as 3d Ising model
We investigate the Binder Cumulant

$$g_r = \frac{\langle L^4 \rangle}{\langle L^2 \rangle^2} - 3$$

Where L is the order parameter, so called Polyakov loop
g_r is an universal quantity at the critical coopling

$$g_r = -1.41 \qquad \text{in the 3d Ising model}$$

The reduced temperature can be approximated by

$$x = \frac{\beta - \beta_c}{\beta_c}$$

The ν critical exponent

The Binder Cumulant in the vicinity of the critical temperature

$$g_r(N_\sigma, x) = Q(xN_\sigma^{1/\nu}).$$

We expand the Q function

$$Q(xN_\sigma^{1/\nu}) = \underbrace{Q(0)}_{g_r(x=0)} + Q'(0) \cdot xN_\sigma^{1/\nu} + O(x^2).$$

$g_r(x = 0)$ is an universal quantity
The slop $S(b \cdot N_\sigma)$ of the $g_r(\beta/\delta)$ will be

$$S(b \cdot N_\sigma) = S(N_\sigma) b^{1/\nu}$$

This way we can estimate the ν exponent

$$1/\nu = \frac{\log\left(\frac{S(b \cdot N_\sigma)}{S(N_\sigma)}\right)}{\log(b)}.$$

Do not need for the critical couplings or the explicit form of the scaling function.

N_τ	b	N_σ	$1/\nu_{\text{topological}}$	$1/\nu_{\text{Wilson}}$
4	5/3	12	1.42(12)	1.54(7)
6	5/3	18	1.46(16)	1.67(24)
8	5/3	24	1.65(25)	1.46(23)
10	4/3	30	1.43(25)	1.74(26)

Table: The ν exponent value for the topological and Wilson action(in the case of the 3d Ising model $1/\nu = 1.59$). Here we use our data to estimate the $1/\nu_{Wilson}$.

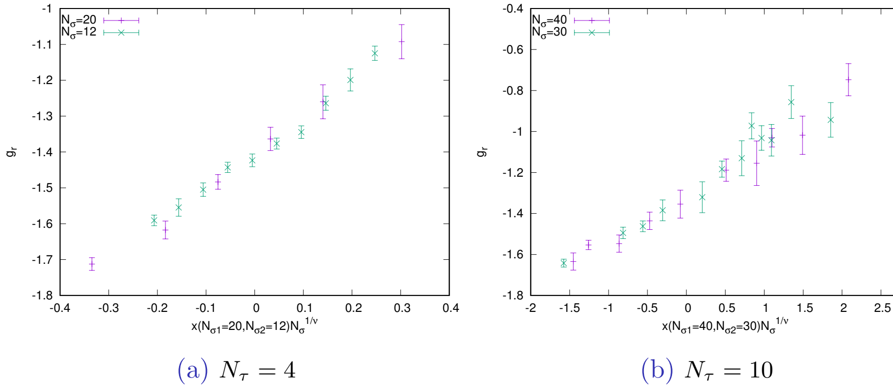


Figure: The scaling of the Binder Cumulant with the topological action.

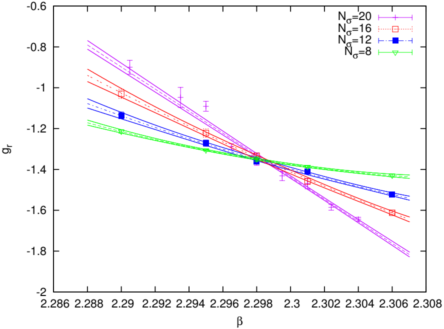
Critical couplings

The finite-volume critical couplings ($\beta_c(N_\sigma, b)$) is determed from the intersection points of the Binder cumulant
N_σ is the spatial size of the smaller lattice
b is the ratio $\frac{N'_\sigma}{N_\sigma}$
To extrapolate infinite volume we use

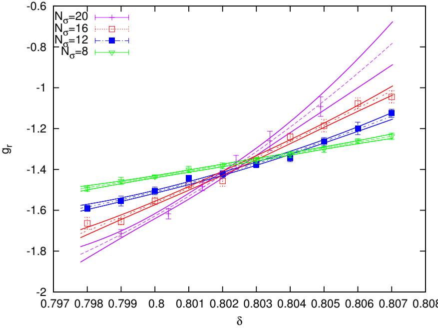
$$\beta_c(N_\sigma, b) = \beta_{c,\infty}(1 - a\epsilon) \tag{1}$$

where

$$\epsilon = N_\sigma^{y_1 - 1/\nu} \frac{1 - b^{y_1}}{b^{1/\nu} - 1} \qquad y_1 = -1 \tag{2}$$



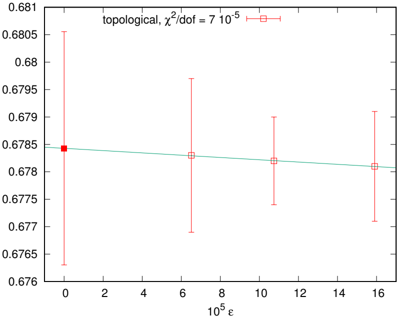
(a) $N_\tau = 4$, Wilson



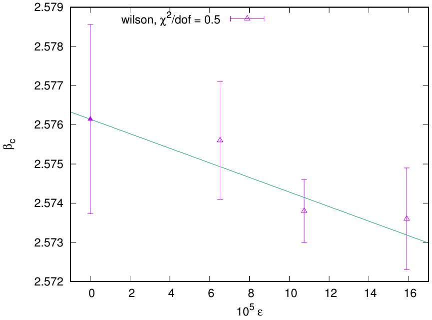
(b) $N_\tau = 4$, topological

Figure: The intersection of the Binder cumulant. We investigate $N_\tau = 4, 6, 8$, with $N_\sigma = 2, 3, 4, 5 \cdot N_\tau$ and $N_\tau = 10$, with $N_\sigma = 2, 3, 4 \cdot N_\tau$

.



(a) $N_\tau = 4$, topological



(b) $N_\tau = 4$, Wilson

Figure: Extrapolation of the critical couplings to infinite volume in the case of $N_\tau = 10$.

Extrapolation to the continuum

We know

$$N_\tau = \frac{1}{T_c a(\beta_c)}$$

We need dimensionless quantity, so we measure

$$\frac{t_0}{a(\beta_c)^2}$$

N_τ	β_c	t_0/a^2	δ_c	t_0/a^2
4	2.2986(6)	1.5723(51)	0.8013(5)	1.6789(166)
6	2.4265(30)	3.368(65)	0.7408(5)	3.5356(261)
8	2.5115(40)	5.84(15)	0.7038(10)	6.0562(994)
10	2.5761(24)	8.86(13)	0.6784(21)	9.1619(3129)

Table: Values of t_0/a^2 at the critical couplings.

Our dimensionless quantity is

$$\left(8\frac{t_0}{a^2}\right)^{0.5} N\tau = \sqrt{8t_0}T_c$$

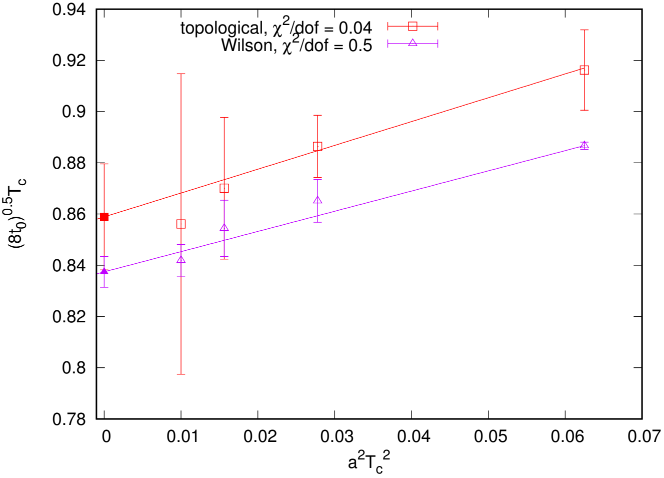


Figure: Continuum extrapolation of the dimensionless ratio $\sqrt{8t_0}T_c$

Summary

Our results agree with the two different action We investigate low and high temperature observables.

- in the continuum
- with finite lattice spacing

In progress: we will check another quantity which connected to the running coupling

Foresight

Fermion lattice action
Nielsen-Ninomiya No-Go Theorem
If we require

- Locality
- one fermion flavor
- correct classical continuum limit

Then we do not have chiral symmetry
but the correct classical continuum limit is really important?