Continuum extrapolated high order baryon fluctuations

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in collaboration with

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Outline

1. **Introduction**
   $\sim$ LQCD, conserved charges, fluctuations, sign problem

2. **Methods**
   $\sim$ Taylor method, computing fluctuations, HRG, volume effects

3. **Results**
   $\sim$ continuum estimates, comparisons with literature, CEP

QCD in grand canonical ensemble (GCE)

Partition function of QCD ($N_f = 2+1$):

$$\mathcal{Z}\big(V, T, \{\mu_B, \mu_Q, \mu_S\}\big) = \mathrm{Tr}\big[e^{-(H-\mu_B B-\mu_Q Q-\mu_S S)/T}\big]$$
$$= \mathrm{Tr}\big[e^{-(H-\mu_u N_u-\mu_d N_d-\mu_s N_s)/T}\big]$$

with

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \,, \qquad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \,, \qquad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

Conserved charges of QCD:

- baryon number B
- electric charge Q
- strangeness S

Conserved exactly in the whole system,
can **fluctuate** in subsystems!

Fluctuations of conserved charges of QCD

Observables $\sim$ derivatives of pressure

$$p = \frac{T}{V} \log \mathcal{Z}$$

Generalized susceptibilities ($\hat{p} = p/T^4$, $\hat{\mu} = \mu/T$)

$$\chi_{ij}^{BS} = \frac{\partial^i}{\partial \hat{\mu}_B^i} \frac{\partial^j}{\partial \hat{\mu}_S^j} \, \hat{p}(V, T, \{\mu_B, \mu_S\}) \quad \propto \quad \text{cumulants of } B, S$$

Examples:

$$\langle B \rangle \propto \chi_1^B \qquad \langle B^2 \rangle - \langle B \rangle^2 \propto \chi_2^B \qquad \langle BS \rangle - \langle B \rangle \langle S \rangle \propto \chi_{11}^{BS}$$

Importance of fluctuations

1. EoS of hot-and-dense QGP [hep-lat:2208.05398]:

$$\hat{p}(\mu_B, \dots) = \hat{p}(0) + \frac{1}{2!}\chi_2^B(T)\hat{\mu}_B^2 + \frac{1}{4!}\chi_4^B(T)\hat{\mu}_B^4 + \frac{1}{6!}\chi_6^B(T)\hat{\mu}_B^6 + \dots$$

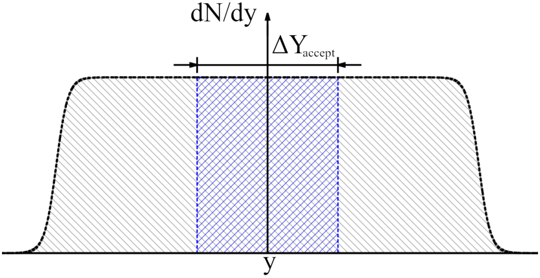
2. CEP searches [nucl-th:2008.04022]

3. sensitivity to effective DoFs [hep-lat:1702.01113]

4. direct comparison of lattice QCD and experimental data

GCE in experiments?

cuts in pseudorapidity $\sim$ sub-volume

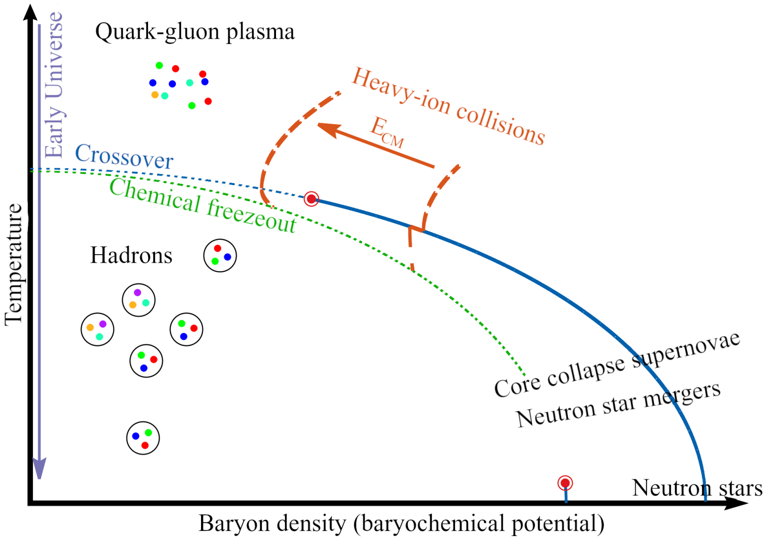


Caveats:

- cuts in $p_{\text{transverse}}$
- proxy $\langle \Delta N_p \rangle \neq \langle B \rangle$
- fluctuating volume
- question of thermalization
- final state interactions

[hep-ph:1203.4529],
[nucl-th:2007.02463]

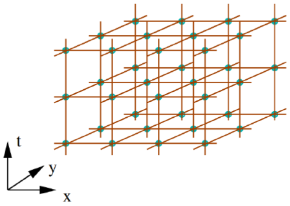
QCD phase diagram



Lattice

- Finite spacetime lattice: $N_x^3 \times N_t$
- Euclidean path integral: $t = -i\tau$

$$\exp(-iHt) = \exp(-H\tau) \implies T = \frac{1}{N_t a}$$



- Continuum limit: for fixed temperature $a \to 0$ and $N_t \to \infty$
- Thermodynamic limit: $V \to \infty \quad \sim \quad$ aspect ratio $LT = N_s/N_t \to \infty$

The sign problem in a nutshell

QCD partition function as a path integral:

$$\mathcal{Z} = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \; e^{-S_{\mathrm{YM}}[U] - \bar{\psi} M[U,m,\mu] \psi} = \int \mathcal{D}U \; \det M[U,m,\mu] e^{-S_{\mathrm{YM}}[U]}$$

Simulations work with **real** and **positive** weights $\det M e^{-S_{\mathrm{YM}}}$

- zero chemical potential: $\mu = 0$
- imaginary chemical potential: $\mu^2 < 0$
- isospin chemical potential: $\mu_u = -\mu_d$

Otherwise: **complex action problem** / **sign problem**

Circumventing the sign problem

1. **Imaginary chemical potential**

$$\text{simulations at } \mu^2 \leq 0 \quad \rightarrow \quad \text{extrapolation to } \mu^2 > 0$$

2. **Taylor method**

$$\text{calculate} \quad \frac{\partial^n}{\partial \mu^n} \langle \mathcal{O} \rangle \quad \text{at } \mu = 0 \quad \rightarrow \quad \text{extrapolation to } \mu > 0$$

3. **Reweighting**

$$\text{simulate a different theory} \quad \rightarrow \quad \text{reweight back to the original theory}$$

Taylor method

Taylor expansion of the pressure

$$\hat{p}(V,T,\{\hat{\mu}_B,\hat{\mu}_S\}) = \sum_{i,j} \frac{\hat{\mu}_B^i \hat{\mu}_S^j}{i!j!} \left[\frac{\partial^i}{\partial \hat{\mu}_B^i} \frac{\partial^j}{\partial \hat{\mu}_S^j} \hat{p}(V,T,\{\hat{\mu}_B,\hat{\mu}_S\}) \right]_{\hat{\mu}_B=\hat{\mu}_S=0}$$

expansion coefficients: $\boxed{\chi_{ij}^{BS}(T) \text{ at } \mu_B = \mu_S = 0}$

$\sim$ possible CEP search:

go to high enough (i,j) and look for signs of a divergence...

Taylor method: study of pion condensation

Instead of $\mu_B \quad \rightarrow \quad$ isospin chemical potential μ_I [hep-lat:2308.06105]

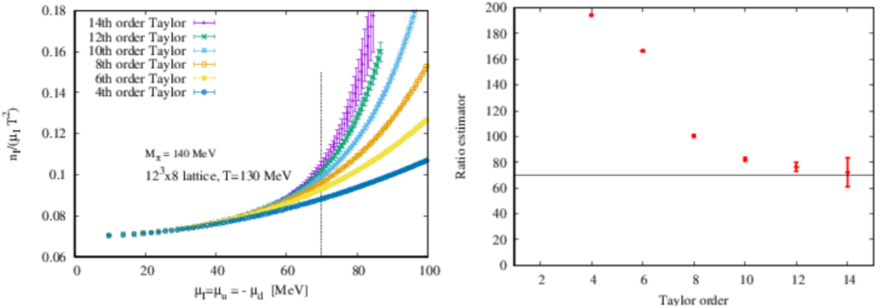


Figure: QM2023 Plenary by A. Pásztor

2nd order transition at low T and $\mu_I \approx m_\pi/2 \approx 70$ MeV [hep-ph:0005225]

How to calculate fluctuations on the lattice

$$\mathcal{Z} = \int \mathcal{D}U \det M_u[U,\mu_u] \det M_d[U,\mu_d] \det M_s[U,\mu_s] e^{-S_{\rm YM}[U]}$$

1. Measuring

$$A_n^{(q)} = \frac{\partial^n}{\partial \hat{\mu}_q^n} \log\det M_q \ , \qquad \text{e.g.} \quad A_1^{(u)} = \mathrm{Tr}\left(\frac{\partial M_u}{\partial \hat{\mu}_u} M_u^{-1} \right)$$

2. Combining and averaging $A_n^{(q)}$-s: $\boxed{A_n^{(q)} \quad \rightarrow \quad \chi_{abc}^{uds}}$,

$$\text{e.g.} \quad \chi_2^u = \frac{1}{VT^3} \frac{\partial^2 \log \mathcal{Z}}{\partial \hat{\mu}_u^2} = \left\langle (A_1^{(u)})^2 + A_2^{(u)} \right\rangle$$

$$\chi_{11}^{ud} = \frac{1}{VT^3} \frac{\partial^2 \log \mathcal{Z}}{\partial \hat{\mu}_u \partial \hat{\mu}_d} = \left\langle (A_1^{(u)})^2 \right\rangle$$

With computer algebra

$\chi_8^u =$

$$
\begin{aligned}
&-630\langle A_1^2\rangle^4 - 2520\langle A_1^2\rangle^3\langle A_2\rangle - 3780\langle A_1^2\rangle^2\langle A_2\rangle^2 - 2520\langle A_1^2\rangle\langle A_2\rangle^3 - 630\langle A_2\rangle^4 + 420\langle A_1^2\rangle^2\langle A_1^4\rangle \\
&+ 840\langle A_1^2\rangle\langle A_1^4\rangle\langle A_2\rangle + 420\langle A_1^4\rangle\langle A_2\rangle^2 + 2520\langle A_1^2\rangle^2\langle A_1^2A_2\rangle + 5040\langle A_1^2\rangle\langle A_1^2A_2\rangle\langle A_2\rangle + 2520\langle A_1^2A_2\rangle\langle A_2\rangle^2 \\
&+ 1680\langle A_1A_3\rangle\langle A_1^2\rangle^2 + 3360\langle A_1A_3\rangle\langle A_1^2\rangle\langle A_2\rangle + 1680\langle A_1A_3\rangle\langle A_2\rangle^2 + 1260\langle A_1^2\rangle^2\langle A_2^2\rangle + 2520\langle A_1^2\rangle\langle A_2\rangle\langle A_2^2\rangle \\
&+ 1260\langle A_2\rangle^2\langle A_2^2\rangle + 420\langle A_1^2\rangle^2\langle A_4\rangle + 840\langle A_1^2\rangle\langle A_2\rangle\langle A_4\rangle + 420\langle A_2\rangle^2\langle A_4\rangle - 28\langle A_1^2\rangle\langle A_1^6\rangle - 28\langle A_1^6\rangle\langle A_2\rangle \\
&- 420\langle A_1^2\rangle\langle A_1^4A_2\rangle - 420\langle A_1^4A_2\rangle\langle A_2\rangle - 560\langle A_1^2\rangle\langle A_1^3A_3\rangle - 560\langle A_1^3A_3\rangle\langle A_2\rangle - 1260\langle A_1^2\rangle\langle A_1^2A_2^2\rangle \\
&- 1260\langle A_1^2A_2^2\rangle\langle A_2\rangle - 35\langle A_1^4\rangle^2 - 420\langle A_1^2A_2\rangle\langle A_1^4\rangle - 280\langle A_1A_3\rangle\langle A_1^4\rangle - 210\langle A_1^4\rangle\langle A_2^2\rangle - 70\langle A_1^4\rangle\langle A_4\rangle \\
&- 420\langle A_1^2\rangle\langle A_1^2A_4\rangle - 420\langle A_1^2A_4\rangle\langle A_2\rangle - 1680\langle A_1A_2A_3\rangle\langle A_1^2\rangle - 1680\langle A_1A_2A_3\rangle\langle A_2\rangle - 420\langle A_1^2\rangle\langle A_2^3\rangle \\
&- 420\langle A_2\rangle\langle A_2^3\rangle - 1260\langle A_1^2A_2\rangle^2 - 1680\langle A_1A_3\rangle\langle A_1^2A_2\rangle - 1260\langle A_1^2A_2\rangle\langle A_2^2\rangle - 420\langle A_1^2A_2\rangle\langle A_4\rangle \\
&- 168\langle A_1A_5\rangle\langle A_1^2\rangle - 168\langle A_1A_5\rangle\langle A_2\rangle - 420\langle A_1^2\rangle\langle A_2A_4\rangle - 420\langle A_2\rangle\langle A_2A_4\rangle - 280\langle A_1^2\rangle\langle A_3^2\rangle - 280\langle A_2\rangle\langle A_3^2\rangle \\
&- 560\langle A_1A_3\rangle^2 - 840\langle A_1A_3\rangle\langle A_2^2\rangle - 280\langle A_1A_3\rangle\langle A_4\rangle - 315\langle A_2^2\rangle^2 - 210\langle A_2^2\rangle\langle A_4\rangle - 28\langle A_1^2\rangle\langle A_6\rangle \\
&- 28\langle A_2\rangle\langle A_6\rangle - 35\langle A_4\rangle^2 + \langle A_1^8\rangle + 28\langle A_1^6A_2\rangle + 56\langle A_1^5A_3\rangle + 210\langle A_1^4A_2^2\rangle + 70\langle A_1^4A_4\rangle \\
&+ 560\langle A_1^3A_2A_3\rangle + 420\langle A_1^2A_2^3\rangle + 56\langle A_1^3A_5\rangle + 420\langle A_1^2A_2A_4\rangle + 280\langle A_1^2A_3^2\rangle + 840\langle A_1A_2^2A_3\rangle \\
&+ 105\langle A_2^4\rangle + 28\langle A_1^2A_6\rangle + 168\langle A_1A_2A_5\rangle + 280\langle A_1A_3A_4\rangle + 210\langle A_2^2A_4\rangle + 280\langle A_2A_3^2\rangle + 8\langle A_1A_7\rangle \\
&+ 28\langle A_2A_6\rangle + 56\langle A_3A_5\rangle + 35\langle A_4^2\rangle + \langle A_8\rangle
\end{aligned}
$$

Change of basis

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q\ , \qquad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q\ , \qquad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

$$\begin{aligned}
\chi_2^B = \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_B^2} &= \frac{\partial}{\partial \hat{\mu}_B}\left[\frac{\partial \hat{p}}{\partial \hat{\mu}_u}\frac{\partial \hat{\mu}_u}{\partial \hat{\mu}_B} + \frac{\partial \hat{p}}{\partial \hat{\mu}_d}\frac{\partial \hat{\mu}_d}{\partial \hat{\mu}_B} + \frac{\partial \hat{p}}{\partial \hat{\mu}_s}\frac{\partial \hat{\mu}_s}{\partial \hat{\mu}_B}\right] \\
&= \frac{1}{3}\Bigg[\frac{\partial^2 \hat{p}}{\partial \hat{\mu}_u^2}\frac{\partial \hat{\mu}_u}{\partial \hat{\mu}_B} + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_u \partial \hat{\mu}_d}\frac{\partial \hat{\mu}_d}{\partial \hat{\mu}_B} + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_u \partial \hat{\mu}_s}\frac{\partial \hat{\mu}_s}{\partial \hat{\mu}_B} \\
&\quad + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_u \partial \hat{\mu}_d}\frac{\partial \hat{\mu}_u}{\partial \hat{\mu}_B} + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_d^2}\frac{\partial \hat{\mu}_d}{\partial \hat{\mu}_B} + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_d \partial \hat{\mu}_s}\frac{\partial \hat{\mu}_s}{\partial \hat{\mu}_B} \\
&\quad + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_u \partial \hat{\mu}_s}\frac{\partial \hat{\mu}_u}{\partial \hat{\mu}_B} + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_d \partial \hat{\mu}_s}\frac{\partial \hat{\mu}_d}{\partial \hat{\mu}_B} + \frac{\partial^2 \hat{p}}{\partial \hat{\mu}_s^2}\frac{\partial \hat{\mu}_s}{\partial \hat{\mu}_B}\Bigg] \\
&= \boxed{\frac{1}{9}\left(2\chi_2^u + \chi_2^s + 4\chi_{11}^{us} + 2\chi_{11}^{ud}\right)}
\end{aligned}$$

With computer algebra

$$\chi_8^B = \frac{1}{6561}\Big[2\chi_8^{u} + 16\chi_{71}^{ud} + 16\chi_{71}^{us} + 56\chi_{62}^{ud} + 112\chi_{611}^{uds} + 56\chi_{62}^{us} + 112\chi_{53}^{ud}$$

$$+ 336\chi_{521}^{uds} + 336\chi_{512}^{uds} + 112\chi_{53}^{us} + 70\chi_{44}^{ud} + 560\chi_{431}^{uds} + 840\chi_{422}^{uds}$$

$$+ 560\chi_{413}^{uds} + 140\chi_{44}^{us} + 560\chi_{332}^{uds} + 1120\chi_{323}^{uds} + 560\chi_{314}^{uds} + 112\chi_{35}^{us}$$

$$+ 420\chi_{224}^{uds} + 336\chi_{215}^{uds} + 56\chi_{26}^{us} + 56\chi_{116}^{uds} + 16\chi_{17}^{us} + \chi_8^{s}\Big]$$

Strangeness neutrality

So far: $\boxed{\mu_S = 0}$

$$\Downarrow$$

$$\chi_1^S(T,\mu_B,\mu_S) \;\propto\; \langle S(T,\mu_B,\mu_S)\rangle = 0 \quad \sim \quad \text{phenomenological relevance}$$

tuning of

$$\mu_S \equiv \mu_S^*(T,\mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + s_7(T)\mu_B^7 + \mathcal{O}(\mu_B^9)$$

[hep-lat:1701.04325]

$s_1, s_3, s_5, s_7 \quad \sim \quad$ from Taylor coefficients order-by-order

Strangeness neutrality: example

To first order:

$$\frac{n_S}{T^3} = \frac{\partial \hat{p}}{\partial \hat{\mu}_S} = \sum_{i,j} \frac{j}{i!j!} \chi_{ij}^{BS}(T) \hat{\mu}_B^i \hat{\mu}_S^{j-1} \approx \chi_1^S(T) + \chi_{11}^{BS}(T)\hat{\mu}_B + \chi_2^S(T)\hat{\mu}_S \stackrel{!}{=} 0$$

from which

$$\hat{\mu}_S^*(T) = s_1(T)\hat{\mu}_B = -\frac{\chi_{11}^{BS}(T)}{\chi_2^S(T)}\hat{\mu}_B \quad \Longrightarrow \quad \boxed{s_1(T) = -\frac{\chi_{11}^{BS}(T)}{\chi_2^S(T)}}$$

(s_3, s_5, s_7 with computer algebra)

Hadron resonance gas (HRG) model

Interacting gas of hadrons ≈ non-interacting gas of hadrons *and* resonances

$$p_{\mathrm{QCD}} = \sum_{\mathrm{h}} p_{\mathrm{h}}^{\mathrm{free}} = \mp \frac{V}{2\pi^2 T^3} \sum_{\mathrm{h}} d_{\mathrm{h}} \int_0^\infty \mathrm{d}p \, p^2 \log\left[1 \mp z_{\mathrm{h}} e^{-\sqrt{m_{\mathrm{h}}^2 + p^2}/T}\right]$$

$$= \frac{VT}{2\pi^2} \sum_{\mathrm{h}} m_{\mathrm{h}}^2 d_{\mathrm{h}} \sum_{n=1}^{\infty} \frac{(\pm 1)^{n+1}}{n^2} z_{\mathrm{h}}^n K_2\left(\frac{n m_{\mathrm{h}}}{T}\right)$$

with $z_{\mathrm{h}} = \exp[(B_{\mathrm{h}}\mu_B + Q_{\mathrm{h}}\mu_Q + S_{\mathrm{h}}\mu_S)/T]$

- $\mathcal{O}(10^3)$ hadrons
- non-critical baseline [nucl-th:2007.02463]
- uses GCE (just like lattice QCD)

Can HRG describe lattice data?

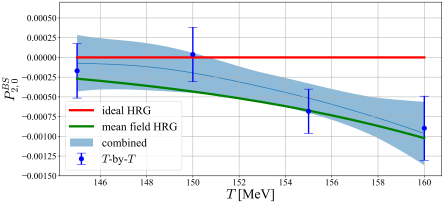
Corrections to HRG from the lattice

Fugacity expansion

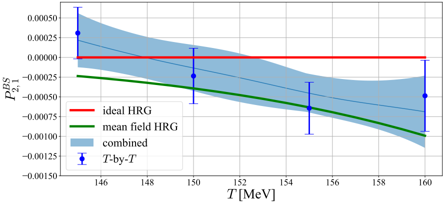
$$\hat{p}(V,T,\{\hat{\mu}_B,\hat{\mu}_S\}) = \sum_{m,n} P^{BS}_{mn}(T)\cosh(m\hat{\mu}_B - n\hat{\mu}_S)$$

$$= P^{BS}_{00}(T) + P^{BS}_{10}(T)\cosh(\hat{\mu}_B) + P^{BS}_{01}\cosh(\hat{\mu}_S) + P^{BS}_{11}(T)\cosh(\hat{\mu}_B - \hat{\mu}_S)$$
$$+ P^{BS}_{20}(T)\cosh(2\hat{\mu}_B) + \dots$$

e.g. $B = 2$ sector is ~ 3 magnitudes larger on the lattice [hep-lat:2102.06625]



$P^{BS}_{20}(T)$



$P^{BS}_{21}(T)$

negative correction $\implies$ repulsive interaction [Phys.Rev. 187 (1969)]

Phenomenology: the range of short-range interactions

Based on

- [hep-ph:1708.00879] $\sim$ mean field model with energy shift $\propto$ density
  $\sim$ S-matrix formalism with NN phase shifts
- [hep-ph:1708.02852] $\sim$ Van der Waals-like extension of HRG

$$\Downarrow$$

repulsive, short-range interaction dominates first order corrections

with typical scale of $v_0 = 1 - 3.5\ \mathrm{fm}^3$

Detour: Roberge-Weiss periodicity

Would expect 2π periodicity from $\mathcal{Z} = \mathrm{Tr}\left[e^{-(H-\mu_u N_u - \mu_d N_d - \mu_s N_s)/T}\right]$

$$\begin{aligned} \mathcal{Z}(\hat{\mu}_q) &= \mathcal{Z}\left(\hat{\mu}_q + \frac{2\pi i k}{3}\right) \qquad \sim \qquad \frac{2\pi}{3} \text{ periodic in } \hat{\mu}_q \\ &= \mathcal{Z}(\hat{\mu}_B + 2\pi i k) \qquad \sim \qquad 2\pi \text{ periodic in } \hat{\mu}_B \end{aligned}$$

Ensures that only integer B terms appear in fugacity expansion

History: current continuum results and estimates

Leading order (since 2012) `[hep-lat:1204.6710]`:

$$\chi_2^B \qquad \chi_2^S \qquad \chi_{11}^{BS}$$

Next-to-leading order (since 2015) `[hep-lat:1507.04627, 2212.09043]`:

$$\chi_4^B \qquad \chi_{31}^{BS} \qquad \chi_{22}^{BS} \qquad \chi_{13}^{BS} \qquad \chi_4^S$$

Next-to-next-to-leading order (continuum results now):

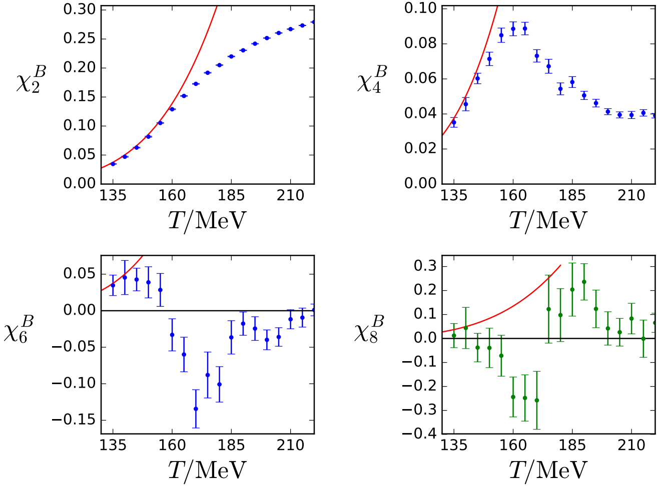
$$\chi_6^B \qquad \chi_{51}^{BS} \qquad \chi_{42}^{BS} \qquad \chi_{33}^{BS} \qquad \chi_{24}^{BS} \qquad \chi_{15}^{BS} \qquad \chi_6^S$$

N^{3}LO (results at finite lattice spacing and cont. at $T = 145$ MeV)

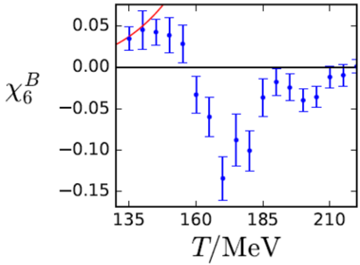
$$\chi_8^B \qquad \chi_{71}^{BS} \qquad \chi_{62}^{BS} \qquad \chi_{53}^{BS} \qquad \chi_{44}^{BS} \qquad \chi_{35}^{BS} \qquad \chi_{26}^{BS} \qquad \chi_{17}^{BS} \qquad \chi_8^S$$

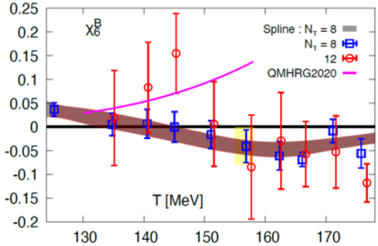
Previous results

[hep-lat:1805.04445] $LT = 4, N_t = 12$



Discrepancies of previous results





[hep-lat:1805.04445]

- $LT = 4,\ N_t = 12$
- $\mu_B = i\mu_B^I$
- Fit to μ_B^I

[hep-lat:2202.09184]

- $LT = 4,\ N_t = 8$
- $\mu_B = 0$
- μ_B definition breaks RW periodicity

Our lattice setup

- $N_f = 2 + 1 + 1$ 4HEX staggered action + DBW2 gauge action
- $T = 130 \ldots 200$ MeV
- $N_t = 8, 10, 12$
- aspect ratio $LT = 2$
- physical point: $m_\pi / f_\pi = 1.0337, m_s/m_{ud} = 27.63, m_c/m_s = 11.85$
- statistics: $\mathcal{O}(10^4)$ - $\mathcal{O}(10^5)$ configuration/ensemble

The question of large enough volume...

Based on phenomenology, $LT = 2$ probably large enough to capture short-range repulsion:

- $T = 145$ MeV

$$L^3 \approx 20\ \text{fm}^3 \approx 6\text{-}20\ v_0$$
$$m_\pi L \approx 1.9$$

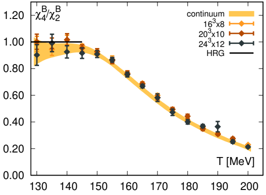
- $T = 130$ MeV

$$L^3 \approx 28\ \text{fm}^3 \approx 8\text{-}28\ v_0$$
$$m_\pi L \approx 2.2$$

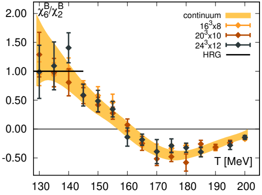
- $T = 120$ MeV

$$L^3 \approx 35\ \text{fm}^3 \approx 10\text{-}35\ v_0$$
$$m_\pi L \approx 2.3$$

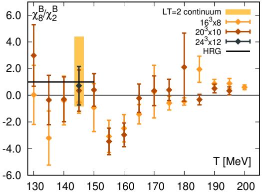
Results - 4HEX continuum



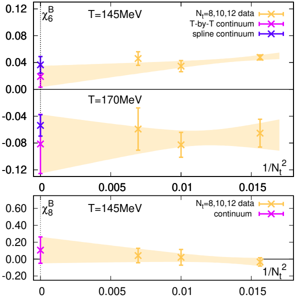
χ_4^B/χ_2^B



χ_6^B/χ_2^B



χ_8^B/χ_2^B



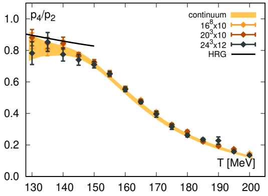
- agreement with HRG for $T < 145$ MeV
- 4HEX: small cut-off effects due to smaller taste-breaking

Results - 4HEX continuum at $n_S = 0$

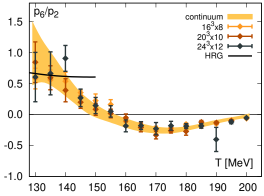
Once s_1, s_3, s_5, s_7 known

$$p_n = \left.\frac{\partial^n \hat{p}}{\partial \hat{\mu}_B^n}\right|_{n_s=0} \qquad \text{of} \qquad \hat{p}_{n_S=0} = \sum_n p_n \hat{\mu}_B^n$$

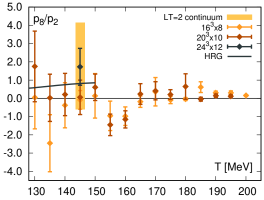
can be computed on strangeness neutral line



p_4/p_2

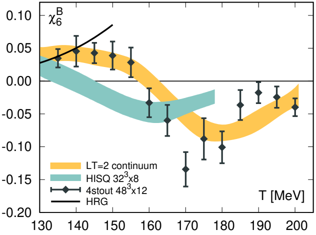


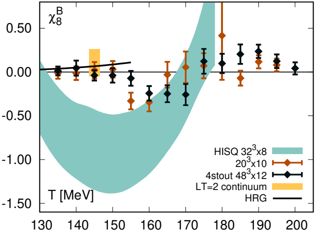
p_6/p_2



p_8/p_2

Results - comparing with literature





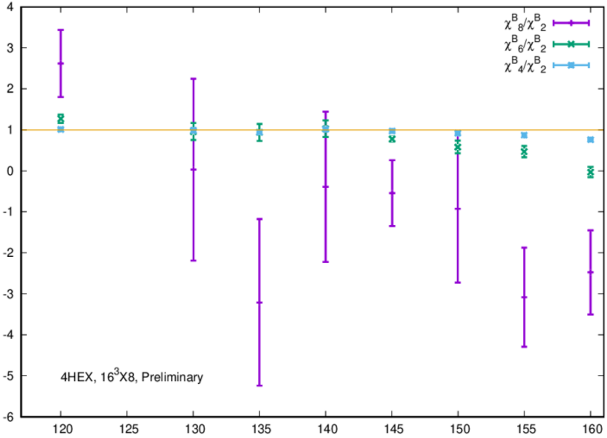
Previous results:

- Pisa (not shown)
- HotQCD: HISQ, $N_t = 8$, $LT = 4$
- WB: 4stout, $N_t = 12$, $LT = 4$
- WB: 4HEX, cont. *and* $N_t = 10$, $LT = 2$

- 4stout $N_t = 12 \approx$ 4HEX cont. for $T < 145$ MeV
  $\implies$ small finite-volume effects
  $\implies$ agreement with HRG
- HISQ $N_t = 8 \not\approx$ 4stout $N_t = 12$
  $\implies$ large cut-off effects
- finite-volume effects for larger T

What can we see at lower temperatures?

Sizeable difference from HRG at $T = 120$ MeV $\quad\sim\quad \chi_4^B < \chi_6^B < \chi_8^B$!



Lee-Yang zeros in a nuthsell

$$\mathcal{Z} = \mathrm{Tr}\left[e^{-(H-\mu_B B)/T}\right] = \sum_n e^{n\mu_B/T}\,\mathrm{Tr}_n\left[e^{-H/T}\right] = \sum_n e^{n\mu_B/T}\,\mathcal{Z}_n$$

$\sim$ polynom in fugacity $e^{\mu_B/T}$ $\Longrightarrow$ roots of $\mathcal{Z}$: Lee-Yang zeros

$\Downarrow$

$p \propto \log \mathcal{Z}$ has logarithmic branch points

As $V \to \infty$:

- an infinite number of zeros tend to the real axis $\Longrightarrow$ **phase transition**
- zeros stay away from the real axis $\Longrightarrow$ **crossover transition**

Near critical point ($V \to \infty$: Lee-Yang edge): $\mathrm{Im}\mu_{\mathrm{LY}} \sim A|T - T_{\mathrm{c}}|^{\beta\delta}$

[hep-lat:1904.01974],[hep-lat:1911.00043],[hep-th:2312.06952],[hep-lat:2401.05651]

Padé resummation

(i.e. rational function approximation)

For all T:

1. For baryon number susceptibility

$$\chi_2^B(\hat{\mu}_B) \approx \chi_2^B(0) + \frac{\chi_4^B(0)}{2}\hat{\mu}_B^2 + \frac{\chi_6^B(0)}{24}\hat{\mu}_B^4 + \frac{\chi_8^B(0)}{720}\hat{\mu}_B^6 \approx \frac{a + b\hat{\mu}_B^2}{1 + c\hat{\mu}_B^2 + d\hat{\mu}_B^4}$$

2. Compute a, b, c, d

$$a = \chi_2^B\ , \qquad b = \frac{(\chi_2^B)^2\chi_8^B - 30\chi_2^B\chi_4^B\chi_8^B + 9(\chi_4^B)^3}{30\big(6(\chi_4^B)^2 - \chi_2^B\chi_6^B\big)}\ ,$$

$$c = \frac{\chi_2^B\chi_8^B - 15\chi_4^B\chi_6^B}{30\big(6(\chi_4^B)^2 - \chi_2^B\chi_6^B\big)}\ , \qquad d = \frac{2\chi_4^B\chi_8^B - 5(\chi_6^B)^2}{120\big(\chi_2^B\chi_6^B - 6(\chi_4^B)^2\big)}$$

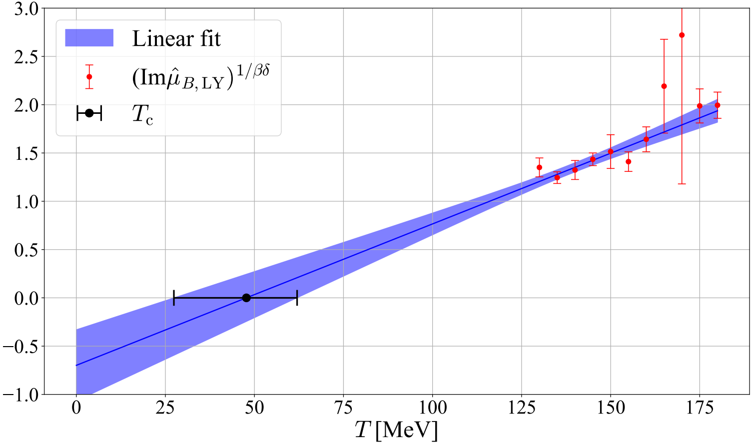
3. Find poles (quadratic formula)

Extrapolate T_c from Lee-Yang edge

$$\mathrm{Im}\mu_{\mathrm{LY}} \sim A|T - T_c|^{\beta\delta} \qquad \text{(lots of caveats...)}$$

Constraining the CEP (PRELIMINARY)

$$T_\mathrm{c} \cong 48^{+15}_{-21}\ \mathrm{MeV}$$



Summary

- Computed generalized susceptibilities up to eighth order, both at $\mu_S = 0$ and at $n_S = 0$
- Made comparisons with previous results, and argued that finite-volume effects are under control for $T < 145$ MeV
- Discussed possible constrains for the position of the CEP

The End

Thank you!

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Fig.: LQCD
Made with `dream.ai`.