old problems and new(?) physics: non-leptonic kaon decay and the ΔI=1/2 rule

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Budapest 2019/11/20

an old problem

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have served[1,2] to set an upper limit of 1/300 for the fraction of K_2^0's which decay into two charged pions. The present experiment, using spark chamber techniques, proposed to extend this limit.

In this measurement, K_2^0 mesons were produced at the Brookhaven AGS in an internal Be target bombarded by 30-BeV protons. A neutral beam was defined at 30 degrees relative to the circulating protons by a $1\frac{1}{2}$-in.$\times 1\frac{1}{2}$-in.$\times$48-in. collimator at an average distance of 14.5 ft. from the internal target. This collimator was followed by a sweeping magnet of 512 kG-in. at ~20 ft. and a 6-in.$\times$6-in.$\times$48-in. collimator at 55 ft. A $1\frac{1}{2}$-in. thickness of Pb was placed in front of the first collimator to attenuate the gamma rays in the beam.

The experimental layout is shown in relation to the beam in Fig. 1. The detector for the decay products consisted of two spectrometers each composed of two spark chambers for track delineation separated by a magnetic field of 178 kG-in. The axis of each spectrometer was in the horizontal plane and each subtended an average solid angle of 0.7×10^{-2} steradians. The spark chambers were triggered on a coincidence between water Cherenkov and scintillation counters positioned immediately behind the spectrometers. When coherent K_1^0 regeneration in solid materials was being studied, an anticoincidence counter was placed immediately behind the regenerator. To minimize interactions K_2^0 decays were observed from a volume of He gas at nearly STP.

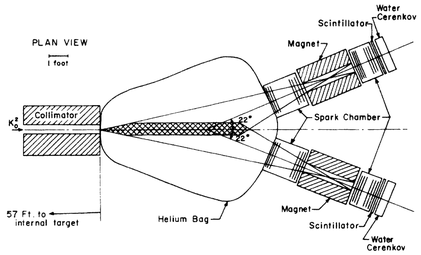


FIG. 1. Plan view of the detector arrangement.

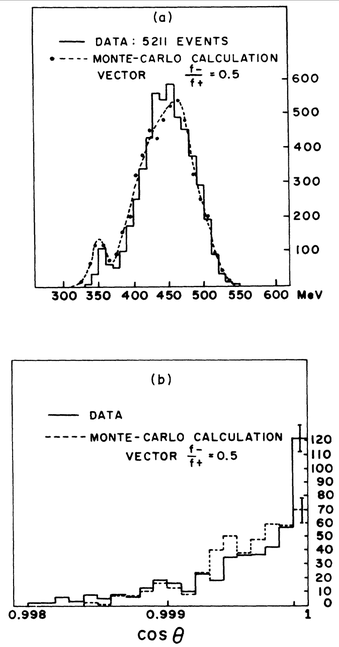


FIG. 2. (a) Experimental distribution in m^* compared with Monte Carlo calculation. The calculated distribution is normalized to the total number of observed events. (b) Angular distribution of those events in the range $490 < m^* < 510$ MeV. The calculated curve is normalized to the number of events in the complete sample.

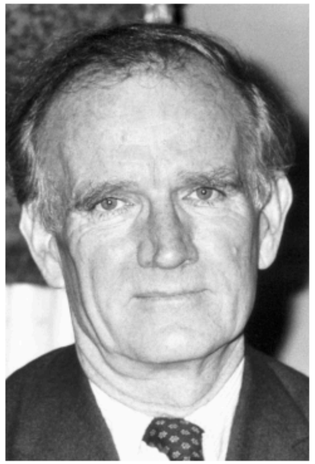
We would conclude therefore that K_2^0 decays to two pions with a branching ratio $R = (K_2 \to \pi^+ + \pi^-)/(K_2^0 \to \text{all charged modes}) = (2.0 \pm 0.4)\times 10^{-3}$ where the error is the standard deviation. As emphasized above, any alternate explanation of the effect requires highly nonphysical behavior of the three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as $K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0)]$ then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$ where τ_1 and τ_2 are the K_1^0 and K_2^0 mean lives and R_T is the branching ratio including decay to two π^0. Using $R_T = \frac{3}{8}R$ and the branching ratio quoted above, $|\epsilon| \cong 2.3\times 10^{-3}$.

The Nobel Prize in Physics 1980

Photo from the Nobel Foundation archive.

James Watson Cronin

Prize share: 1/2



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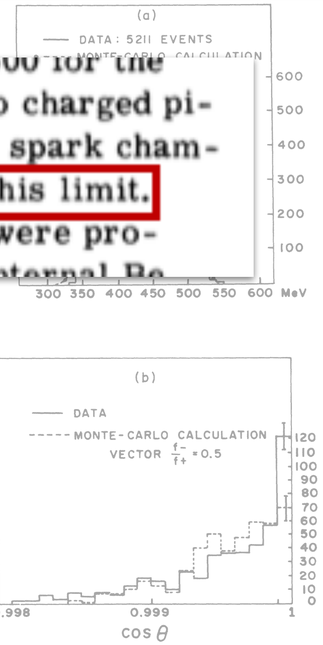

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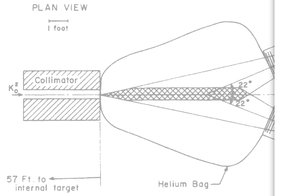

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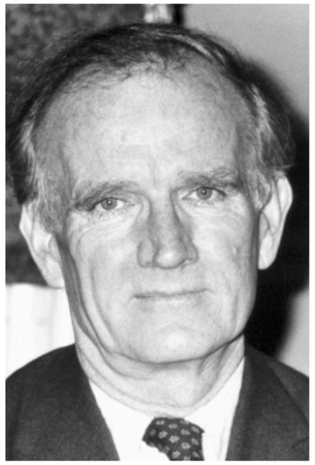

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a persistent problem

Lattice 19

Fri 21/6

14:00	TKNN formula for general lattice Hamiltonian in odd dimensions — *Tetsuya Onogi*	Resonance study of SU(2) model with 2 fundamental flavours of ferm... — *Tadeusz Janowski*
	Spectral Methods and Running Scales in Causal Dynamical Triang... — *Giuseppe Clemente*	Meson spectrum of Sp(4) lattice gauge theory with two fundamental Dirac fe... — *Jong-Wan Lee*
	The meson spectrum of large N gauge theories — *Prof. Antonio Gonzalez-Arroyo*	Sp(2N) Yang-Mills towards large N. — *Jack Holligan* — *Shimao 3B* 14:40 - 15:00
15:00	Numerical study of ADE-type $\mathcal{N}=2$ Landau--Ginzburg m... — *Okuto Morikawa*	Towards a composite Higgs and a partially composite top quark — *Benjamin Svetitsky*
	Gauge-invariant path-integral measure for the overalp Weyl fermions in 16 of SO(10) and the SM — *Prof. Yoshio Kikukawa* — *Shimao 3B* 15:20 - 15:40	
	Coffee/Tea break 15:40 - 16:10	
16:00	Calculation of the $K_L - K_S$ mass difference for physical quark masses — *Mr Bigeng Wang* — *Shimao 5* 16:10 - 16:30	
	Investigating Rare Kaon Decays with the All-to-All Method — *Fionn O hOgain* — *Shimao 5* 16:30 - 16:50	
	S-wave pi-pi I=0 and I=2 scattering at physical pion mass — *Tianle Wang* — *Shimao 5* 16:50 - 17:10	
17:00	Update on the improved lattice calculation of direct CP-violation in K decays — *Dr Christopher Kelly* — *Shimao 5* 17:10 - 17:30	
	Charm CP & the lattice — *Amarjit Soni* — *Shimao 5* 17:30 - 17:50	

16th Conference on Flavor Physics & CP Violation

FPCP 2018

14 -18 July 2018, Hyderabad, India

Sun 15/7

	Kaon decays: epsilon/epsilon', new physics — *Satoshi Mishima* — *Hyderabad* 14:15 - 14:45	
15:00	Search for ultra-rare Kaon decay K-> pi nu nu — *Jurgen Engelfried* — *Hyderabad* 14:45 - 15:15	
	Tea/Coffe — *Hyderabad* 15:15 - 15:45	
16:00	Latest results from KOTO — *Kota Nakagiri* — *Hyderabad* 15:45 - 16:15	
	Latest results on the KLOE data and status of analysis of the KLOE-2 data — *Eryk Czerwiński* — *Hyderabad* 16:15 - 16:45	

prime time attention by hep-ph and hep-lat communities for 55 years — and running

a persistent problem

ε'/ε-2018: A Christmas Story

Andrzej J. Buras

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Physik Department, TU München, James-Franck-Straße, D-85748 Garching, Germany
E-mail: aburas@ph.tum.de

Abstract

I was supposed to review the status of ε'/ε both at the CKM Workshop in September in Heidelberg and recently at the Discrete 2018 Conference in Vienna. Unfortunately I had to cancel both talks for family reasons. My main goal in these talks was to congratulate NA48 and KTeV collaborations for the discovery of new sources of CP violation through their heroic efforts to measure the ratio ε'/ε in the 1980s and 1990s with final results presented roughly 16 years ago. As I will not attend any other conferences this year I will reach this goal in this writing. In this context I will give arguments, why I am convinced about the presence of new physics in ε'/ε on the basis of my work with Jean-Marc Gérard within the context of the Dual QCD (DQCD) approach and why RBC-UKQCD lattice QCD collaboration and in particular Chiral Perturbation Theory practitioners are still unable to reach this conclusion. I will demonstrate that even in the presence of pion loops, as large as advocated recently by Gisbert and Pich, the value of ε'/ε is significantly below the data, when the main non-factorizable QCD dynamics at long distance scales, represented in DQCD by *the meson evolution*, is taken into account. As appropriate for a Christmas story, I will prophesy the final value of ε'/ε within the SM, which should include in addition to the correct matching between long and short distance contributions, isospin breaking effects, NNLO QCD corrections to both QCD penguin and electroweak penguin contributions and final state interactions. Such final SM result will probably be known from lattice QCD only in the middle of the 2020s, but already in 2019 we should be able to see some signs of NP in the next result on ε'/ε from RBC-UKQCD. In this presentation I try to avoid, as much as possible, the overlap with my recent review of Dual QCD in [1].

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plan

- phenomenology of neutral kaon decay

- understanding non-leptonic kaon decay within the SM
  - electroweak effective Hamiltonian analysis
  - exact (lattice) vs. approximate (effective theory/large N/models)
  - why is it so hard?
  - state-of-the-art quantitative results

- understanding the anatomy of ΔI=1/2
  - the strategy
  - (old) results for QCD amplitudes
  - large N_c
  - insight into light meson physics

- outlook

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neutral kaon decay

K^0_S $I(J^P) = \frac{1}{2}(0^-)$

Mean life $\tau = (0.8954 \pm 0.0004) \times 10^{-10}$ s (S = 1.1) Assuming *CPT*

Mean life $\tau = (0.89564 \pm 0.00033) \times 10^{-10}$ s Not assuming *CPT*

K^0_S DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Hadronic modes			
$\pi^0\pi^0$	(30.69 ± 0.05) %		209
$\pi^+\pi^-$	(69.20 ± 0.05) %		206

K^0_L $I(J^P) = \frac{1}{2}(0^-)$

$m_{K_L} - m_{K_S}$
$= (0.5293 \pm 0.0009) \times 10^{10}\ \hbar\ \mathrm{s}^{-1}$ (S = 1.3) Assuming *CPT*
$= (3.484 \pm 0.006) \times 10^{-12}$ MeV Assuming *CPT*
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Mean life $\tau = (5.116 \pm 0.021) \times 10^{-8}$ s (S = 1.1)

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$\pi^\pm \mu^\mp \nu_\mu$		[o]	(27.04 ± 0.07) %	S=1.1	216
Hadronic modes, including Charge conjugation×Parity Violating (*CPV*) modes					
$3\pi^0$			(19.52 ± 0.12) %	S=1.6	139
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$\pi^+\pi^-$	*CPV*	[q]	$(1.967 \pm 0.010) \times 10^{-3}$	S=1.5	206
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[PDG 2019]

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$$|K_\mathrm{S}\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_1\rangle + \bar{\varepsilon}|K_2\rangle)\,, \quad |K_\mathrm{L}\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_2\rangle + \bar{\varepsilon}|K_1\rangle)\,, \quad \bar{\varepsilon} = \frac{p-q}{p+q}$$

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CP-violation parameters accessible via decay amplitudes into two pions

$$-iT[K^0 \to (\pi\pi)_I] = A_i e^{i\delta_I} \qquad T[(\pi\pi)_I \to (\pi\pi)_I]_{l=0} = 2e^{i\delta_I}\sin\delta_I$$

$$\varepsilon = \frac{T[K_\mathrm{L} \to (\pi\pi)_0]}{T[K_\mathrm{S} \to (\pi\pi)_0]} \simeq \bar{\varepsilon} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$

$$\varepsilon' = \frac{\varepsilon}{\sqrt{2}}\left(\frac{T[K_\mathrm{L} \to (\pi\pi)_2]}{T[K_\mathrm{L} \to (\pi\pi)_0]} - \frac{T[K_\mathrm{S} \to (\pi\pi)_2]}{T[K_\mathrm{S} \to (\pi\pi)_0]}\right) \simeq \frac{1}{\sqrt{2}}\,e^{i(\delta_2-\delta_0+\pi/2)}\,\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

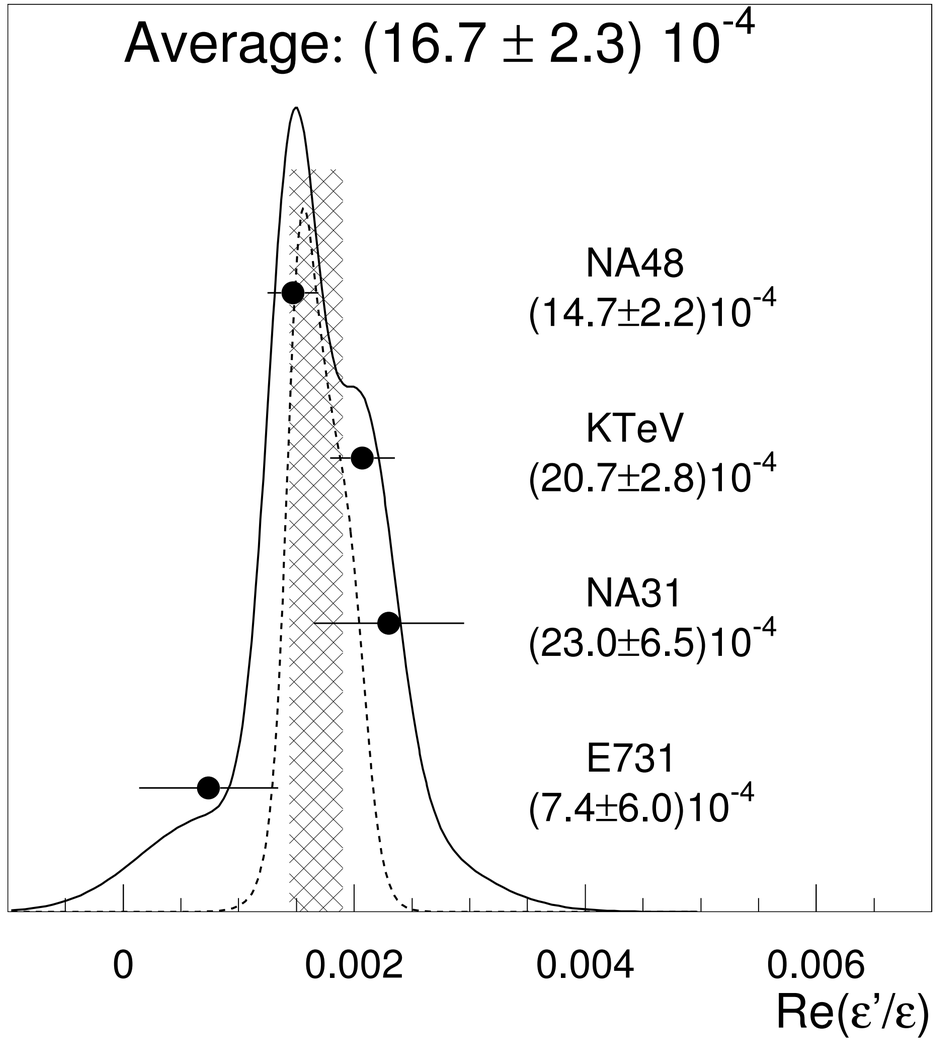
neutral kaon decay

experiment:

$$|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (16.5 \pm 2.6) \times 10^{-4}$$

$$\left|\frac{A_0}{A_2}\right| = 22.35$$



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$$-iT[K^0 \to (\pi\pi)_I] = A_i e^{i\delta_I}$$

(similar observations in baryon sector, e.g., $\Lambda/\Sigma \to N\pi$, heavy meson decay, ...)

[fully?] satisfactory understanding of result within SM lacking for 45 years

plan

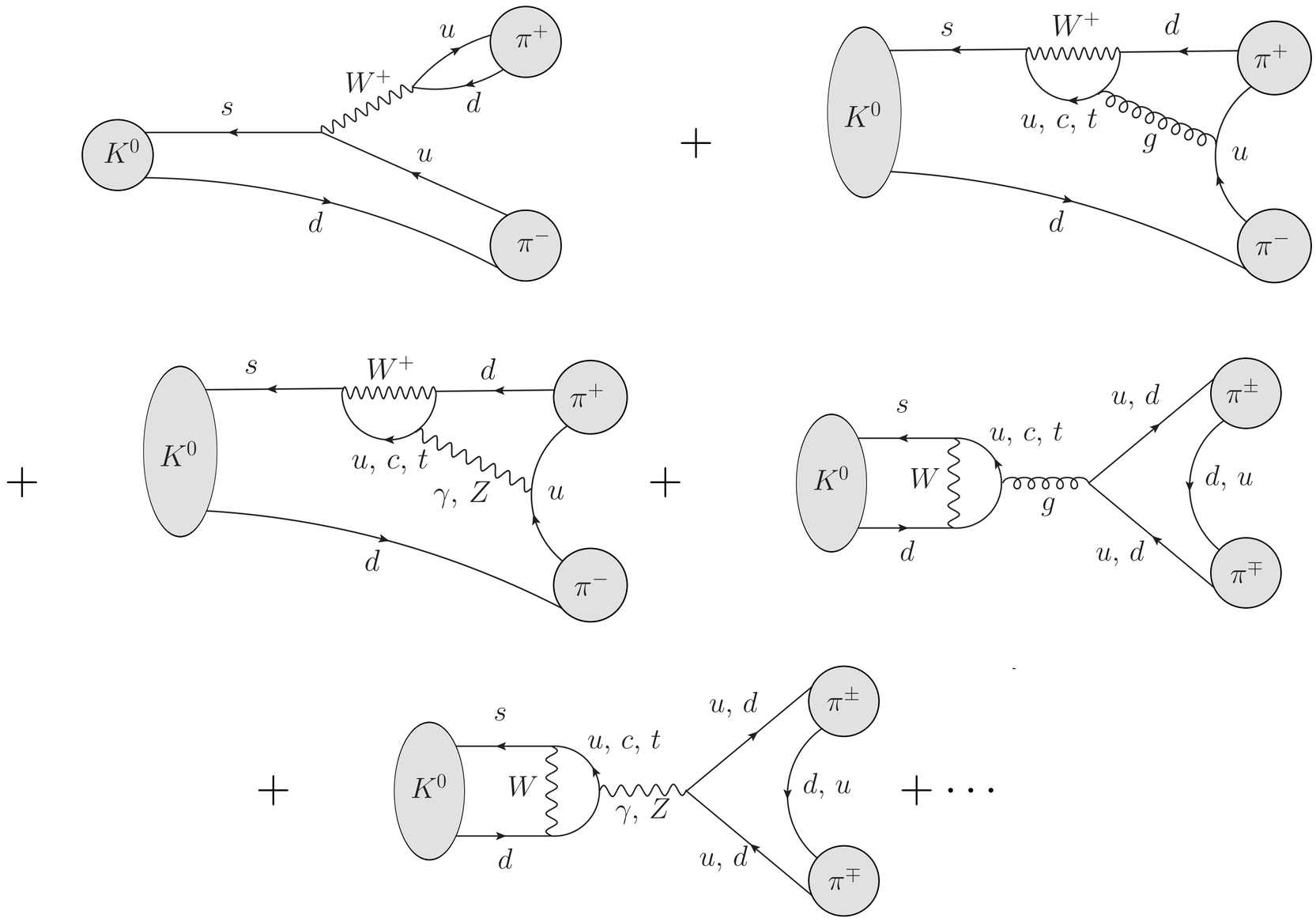
- phenomenology of neutral kaon decay

- understanding non-leptonic kaon decay within the SM
  - electroweak effective Hamiltonian analysis
  - exact (lattice) vs. approximate (effective theory/large N/models)
  - why is it so hard?
  - state-of-the-art quantitative results

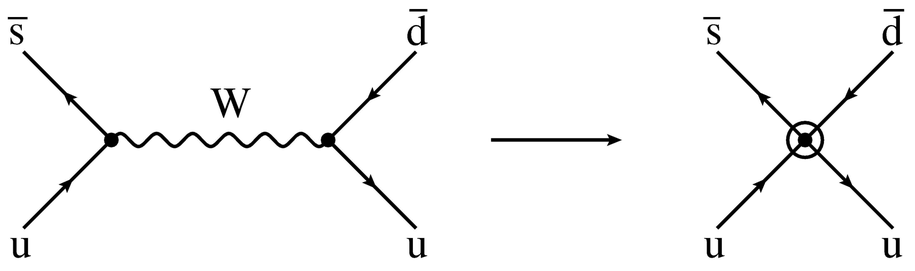
- understanding the anatomy of ΔI=1/2
  - the strategy
  - (old) results for QCD amplitudes
  - large N_c
  - insight into light meson physics

- outlook

effective weak Hamiltonian analysis



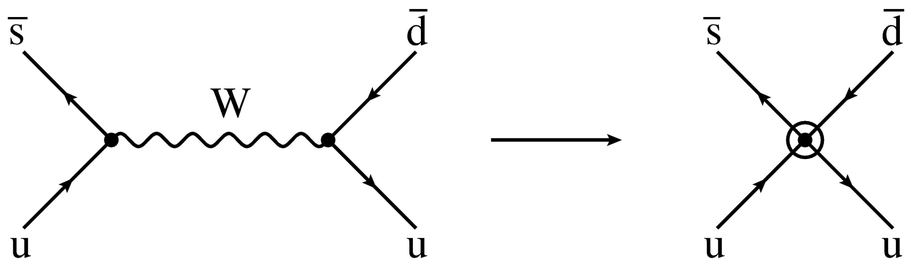
effective weak Hamiltonian analysis



$$m_K^2 \ll M_W^2,\ m_t^2,\ m_c^2(?) \quad \Rightarrow$$

$$\frac{1}{p^2 - m_X^2} \simeq -\frac{1}{m_X^2}\left[1 + \mathcal{O}\left(\frac{p^2}{m_X^2}\right)\right]$$

effective weak Hamiltonian analysis



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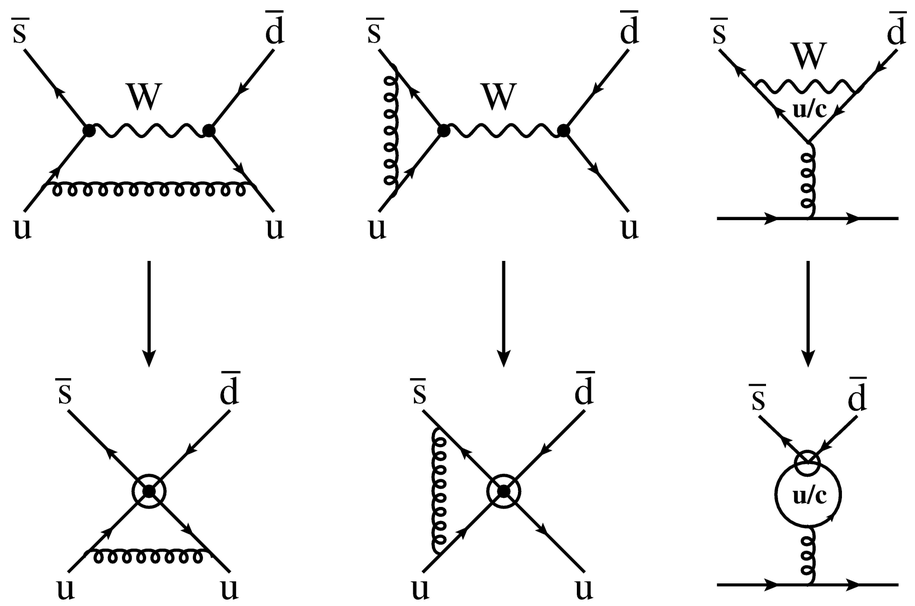

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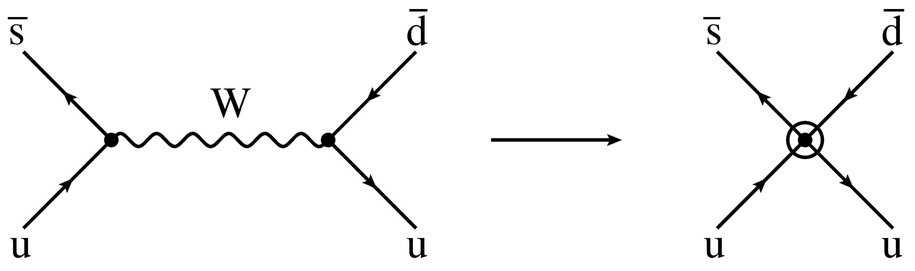

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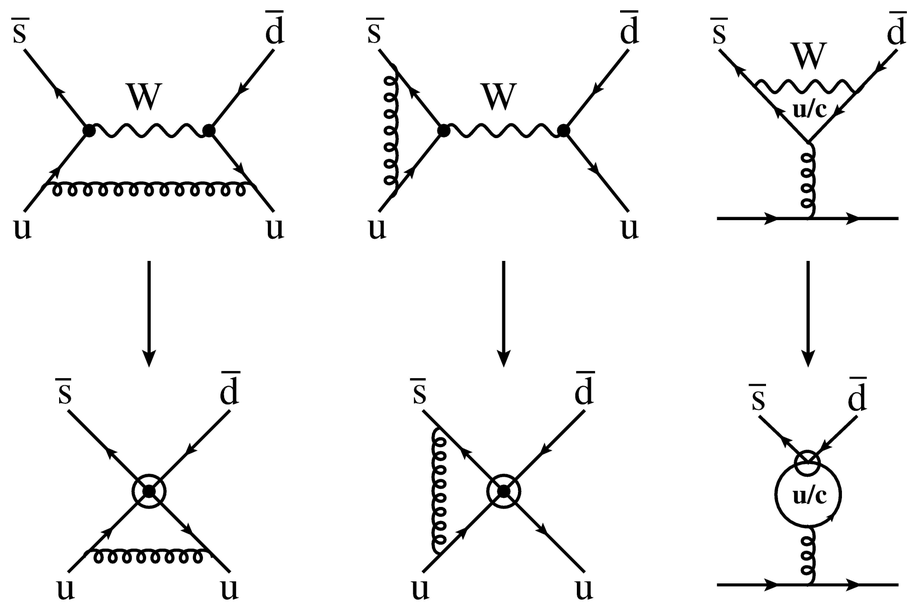

$$T[K \to \pi\pi] \approx \langle \pi\pi | \mathcal{H}_{\rm w}^{\rm eff} | K \rangle + \mathcal{O}\left(\frac{p^2}{M_W^2}\right)$$

$$\mathcal{H}_{\rm w}^{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \sum_k f_k(V_{\rm CKM})\, C_k(\mu/M_W)\, \bar{O}_k(\mu)$$

Wilson coefficients
(short-distance physics)

four-quark operators
(long-distance physics)

effective weak Hamiltonian analysis

CP-violation effects neglected ($\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim 10^{-3}$), keep active charm quark:

$$\mathcal{H}_{\rm w}^{\rm eff} = \frac{g_{\rm w}^2}{2M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} \{k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2^\sigma\}$$

$$Q_1^\pm = (\bar{s}_{\rm L}\gamma_\mu u_{\rm L})(\bar{u}_{\rm L}\gamma_\mu d_{\rm L}) \pm (\bar{s}_{\rm L}\gamma_\mu d_{\rm L})(\bar{u}_{\rm L}\gamma_\mu u_{\rm L}) - [u \leftrightarrow c]$$

$$Q_2^\pm = (m_u^2 - m_c^2)\{m_d(\bar{s}_{\rm L} d_{\rm R}) + m_s(\bar{s}_{\rm R} d_{\rm L})\}$$

(do not contribute to physical $K \to \pi\pi$ transitions)

effective weak Hamiltonian analysis

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(do not contribute to physical $K \to \pi\pi$ transitions)

$$\left|\frac{A_0}{A_2}\right| = \frac{k_1^-(M_W)}{k_1^+(M_W)} \frac{\langle(\pi\pi)_{I=0}|Q_1^-|K\rangle}{\langle(\pi\pi)_{I=2}|Q_1^+|K\rangle} \qquad \frac{k_1^-(M_W)}{k_1^+(M_W)} \simeq 2.8$$

- bulk of effect should come from long-distance QCD contribution

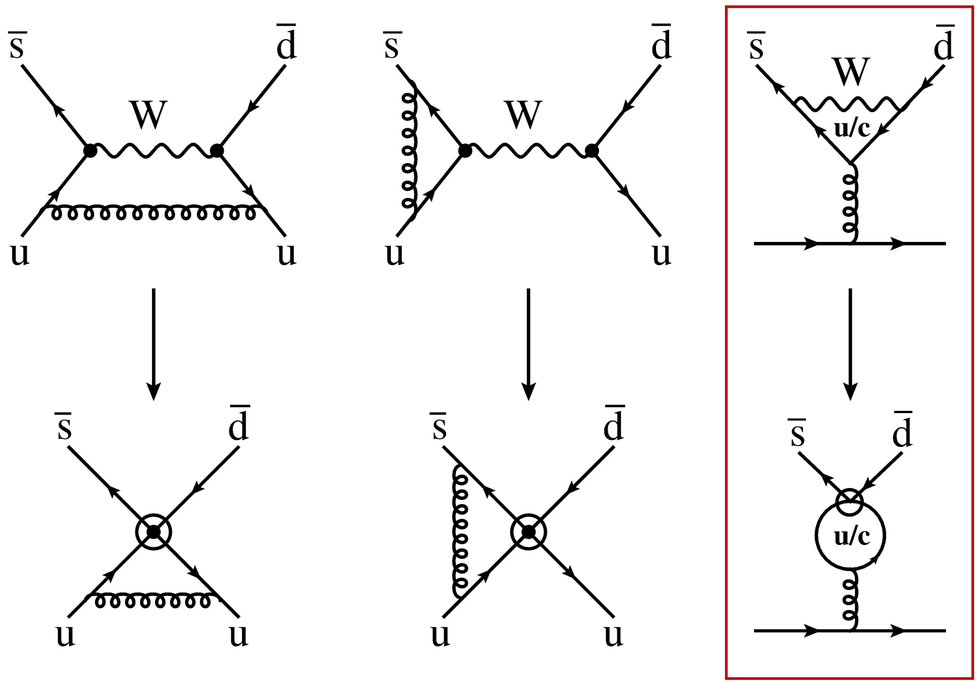
[Gaillard, Lee; Altarelli, Maiani 1974]

- reliable non-perturbative determination mandatory

[Cabibbo, Martinelli, Petronzio; Brower, Maturana, Gavela, Gupta 1984]

effective weak Hamiltonian analysis

if charm quark is also integrated out (perturbation theory at m_c?):

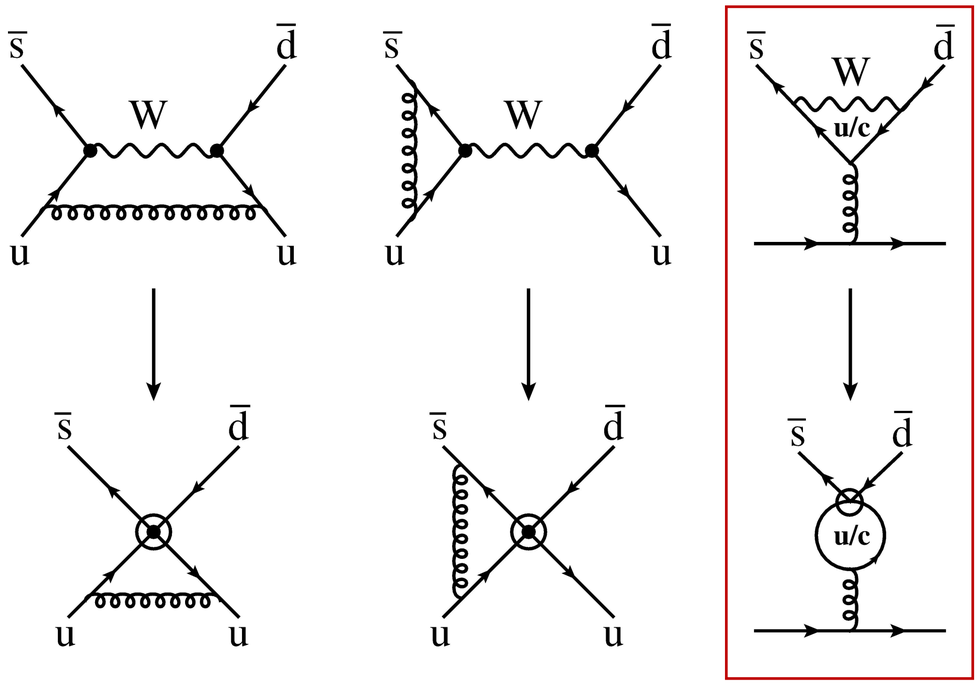


$$\mathcal{H}_{\mathrm{w}}^{\mathrm{eff}} = \frac{g_{\mathrm{w}}^2}{2M_W^2} V_{ud}V_{us}^* \sum_{i=1}^{10} \left[z_i + \tau y_i\right] Q_i$$

$$\tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}$$

effective weak Hamiltonian analysis

if charm quark is also integrated out (perturbation theory at m_c?):



$$\mathcal{H}_{\rm w}^{\rm eff} = \frac{g_{\rm w}^2}{2M_W^2} V_{ud}V_{us}^* \sum_{i=1}^{10} [z_i + \tau y_i]\, Q_i \qquad \tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}$$

- several four-quark operators (2xcurrent², 4xQCD/EW penguins), **includes "CP-violating" structures**
- missing GIM mechanism quadratic divergences in penguin operators (log if charm active)
- suggests enhancement mechanism due to peculiar role of charm scale

$$\sim \frac{m_c^2 - m_u^2}{\mu^2}\,, \quad m_c \ll \mu \ll M_W$$

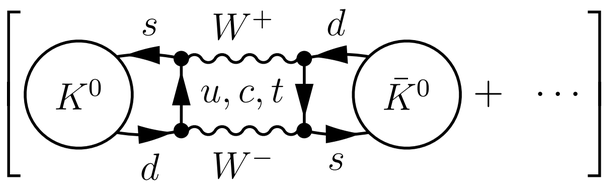
$$\sim \ln \frac{m_c^2}{\mu^2}\,, \quad \mu \lesssim m_c$$

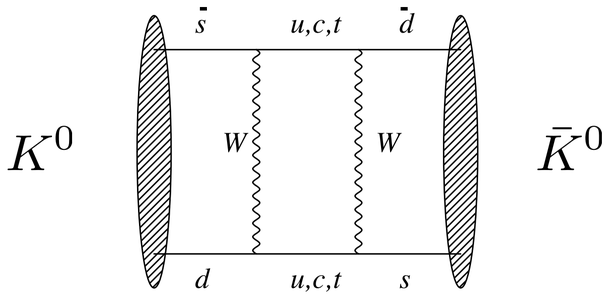
[Shifman, Vainshtein, Zakharov 1975-77]

effective weak Hamiltonian analysis

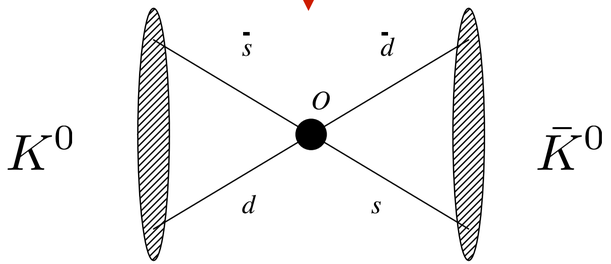
useful relation to neutral kaon mixing:

$\varepsilon \propto \mathrm{Im}$





$$\varepsilon \propto \hat{B}_K = \frac{\langle \bar{K}^0 | (\bar{s}\gamma^{\mathrm{L}}_\mu d)(\bar{s}\gamma^{\mathrm{L}}_\mu d) | K^0 \rangle}{\frac{8}{3} F_K^2 m_K^2}$$

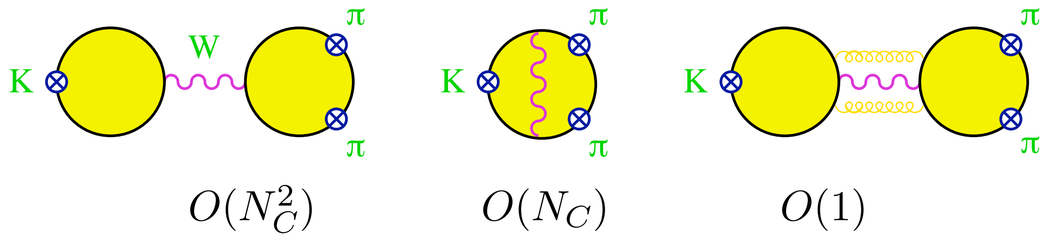


in the chiral limit, this amplitude is the same as the contribution to kaon decay in the I=3/2 channel (with active charm)

how to tackle it

approximate methods/effective theory

- spectacular failure of naive $1/N_c$ expansion



$$T[K^0 \to \pi^0\pi^0] \sim 0 \quad \Rightarrow \quad \left|\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$

[Fukugita et al. 1977]
[Chivukula, Flynn, Georgi 1986]

- elaborate approaches that combine $1/N_c$, chiral perturbation theory+vector dominance, and quark-hadron duality claim success

[Buras, Gérard, Bardeen 2014]
[Gisbert, Pich 2017]

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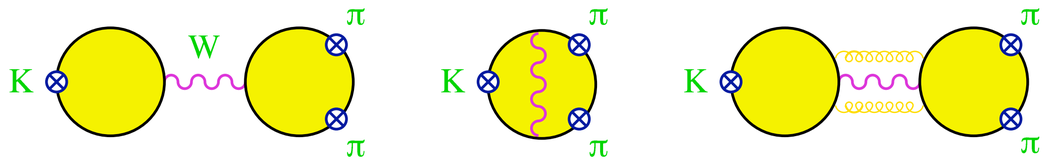

$O(N_C^2)$ $O(N_C)$ $O(1)$

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[Buras, Gérard, Bardeen 2014]
[Gisbert, Pich 2017]

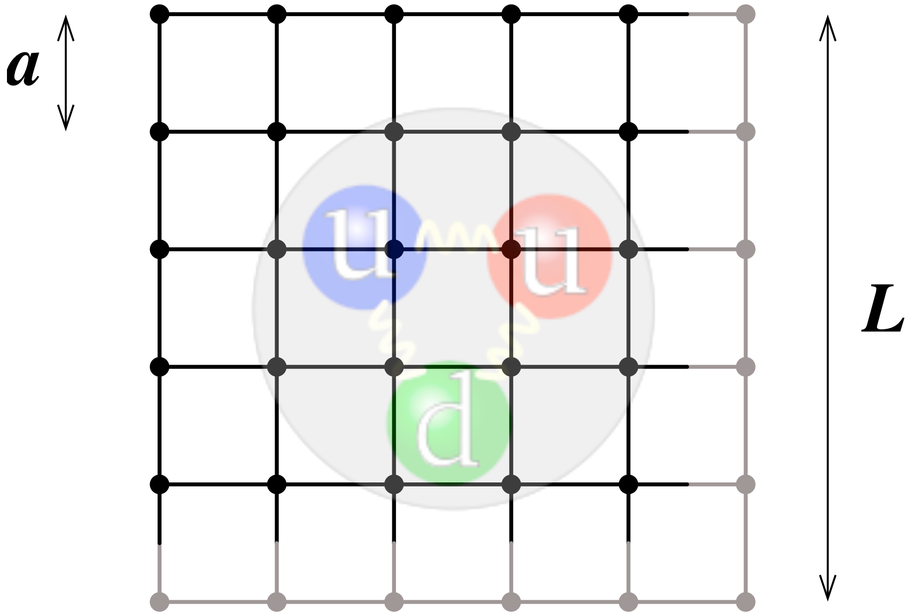
lattice QCD [rest of this talk]

- first-principles approach, uncertainties can be systematically improved
- has reached precision era, main player in flavour physics — e.g., B_K
- however, ΔI=1/2 and ε'/ε remain very difficult problems

lattice QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g^2}\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \sum_{q=1}^{N_\mathrm{f}} \bar{\psi}_q \left[i\not{D} - m_q\right]\psi_q \underbrace{+\frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[F_{\mu\nu}F_{\rho\sigma}\right]}_{\not{C}\mathrm{P}}$$

first-principles approach = control all systematic uncertainties



[Wilson 1974]

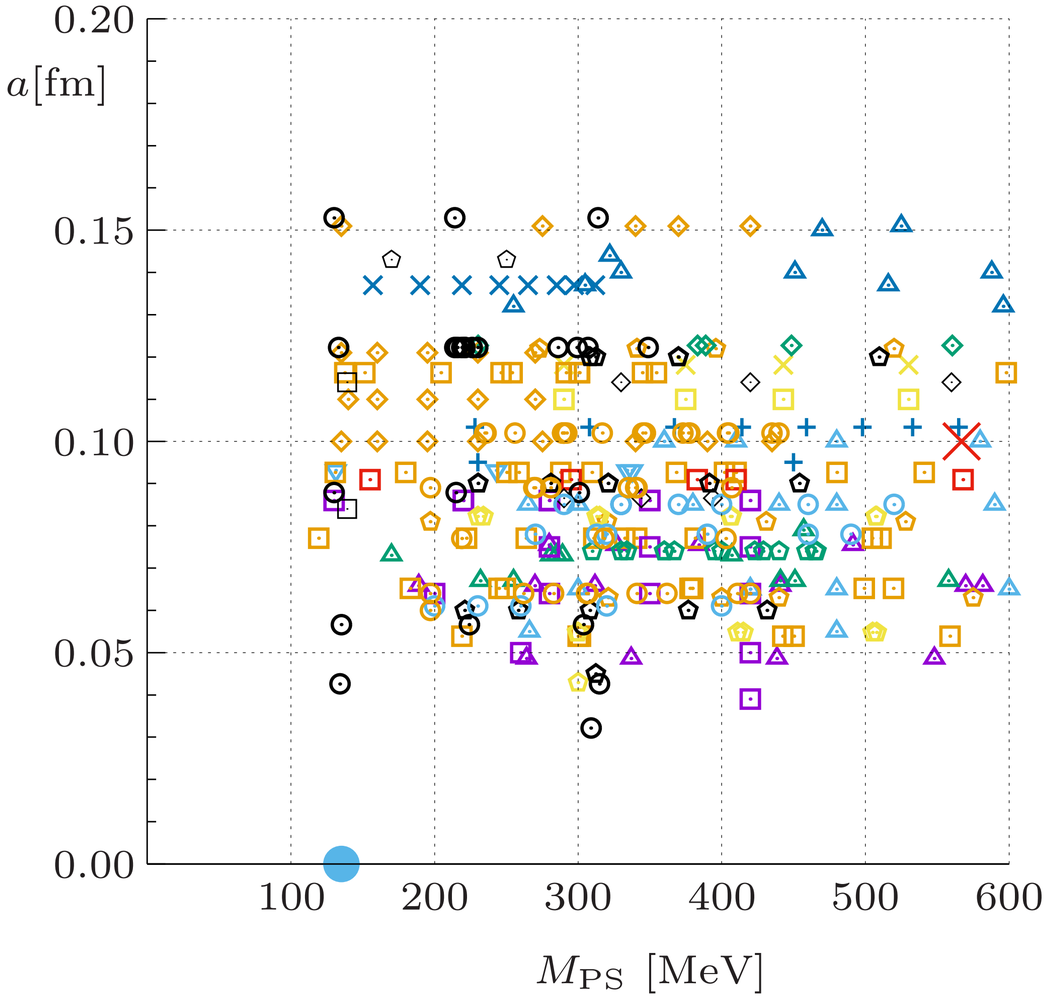
- spacetime = Euclidean lattice
- allows to define path integral rigorously *and* compute it via Monte Carlo methods
- QCD recovered by removing cutoffs at physical kinematics
- values of Lagrangian parameters fixed by N_f+1 hadron masses/decay constants — everything else are **predictions**

lattice QCD

[SMBC]

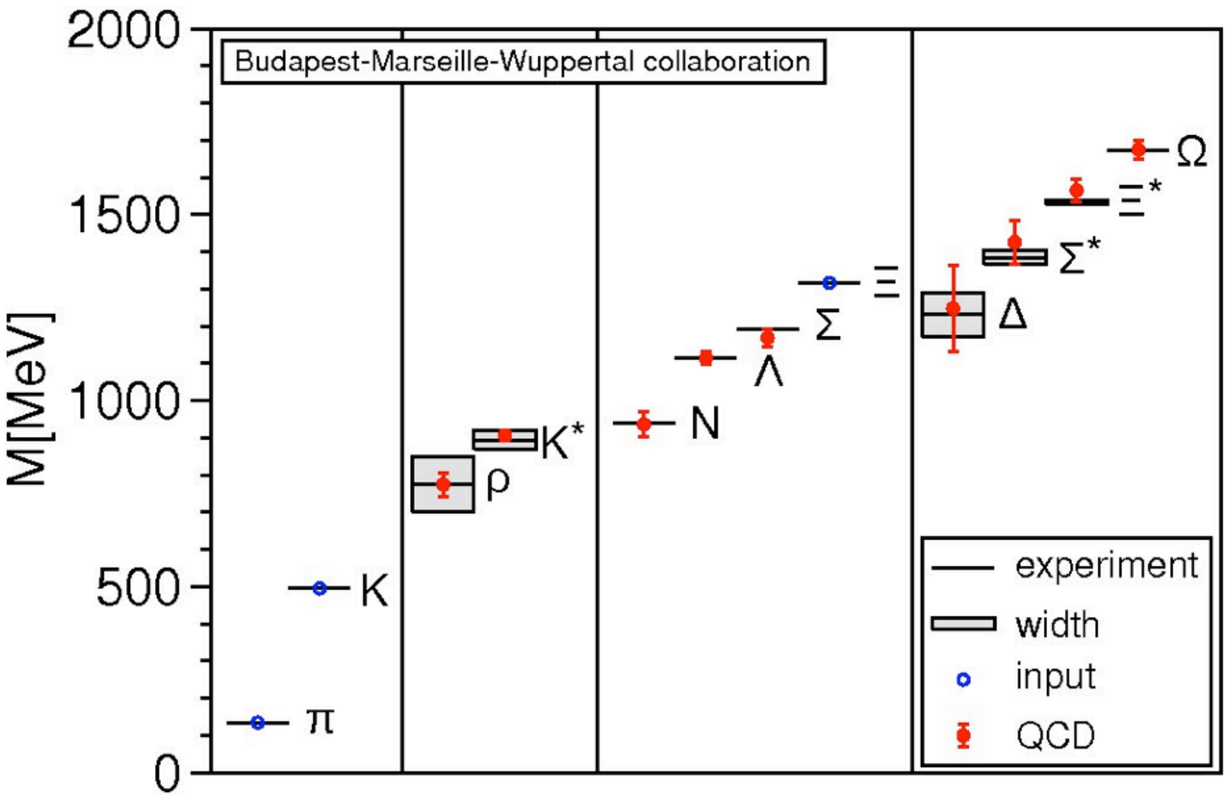
lattice QCD

CLS	$N_f = 2$	
ETMC	$N_f = 2$	
(clover) ETMC	$N_f = 2$	
QCDSF	$N_f = 2$	
BGR	$N_f = 2$	
JLQCD	$N_f = 2$	
(plaq) TWQCD	$N_f = 2$	
(Iwa) TWQCD	$N_f = 2$	
(HEX) BMW	$N_f = 2+1$	
(stout) BMW	$N_f = 2+1$	
(stout-stag) BMW	$N_f = 2+1$	
CLS	$N_f = 2+1$	
HSC	$N_f = 2+1$	
PACS-CS	$N_f = 2+1$	
QCDSF	$N_f = 2+1$	
JLQCD	$N_f = 2+1$	
(Möbius) JLQCD	$N_f = 2+1$	
RBC-UKQCD	$N_f = 2+1$	
(DSDR) RBC-UKQCD	$N_f = 2+1$	
(Möbius) RBC-UKQCD	$N_f = 2+1$	
MILC	$N_f = 2+1$	
MILC	$N_f = 2+1+1$	
ETMC	$N_f = 2+1+1$	
BMW	$N_f = 1+1+1+1$	
JLQCD/CP-PACS 01	$N_f = 2$	
	M_π (experiment)	



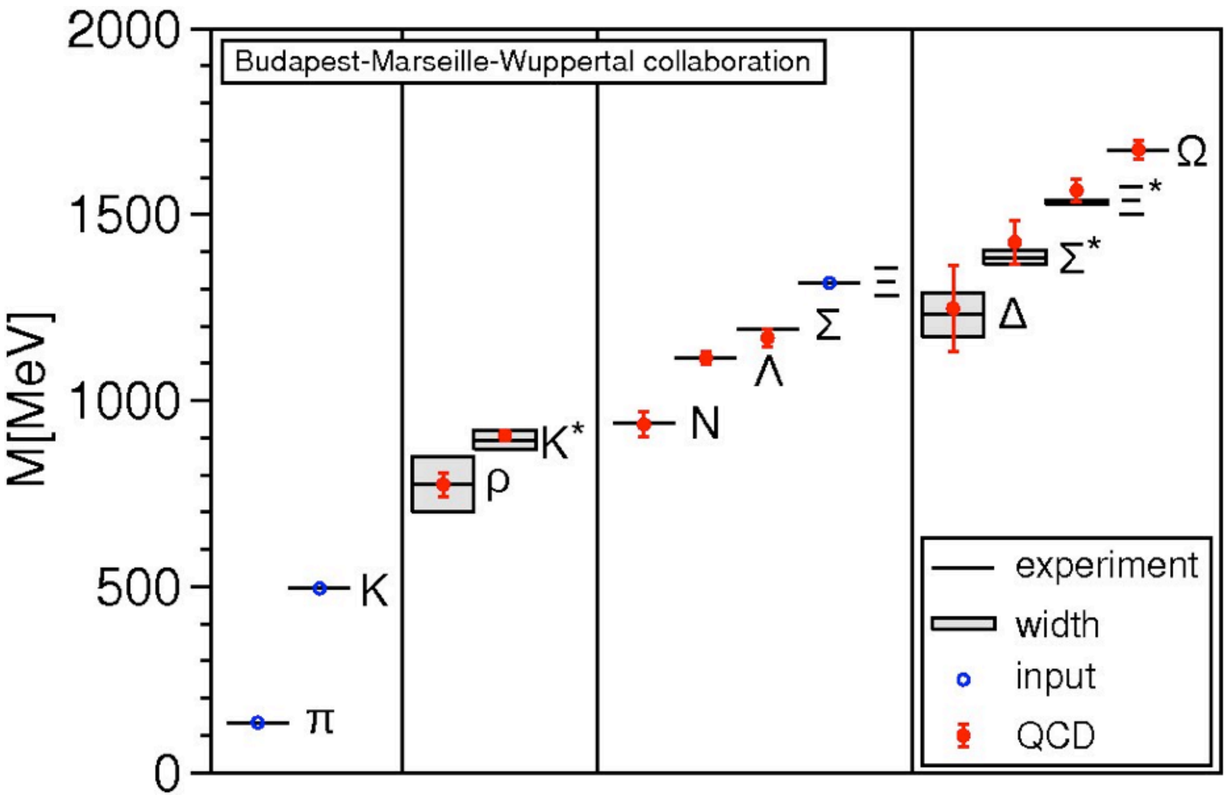
[Herdoíza summer 2015 + partial updates]

lattice QCD



[BMW Collaboration 2008]

lattice QCD



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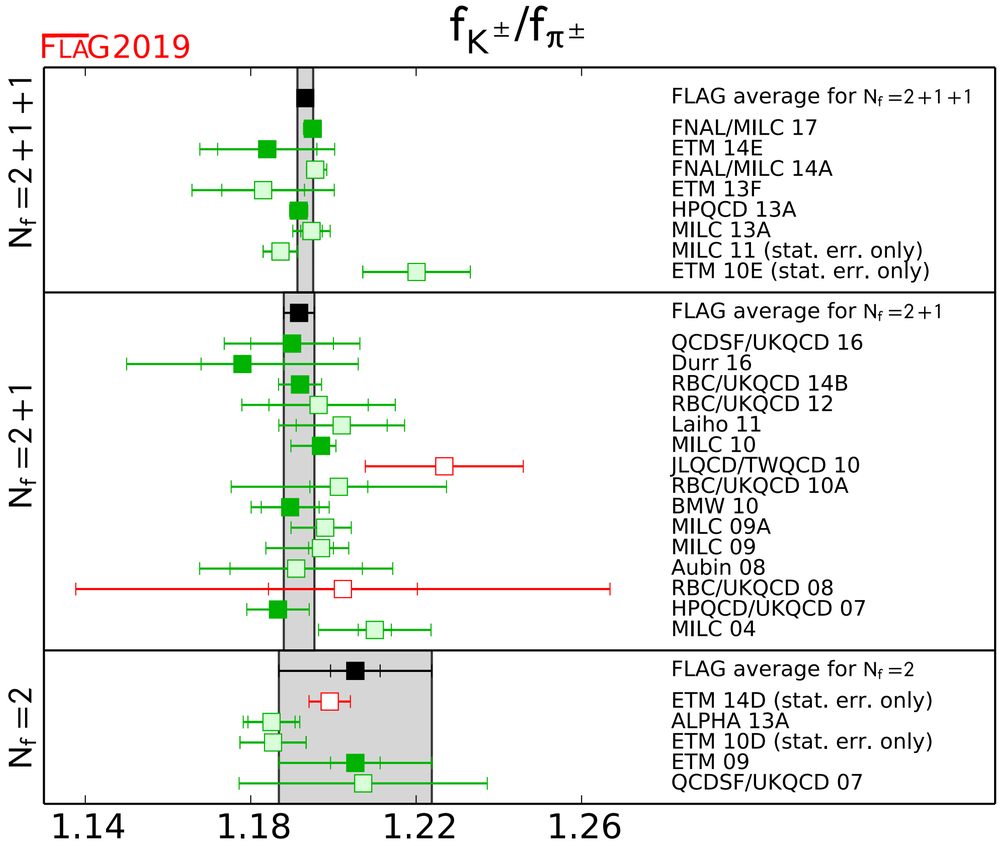
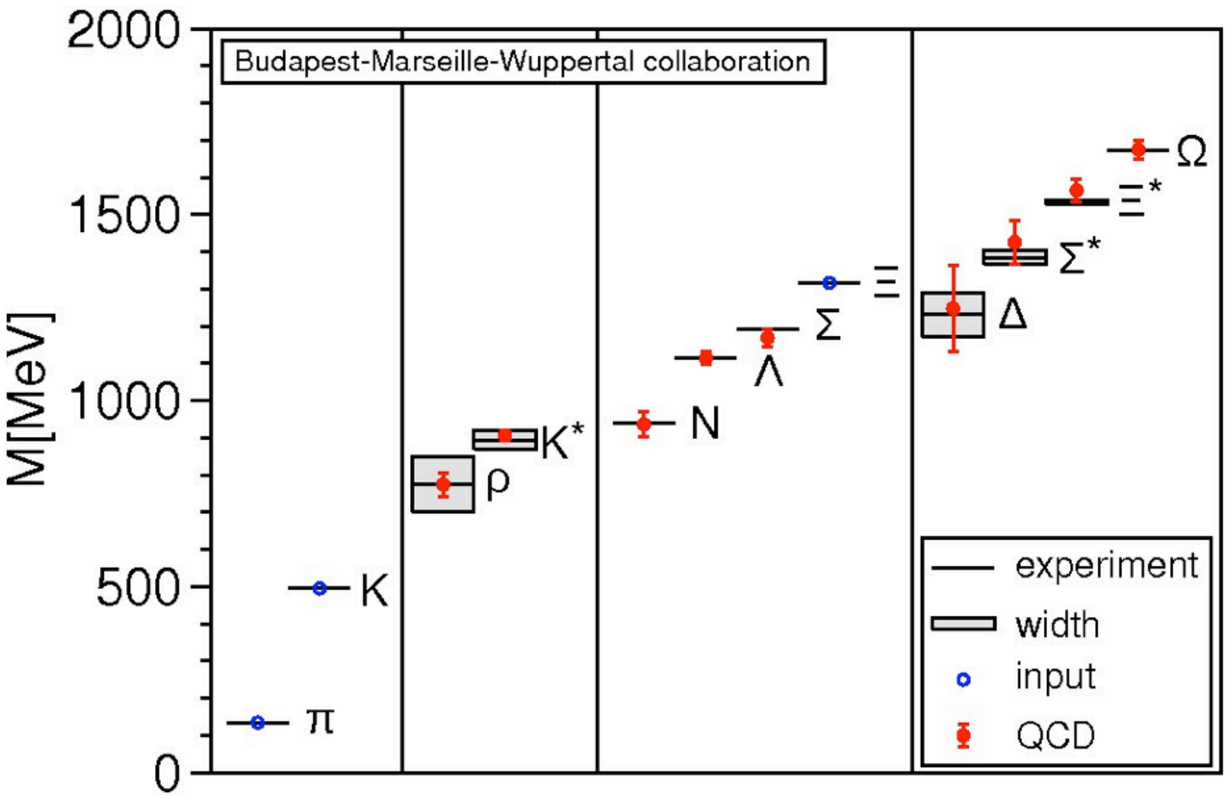


[Flavour Lattice Averaging Group 2019]

lattice QCD



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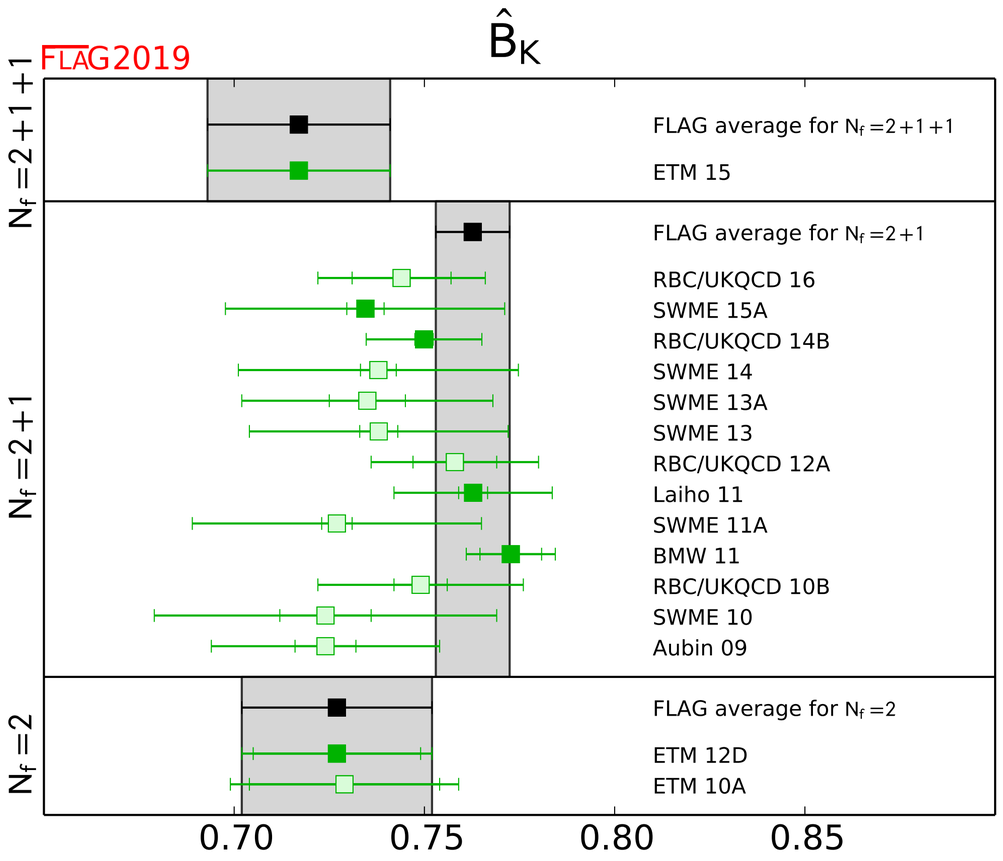

[Flavour Lattice Averaging Group 2019]

$K \to \pi\pi$ in lattice QCD: why is it so difficult?

two no-go theorems stand in our way

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Nielsen-Ninomiya: no ultralocal lattice regularisation preserves the full chiral symmetry group

[Nielsen, Ninomiya 1982]

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- use regularisations with exact chiral symmetry (not ultralocal), or better chiral properties

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- avoid by working at large finite volume to disentangle pion rescattering effects (requires volumes being reached only recently)

  [Lellouch, Lüscher 1998]
  [Lin, Martinelli, Sachrajda, Testa 2001]

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[Maiani, Testa 1990]

- use effective low-energy description of H_{eff} in χPT to relate $K \to \pi\pi$ amplitudes to computable quantities

  [Bernard et al. 1985]

- avoid by working at large finite volume to disentangle pion rescattering effects (requires volumes being reached only recently)

  [Lellouch, Lüscher 1998]
  [Lin, Martinelli, Sachrajda, Testa 2001]

$K \to \pi\pi$: state of the art

far-reaching effort by RBC/UKQCD collaboration

- direct CP violation:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{aligned} = \quad & 1.38(5.15)(4.43) \times 10^{-4}, && \text{(this work)} \\ & 16.6(2.3) \times 10^{-4} && \text{(experiment)} \end{aligned}}$$

[Bai et al., PRL 115 (2015) 212001]

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[Bai et al., PRL 115 (2015) 212001]

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (15 \pm 7) \times 10^{-4}$$

χPT-based, large log effect from final state interactions

[Gisbert, Pich 2017]

$$= (14 \pm 5) \times 10^{-4}$$

[Cirigliano, Gisbert, Pich, Rodríguez Sánchez today]

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.9 \pm 4.5) \times 10^{-4}$$

dual QCD-based

[Buras, Gorbahn, Jäger, Jamin 2015]

$K \to \pi\pi$: state of the art

ε'/ε-2018: A Christmas Story

Andrzej J. Buras

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Physik Department, TU München, James-Franck-Straße, D-85748 Garching, Germany
E-mail: aburas@ph.tum.de

Abstract

I was supposed to review the status of ε'/ε both at the CKM Workshop in September in Heidelberg and recently at the Discrete 2018 Conference in Vienna. Unfortunately I had to cancel both talks for family reasons. My main goal in these talks was to congratulate NA48 and KTeV collaborations for the discovery of new sources of CP violation through their heroic efforts to measure the ratio ε'/ε in the 1980s and 1990s with final results presented roughly 16 years ago. As I will not attend any other conferences this year I will reach this goal in this writing. In this context I will give arguments, why I am convinced about the presence of new physics in ε'/ε on the basis of my work with Jean-Marc Gérard within the context of the Dual QCD (DQCD) approach and why RBC-UKQCD lattice QCD collaboration and in particular Chiral Perturbation Theory practitioners are still unable to reach this conclusion. I will demonstrate that even in the presence of pion loops, as large as advocated recently by Gisbert and Pich, the value of ε'/ε is significantly below the data, when the main non-factorizable QCD dynamics at long distance scales, represented in DQCD by *the meson evolution*, is taken into account. As appropriate for a Christmas story, I will prophesy the final value of ε'/ε within the SM, which should include in addition to the correct matching between long and short distance contributions, isospin breaking effects, NNLO QCD corrections to both QCD penguin and electroweak penguin contributions and final state interactions. Such final SM result will probably be known from lattice QCD only in the middle of the 2020s, but already in 2019 we should be able to see some signs of NP in the next result on ε'/ε from RBC-UKQCD. In this presentation I try to avoid, as much as possible, the overlap with my recent review of Dual QCD in [1].

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$K \to \pi\pi$: state of the art

far-reaching effort by RBC/UKQCD collaboration

- direct CP violation:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{aligned} = \quad & 1.38(5.15)(4.43) \times 10^{-4}, && \text{(this work)} \\ & 16.6(2.3) \times 10^{-4} && \text{(experiment)} \end{aligned}}$$

[Bai et al., PRL 115 (2015) 212001]

- ΔI=3/2 amplitude (Re(A_2)proportional to B_K in the chiral limit):

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

[Blum et al., PRD 91 (2015) 074502]

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- "emerging understanding of ΔI=1/2 rule"

[Boyle et al., PRL 110 (2013) 152001]

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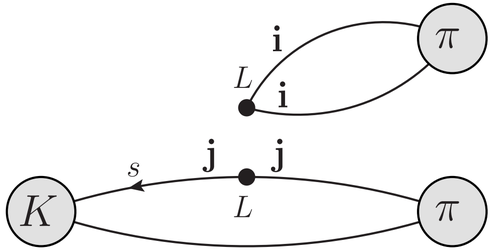
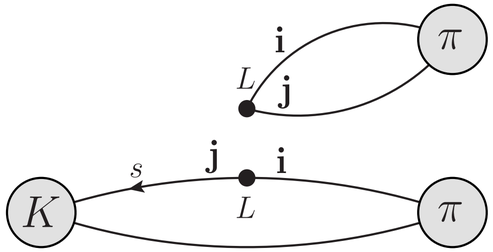


Contraction ①



Contraction ②

- Naive factorisation approach: ② $\sim 1/3$①
- Our computation: ② $\sim -0.7$①

- "emerging understanding of $\Delta I{=}1/2$ rule"

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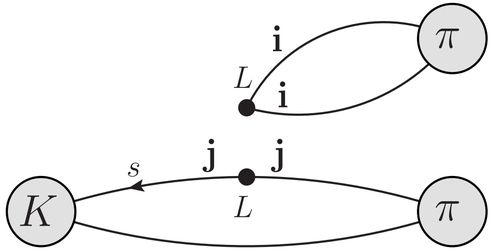

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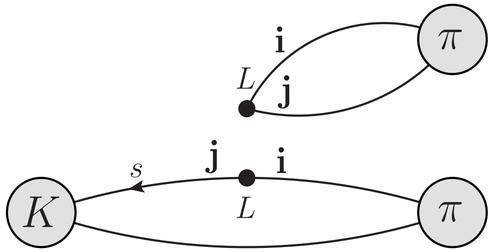

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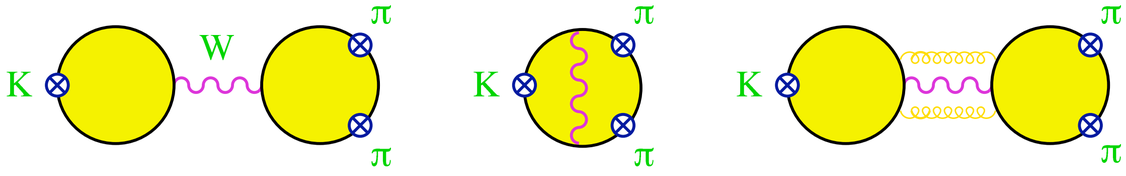

$O(N_C^2)$ $O(N_C)$ $O(1)$

$$T[K^0 \to \pi^0\pi^0] \sim 0 \quad \Rightarrow \quad \left|\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$

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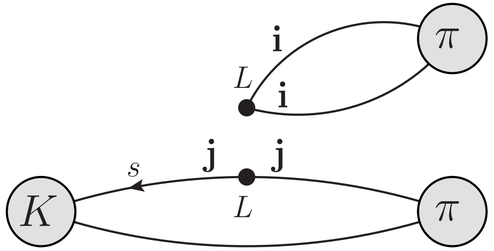

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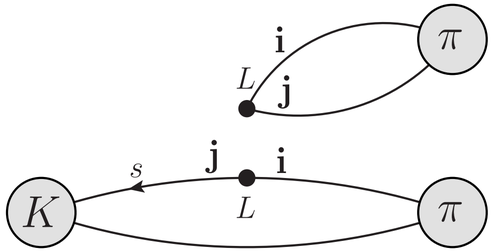

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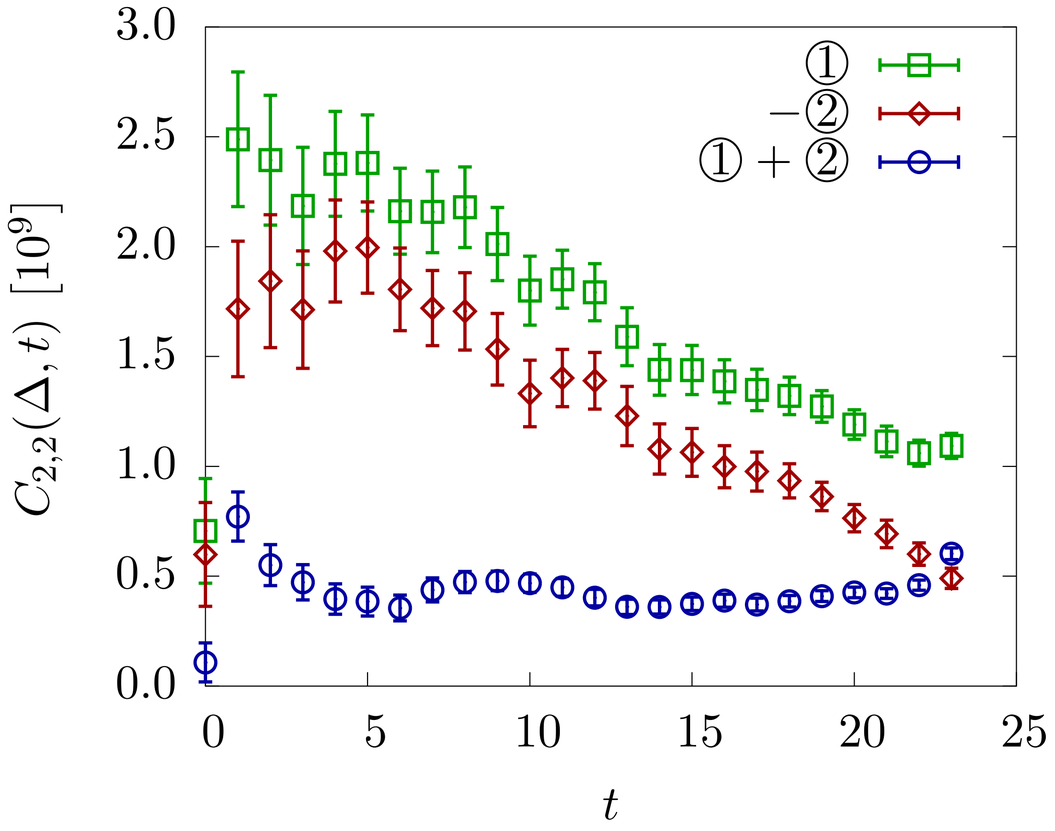

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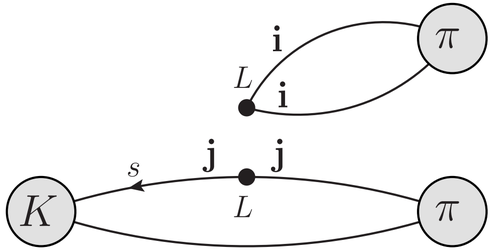

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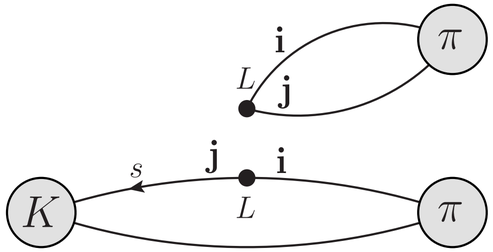

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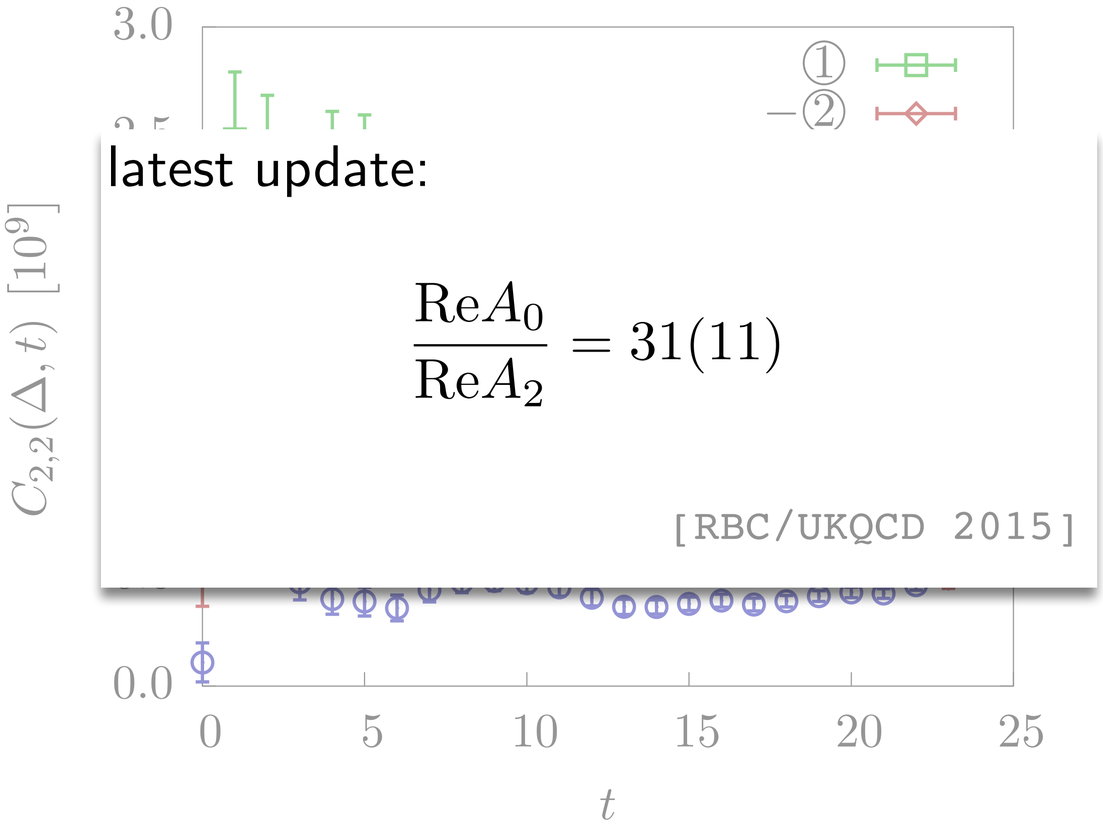

latest update:

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 31(11)$$

[RBC/UKQCD 2015]

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plan

- phenomenology of neutral kaon decay

- understanding non-leptonic kaon decay within the SM
  - electroweak effective Hamiltonian analysis
  - exact (lattice) vs. approximate (effective theory/large N/models)
  - why is it so hard?
  - state-of-the-art quantitative results

- understanding the anatomy of ΔI=1/2
  - the strategy
  - (old) results for QCD amplitudes
  - large N_c
  - insight into light meson physics

- outlook

based on:

- A. Donini, P. Hernández, CP, **F. Romero-López**, to appear.
- P. Hernández, CP, **F. Romero-López** , *Large N_c scaling of meson masses and decay constants*, EPJC 79 (2019) 865.
- A. Donini, P. Hernández, CP, **F. Romero-López**, Nonleptonic kaon decays at large N_c, PRD 94 (2016) 114511.

- E. Endress, CP, *Exploring the role of the charm quark in the ΔI=1/2 rule,* PRD 90 (2014) 094504.
- P. Hernandez, M. Laine, CP, E. Torró, J. Wennekers, H. Wittig, *Determination of the ΔS = 1 weak Hamiltonian in the SU(4) chiral limit through topological zero-mode wave functions*, JHEP 0805 (2008) 043.
- L. Giusti, P. Hernández, M. Laine, CP, J. Wennekers, H. Wittig, *On $K\to\pi\pi$ amplitudes with a light charm quark,* PRL 98 (2007) 082003.

anatomy of ΔI=1/2

[Giusti, Hernández, Laine, Weisz, Wittig 2004]

several possible sources for ΔI=1/2 enhancement:

- physics at charm scale (penguins)
- physics at "intrinsic" QCD scale $\sim \Lambda_{\rm QCD}$
- final state interactions
- all of the above (no dominating "mechanism")

separate low-energy QCD and charm-scale physics: consider amplitudes as a function of charm mass for fixed u,d,s masses

$$m_c = m_u = m_d = m_s \quad \longrightarrow \quad m_c \gg m_u = m_d \leq m_s$$

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implementation (Mark I):

- active charm
- use chiral fermions (good renormalisation, access to very low masses)
- $\Rightarrow$ give up (too expensive) direct computation, use ChiPT $\Rightarrow$ no FSI

anatomy of ΔI=1/2

dynamics of Goldstone bosons at LO given by chiral Lagrangian

$$\mathcal{L} = \tfrac{1}{4}F^2 \mathrm{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] \; - \; \tfrac{1}{2}\Sigma \mathrm{Tr}\left[U M^\dagger e^{i\theta/N_\mathrm{f}} \; + \; \mathrm{h.c.}\right]$$

weak interactions accounted for by low-energy version of effective Hamiltonian

light charm:

$$\mathcal{H}_\mathrm{w}^{(4)} = \frac{g_\mathrm{w}^2}{4M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} \{g_1^\sigma \mathcal{Q}_1^\sigma \; + \; g_2^\sigma \mathcal{Q}_2^\sigma\}$$

$$\mathcal{Q}_1^\pm = \mathcal{J}_\mu^{su}\mathcal{J}_\mu^{ud} \pm \mathcal{J}_\mu^{sd}\mathcal{J}_\mu^{uu} \; - \; [u \leftrightarrow c] \qquad \mathcal{J}_\mu = \frac{F^2}{\sqrt{2}} U \partial_\mu U^\dagger$$

heavy charm:

$$\mathcal{H}_\mathrm{w}^{(3)} = \frac{g_\mathrm{w}^2}{4M_W^2} V_{us}^* V_{ud} \left\{g_{27}\mathcal{Q}_{27} \; + \; g_8 \mathcal{Q}_8 \; + \; g_8' \mathcal{Q}_8'\right\}$$

$$\mathcal{Q}_{27} = \frac{2}{5}\mathcal{J}_\mu^{su}\mathcal{J}_\mu^{ud} + \frac{3}{5}\mathcal{J}_\mu^{sd}\mathcal{J}_\mu^{uu}\,,$$

$$\mathcal{Q}_8 = \frac{1}{2}\sum_{q=u,d,s} \mathcal{J}_\mu^{sq}\mathcal{J}_\mu^{qd}\,,$$

$$\mathcal{Q}_8' = m_l \Sigma F^2 \left[U e^{i\theta/N_\mathrm{f}} + U^\dagger e^{-i\theta/N_f}\right]^{sd}\,,$$

anatomy of ΔI=1/2

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$$g_{27}(0) = g_1^+ \,, \qquad g_8(0) = g_1^- + \frac{1}{5} g_1^+$$

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$$|g_{27}^\mathrm{exp}(\overline{m}_c)| \sim 0.50 \,, \qquad |g_8^\mathrm{exp}(\overline{m}_c)| \sim 10.5$$

match to experiment @ LO in χPT

determining weak LECs: $m_u = m_c$

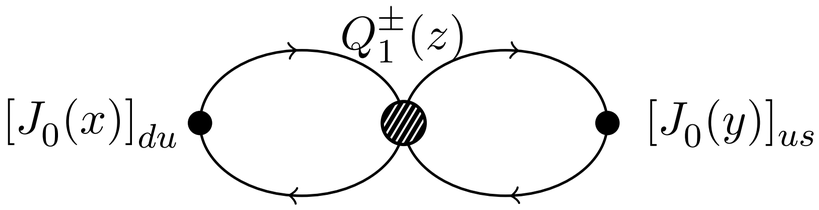
match suitable correlation functions in QCD and ChPT (infinite volume: $K \to \pi$ amplitudes)

QCD

$$R_i^{\pm}(x_0, y_0) = \frac{C_i^{\pm}(x_0, y_0)}{C(x_0)C(y_0)}$$

$$C_i^{\pm}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle J_0^{du}(x) \, Q_i^{\pm}(0) \, J_0^{us}(y) \rangle$$

$$C(x_0) = \int \mathrm{d}^3x \, \langle J_0^{\alpha\beta}(x) \, J_0^{\beta\alpha}(0) \rangle \, ,$$



SU(4) χPT

$$\mathcal{R}_i^{\pm}(x_0, y_0) = \frac{\mathcal{C}_i^{\pm}(x_0, y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

$$\mathcal{C}(x_0) = \int \mathrm{d}^3x \, \langle \mathcal{J}_0^{ud}(x) \, \mathcal{J}_0^{du}(0) \rangle_{\mathrm{SU}(4)} \, ,$$

$$\mathcal{C}_i^{\pm}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x) \, \mathcal{Q}_i^{\pm}(0) \, \mathcal{J}_0^{us}(y) \rangle_{\mathrm{SU}(4)}$$

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$$\mathcal{C}_i^{\pm}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x) \, \mathcal{Q}_i^{\pm}(0) \, \mathcal{J}_0^{us}(y) \rangle_{\mathrm{SU}(4)}$$

$$\mathcal{Z}_1^{\pm} R_1^{\pm}(x_0, y_0) = g_1^{\pm} \mathcal{R}_1^{\pm}(x_0, y_0)$$

determining weak LECs: $m_u \neq m_c$

match suitable correlation functions in QCD and ChPT (infinite volume: $K \to \pi$ amplitudes)

QCD

$$R_{27} = \mathcal{Z}_1^+ R_u^+ \,,$$
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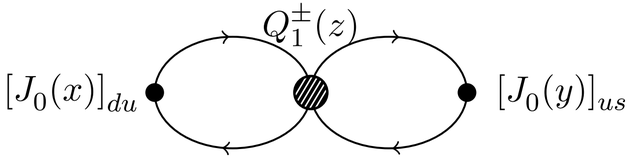

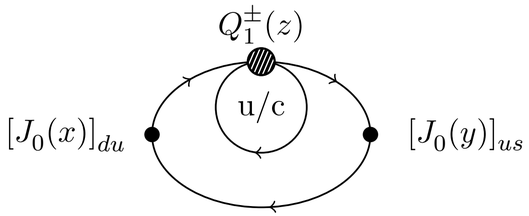

SU(3) χPT

$$\mathcal{R}_i^\pm(x_0, y_0) = \frac{\mathcal{C}_i^\pm(x_0, y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

$$\mathcal{C}_{27}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_{27}(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}} \,,$$
$$\mathcal{C}_8(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_8(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}} \,,$$
$$\mathcal{C}_8'(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_8'(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}} \,,$$

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$$C_i^\pm(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle J_0^{du}(x)\, Q_i^\pm(0)\, J_0^{us}(y)\rangle$$
$$C(x_0) = \int \mathrm{d}^3x \, \langle J_0^{\alpha\beta}(x)\, J_0^{\beta\alpha}(0)\rangle \,,$$
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SU(3) χPT

$$\mathcal{R}_i^\pm(x_0, y_0) = \frac{\mathcal{C}_i^\pm(x_0, y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

$$\mathcal{C}_{27}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_{27}(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}} \,,$$
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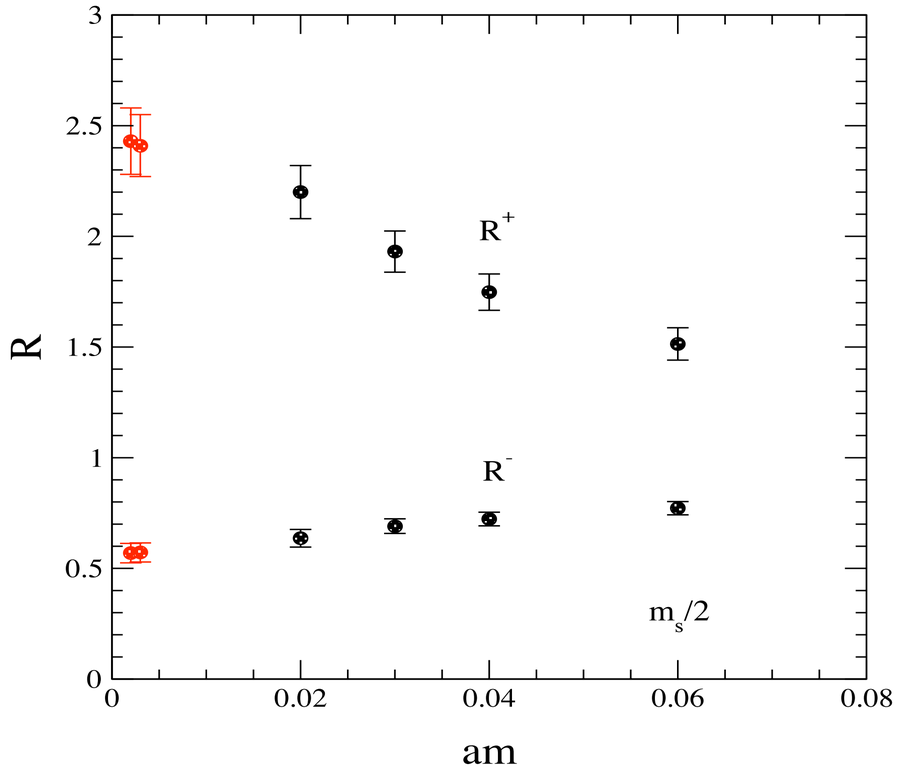
$$R_{27}(x_0, y_0) = g_{27} \mathcal{R}_{27}(x_0, y_0) \,,$$
$$R_8(x_0, y_0) = g_8 \mathcal{R}_8(x_0, y_0) + g_8' \mathcal{R}_8'(x_0, y_0)$$

quenched overlap results

[Giusti, Hernández, Laine, CP, Wennekers, Wittig 2007]

- fixed $a \sim 0.12$ fm
- sophisticated variance reduction techniques
- computations spanning both *p*- and *ε*-regime

	g^+	g^-
This work	$0.51(3)(5)(6)$	$2.6(1)(3)(3)$
"Exp"	~ 0.5	~ 10.4
Large N_c	1	1

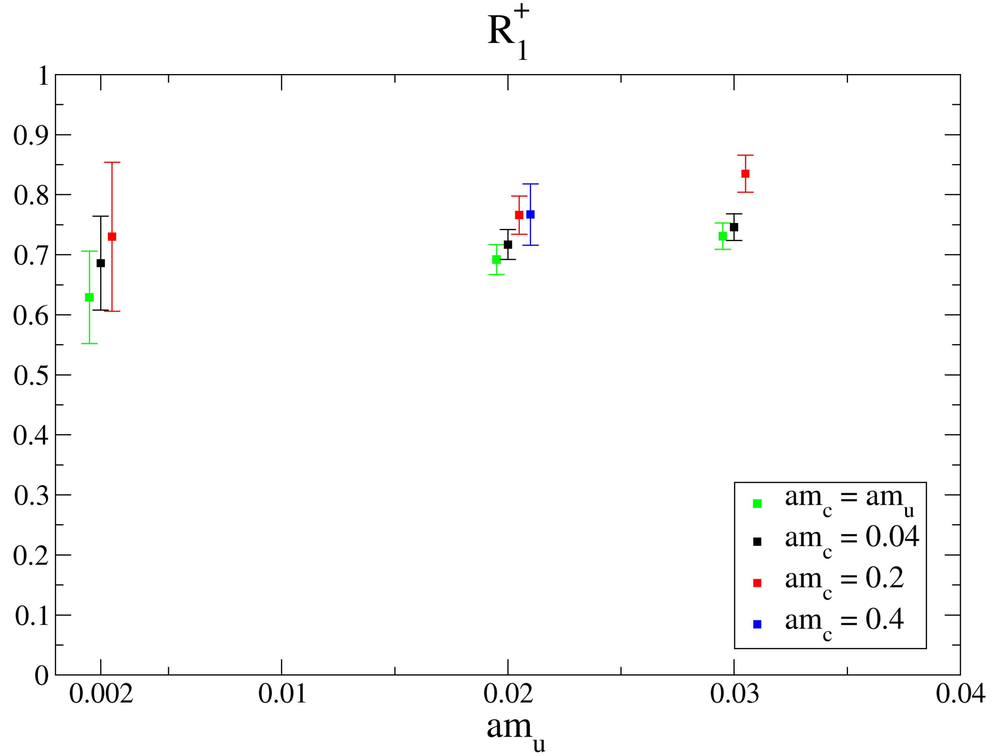


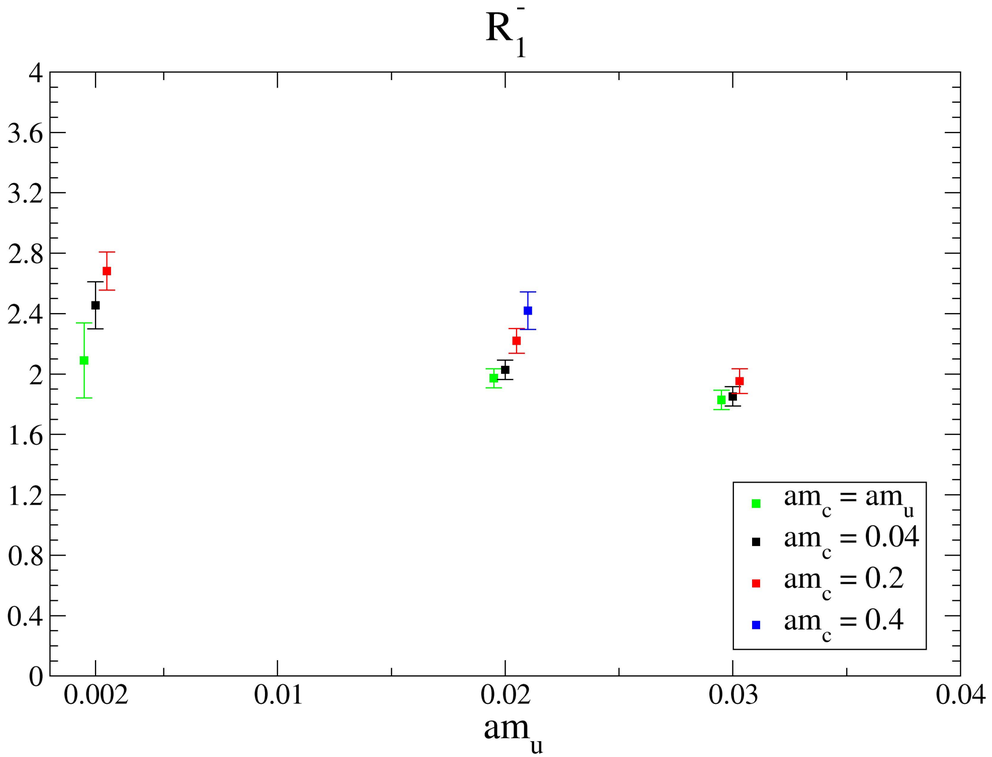
- large chiral corrections, consistent with χPT prediction
- ΔI=3/2 in the right ballpark (n.b. charm enters only via loops / quenching subdominant [?])
- ΔI=1/2 about a factor 4 too small to reproduce physical enhancement
- remarkable enhancement of ΔI=1/2 channel present for light charm: pure "no-penguin" effect

quenched overlap results

[Endress, CP 2014]

- fixed $a \sim 0.12$ fm
- sophisticated variance reduction techniques
- computations spanning both p- and ε-regime
- add heavy(-ish) charm + perturbative operator mixing

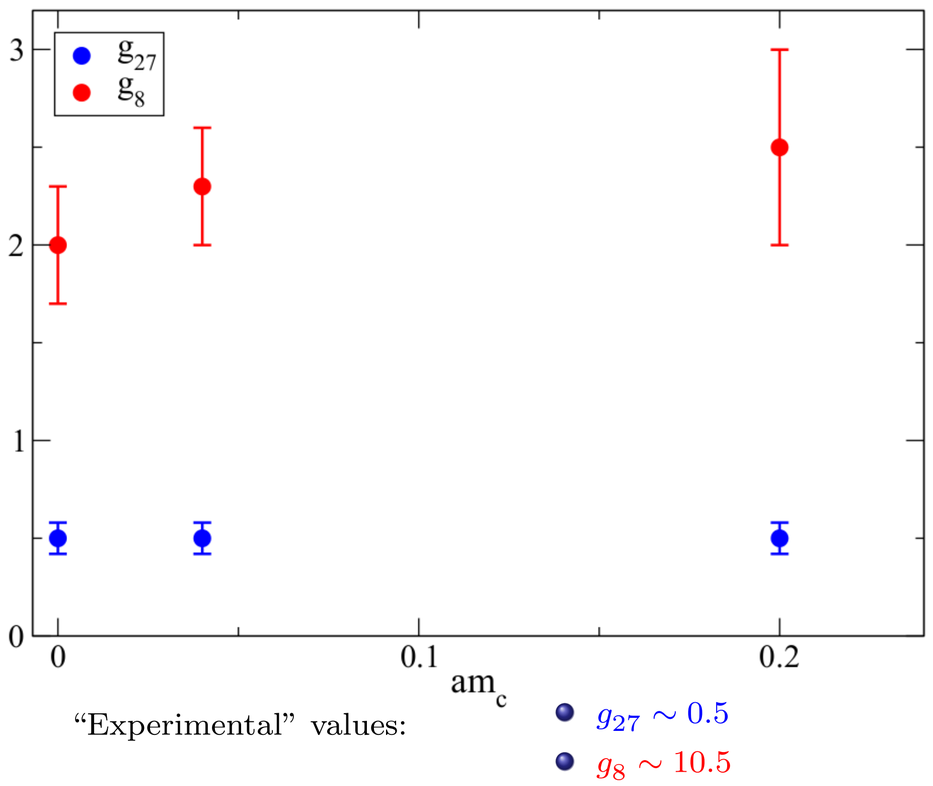




quenched overlap results

[Endress, CP 2014]

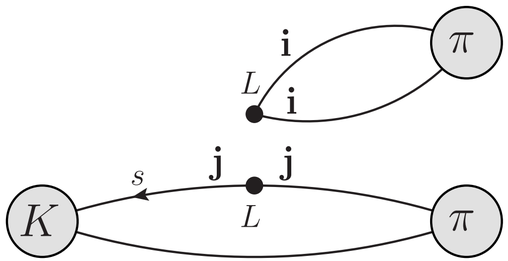
- fixed $a \sim 0.12$ fm
- sophisticated variance reduction techniques
- computations spanning both *p*- and *ε*-regime
- add heavy(-ish) charm + perturbative operator mixing



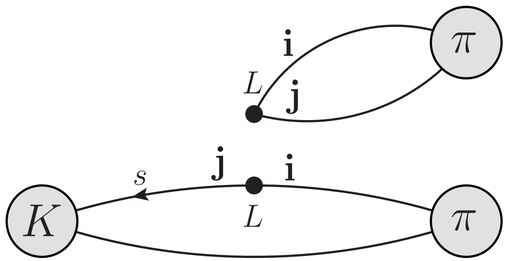
- large chiral corrections, consistent with χPT prediction
- Δ*I*=3/2 in the right ballpark (n.b. charm enters only via loops / quenching subdominant [?])
- Δ*I*=1/2 about a factor ~~4~~ 3.0-3.5 too small to reproduce physical enhancement
- heavy charm adds to the enhancement, but effect is moderate up to $m_c^{phys}/4$ - $m_c^{phys}/2$

ΔI=1/2 @ large N_c

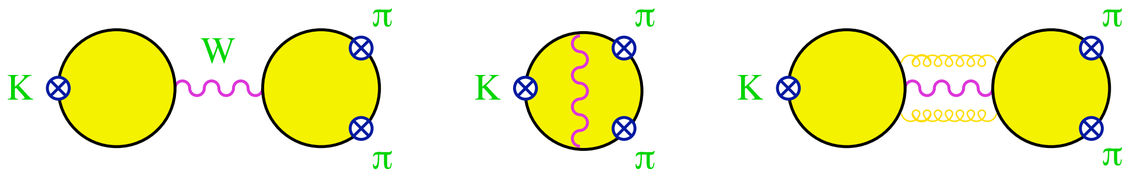
[RBC/UKQCD 2013-15]



Contraction ①

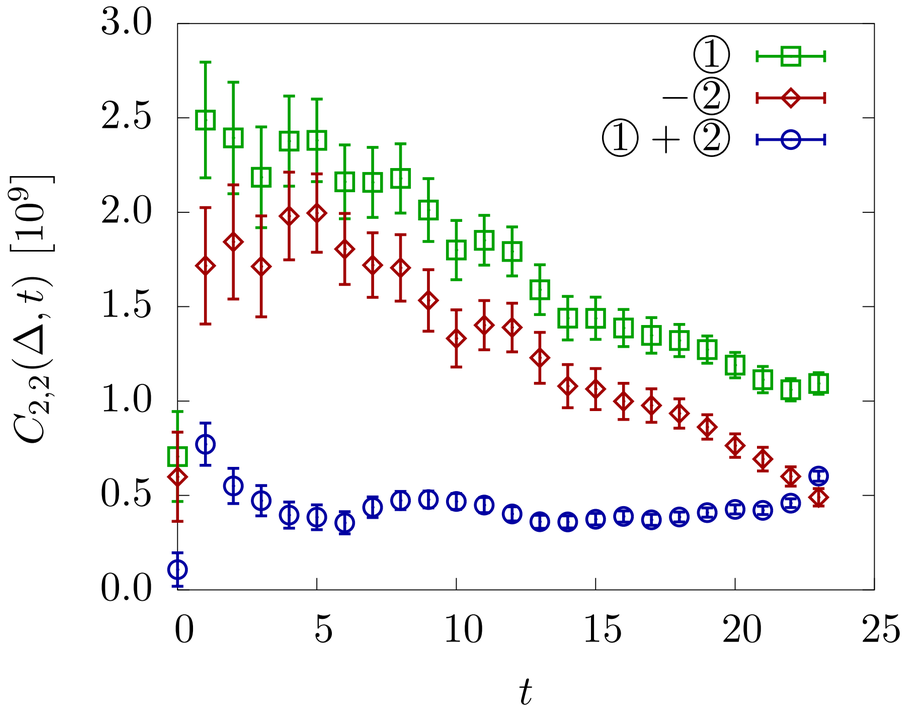


Contraction ②



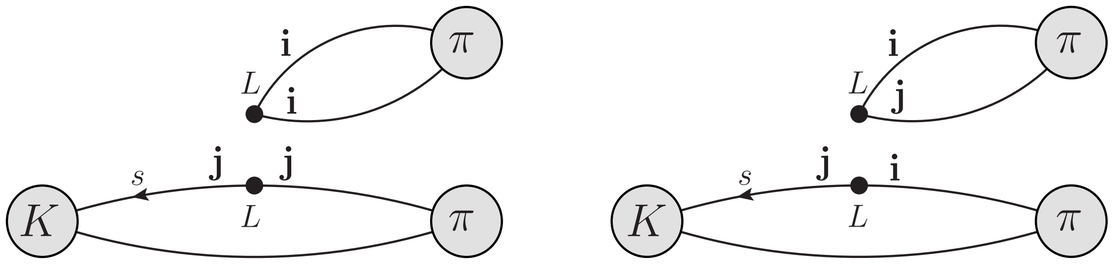
$O(N_C^2)$ $O(N_C)$ $O(1)$

$$T[K^0 \to \pi^0\pi^0] \sim 0 \quad \Rightarrow \quad \left|\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$



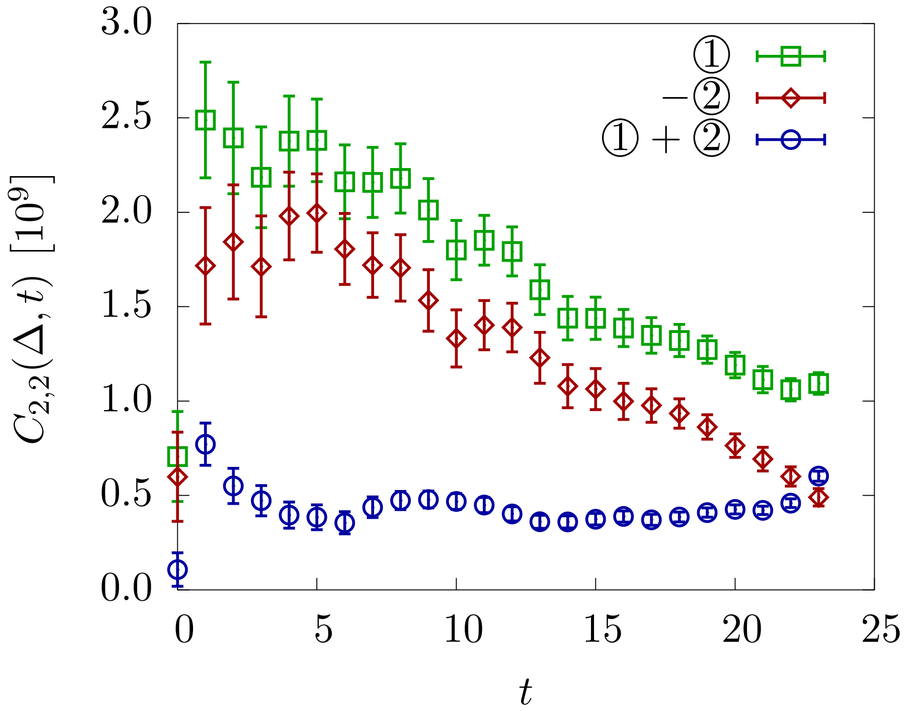
ΔI=1/2 @ large N_c

[RBC/UKQCD 2013-15]



Contraction ① Contraction ②

understanding by comparing connected-disconnected contributions to three-point functions difficult to interpret physically

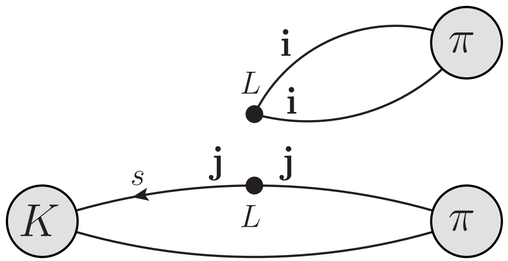


the leading large N_c scaling of each contribution is however different: connection with the physical amplitudes can be established by studying the N_c dependence

[Donini, Hernández, CP, Romero-López 2016]
[cf. also Blum et al., PRD 91 (2015) 074502]

ΔI=1/2 @ large N_c

[RBC/UKQCD 2013-15]



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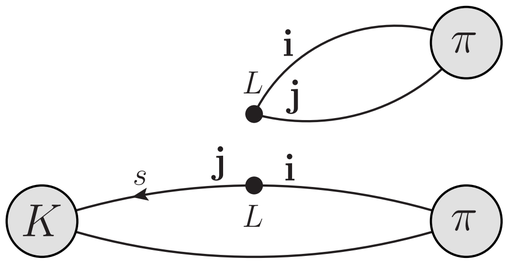

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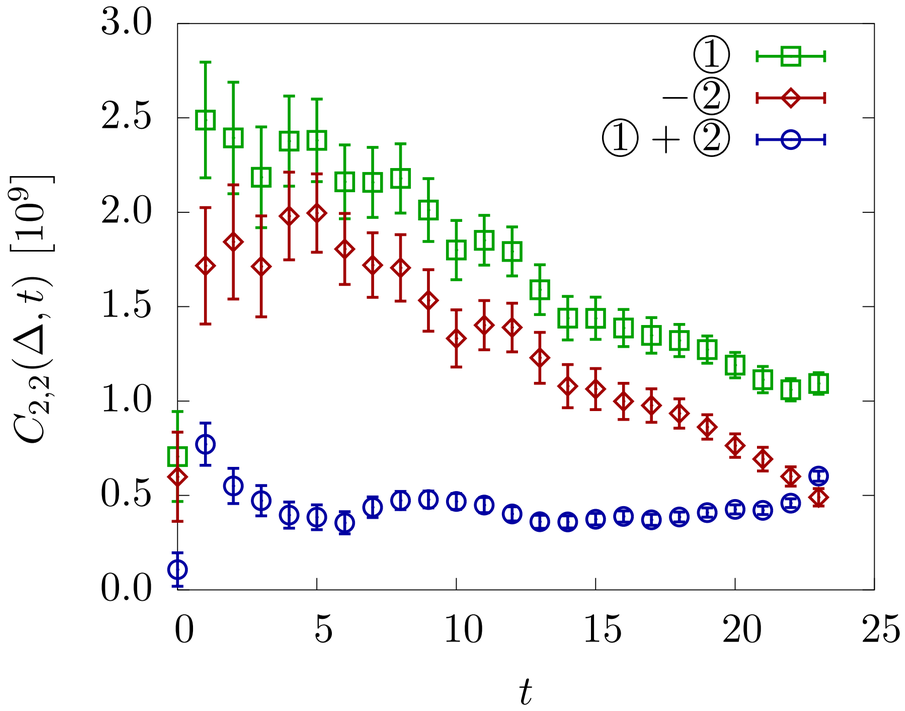
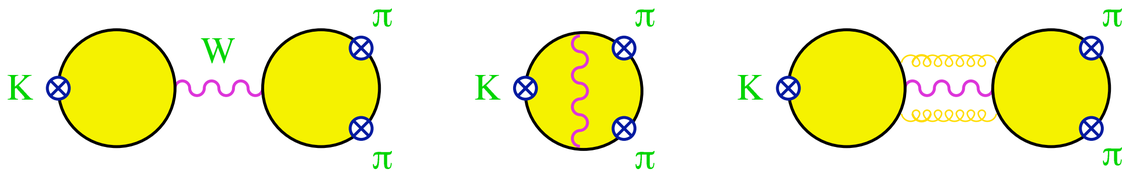
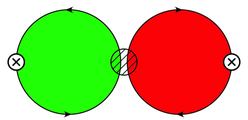




$O(N_C^2)$ $O(N_C)$ $O(1)$

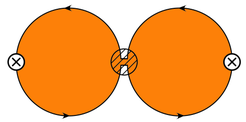
$$T[K^0 \to \pi^0\pi^0] \sim 0 \quad \Rightarrow \quad \left|\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$

$g^{\pm} \propto$



Color-disconnected $O(N_c^2)$

$\mp$

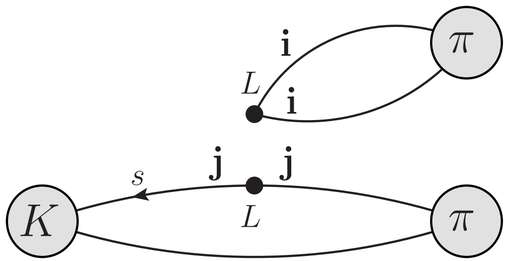


Color-connected $O(N_c)$

$$\frac{A_0}{A_2} = \frac{1}{2\sqrt{2}}\left(1 + 3\frac{g^-}{g^+}\right) \xrightarrow[g^+=g^-]{\text{Large } N_c} \sqrt{2}$$

ΔI=1/2 @ large N_c

[RBC/UKQCD 2013-15]



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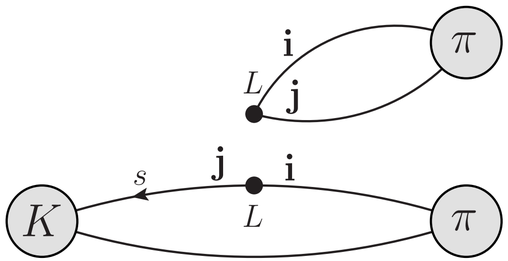

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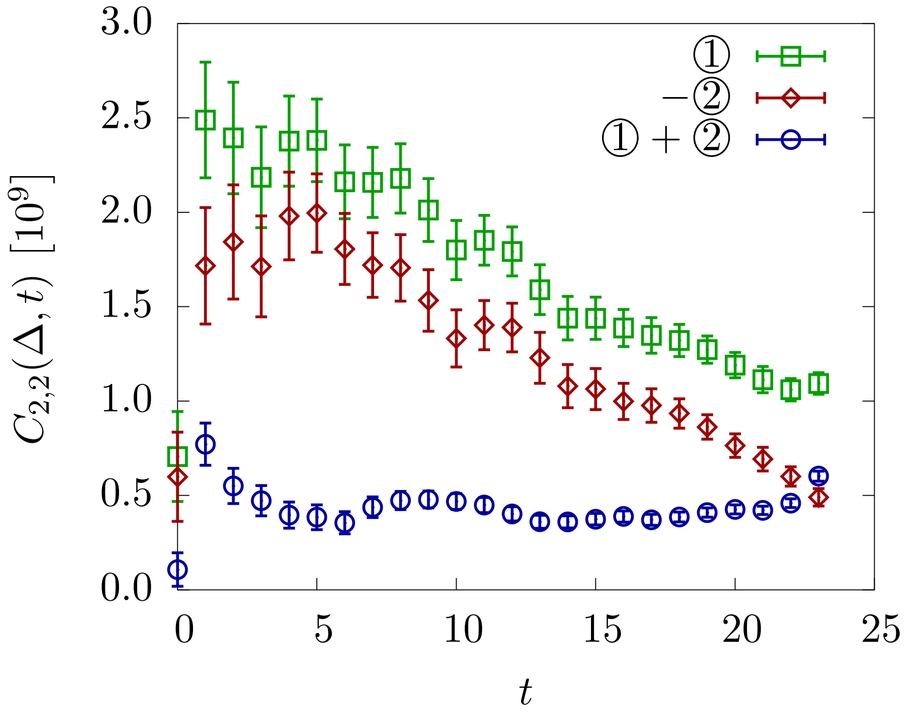

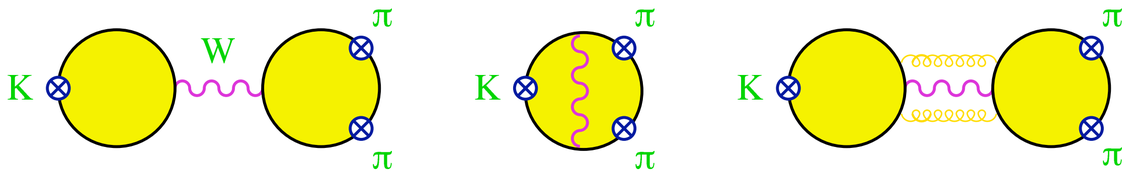

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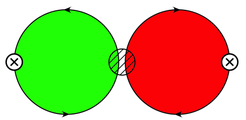
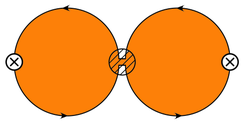


Color-disconnected $O(N_c^2)$

$\mp$



Color-connected $O(N_c)$

m_u=m_d=m_s limit: $\hat{B}_K = \frac{3}{4}\hat{R}^+$

ΔI=1/2 @ large N_c

n.b.: relation between kaon mixing and ΔI=3/2 decay amplitude holds outside the chiral limit for m_u=m_d=m_s, since in that case chiral logs coincide - at leading log

$$\left.\frac{\langle\pi^+\pi^0|H_W|K\rangle}{m_K^2-m_\pi^2}\right|_{m_s=m_d} = \frac{iF}{\sqrt{2}}A^+G_FV_{ud}V_{us}^*$$

$$\langle\pi^+\pi^0|H_W|K^+\rangle_{m_\pi\to 0} \;=\; m_K^2\,\left.\frac{\langle\pi^+\pi^0|H_W|K^+\rangle}{m_K^2-m_\pi^2}\right|_{m_s=m_d}\left(1+\frac{9}{4}\frac{m_K^2}{(4\pi F)^2}\log\frac{m_K^2}{(4\pi F)^2}\right)$$

[Golterman, Leung 1997]

thus, large N corrections to the physical amplitude are fixed by those in A^+

[Donoghue, Golowich, Holstein 1982; Bijnens, Sonoda, Wise 1984]

- caveat 1: in physical kinematics, chiral logs much larger for mixing amplitude
- caveat 2: higher-order ChiPT effects argued to be larger

[Truong 1988; Isgur, Maltman, Weinstein, Barnes 1990; Kambor, Missimer, Wyler 1991; Pallante, Pich 1998]

ΔI=1/2 @ large N_c: numerical study

- simulate for N_c=3,...,8 at fixed lattice spacing, change quark mass along m_u=m_d=m_s=m_c
  - **quenched:** use line of constant physics provided by Regensburg+Scotland+Wales study of meson physics [Bursa et al. 2013]
  - **dynamical:** use gradient flow scale t_0 to set constant physics

- use Wilson fermions for sea (HiRep code), twisted-mass QCD for valence [Hansen 2017]
  - twisted valence à la Frezzotti-Rossi allows to avoid mixing with wrong-chirality operators [Frezzotti, Rossi 2004]
  - mixed-action approach requires matching of valence and sea, performed with meson mass
  - check for residual cutoff effects by changing value of c_{sw} + ongoing simulation on finer lattice

- develop necessary SU(4) χPT to better understand meson dynamics
  - bonus: get large-N_c insight on LECs and meson interactions

[Hernández, CP, Romero-López 2016-2019]

ΔI=1/2 @ large N_c: numerical study

quenched simulations in 16^3 lattices at (roughly) constant PS mass

N_c	T/a	β	$am_{\rm PCAC}$	$am_{\rm PS}$	$R^+_{\rm bare}$	$R^-_{\rm bare}$
3	48	6.0175	-0.002(14)	0.2718(61)	0.774(21)	1.218(31)
4	48	11.028	-0.0015(11)	0.2637(39)	0.783(15)	1.198(19)
5	48	17.535	0.0028(9)	0.2655(31)	0.839(8)	1.145(12)
6	32	25.452	0.0013(7)	0.2676(28)	0.871(6)	1.125(7)
7	32	34.8343	-0.0034(6)	0.2819(19)	0.880(5)	1.122(5)

renormalisation (RI scheme) at scale around 2 GeV performed using one-loop P.T.

[Constantinou et al. 2011]
[Alexandrou et al. 2012]

perturbative two-loop RG running in RI to connect to RGIs

[Ciuchini et al. 1998]
[Buras et al. 2000]

ΔI=1/2 @ large N_c: numerical study

quenched simulations in 16^3 lattices at (roughly) constant PS mass, string tension

N_c	T/a	β	$am_{\rm PCAC}$	$am_{\rm PS}$	$R^+_{\rm bare}$	$R^-_{\rm bare}$
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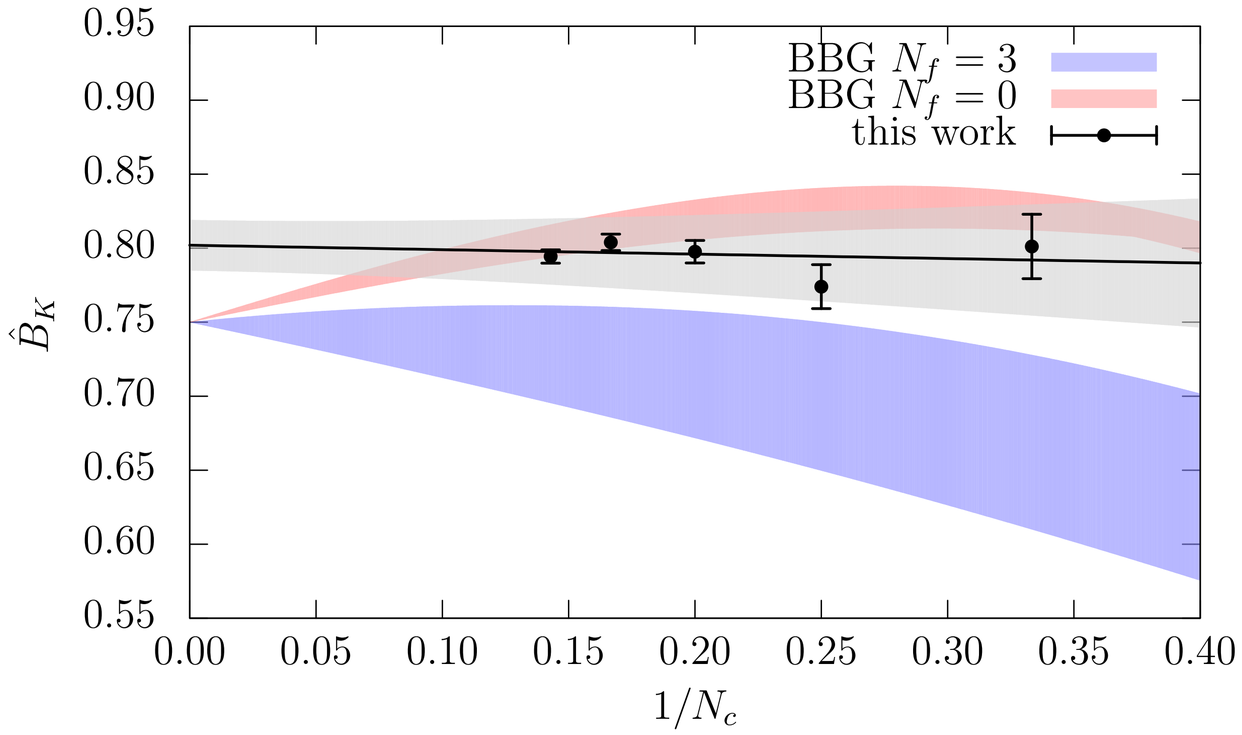

essentially flat scaling, consistent with model by Buras, Gérard, Bardeen

[Bardeen, Buras, Gérard 2014]

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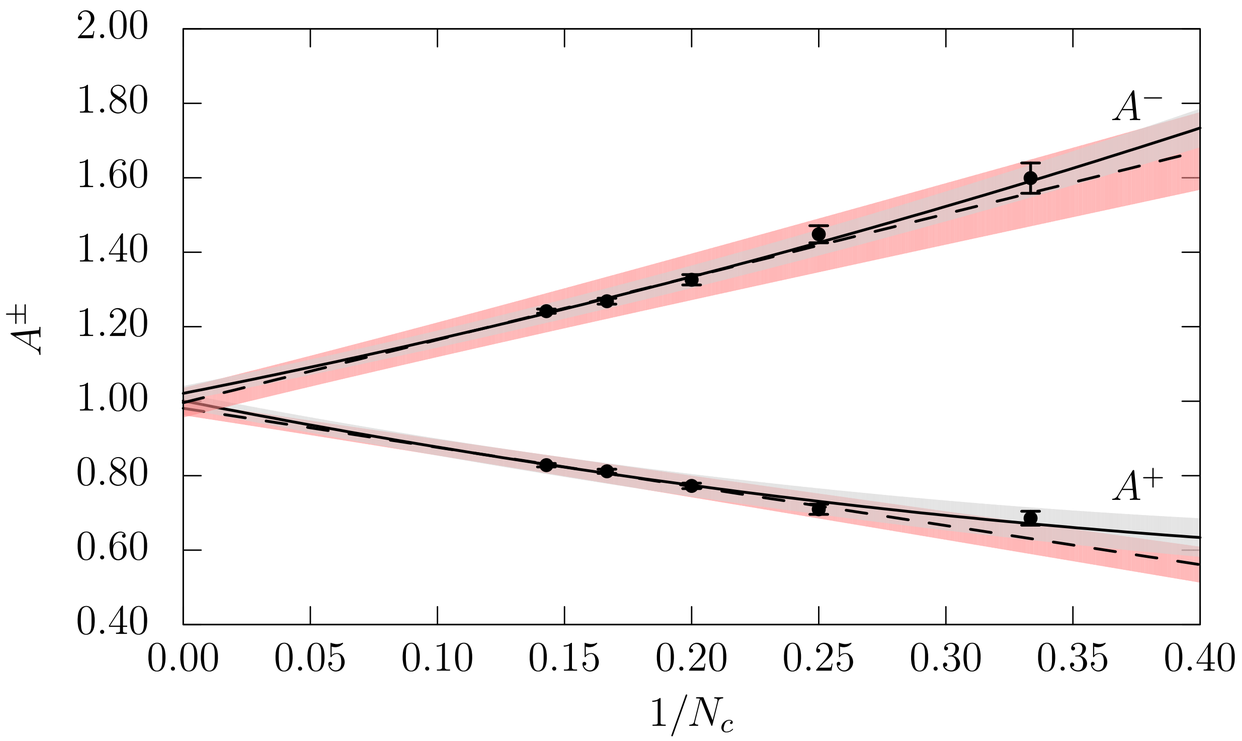

- very linear behaviour in N_c
- expected large N_c limit, corrections at N_c=3 in 30% ballpark
- strong anticorrelation of corrections for the two amplitudes

ΔI=1/2 @ large N_c: numerical study

quenched simulations in 16^3 lattices at (roughly) constant PS mass, string tension

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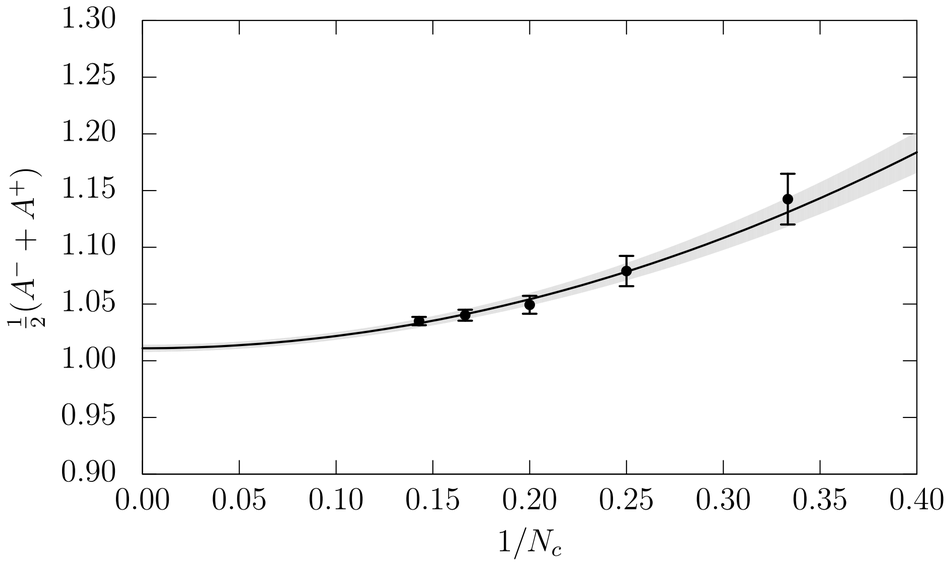

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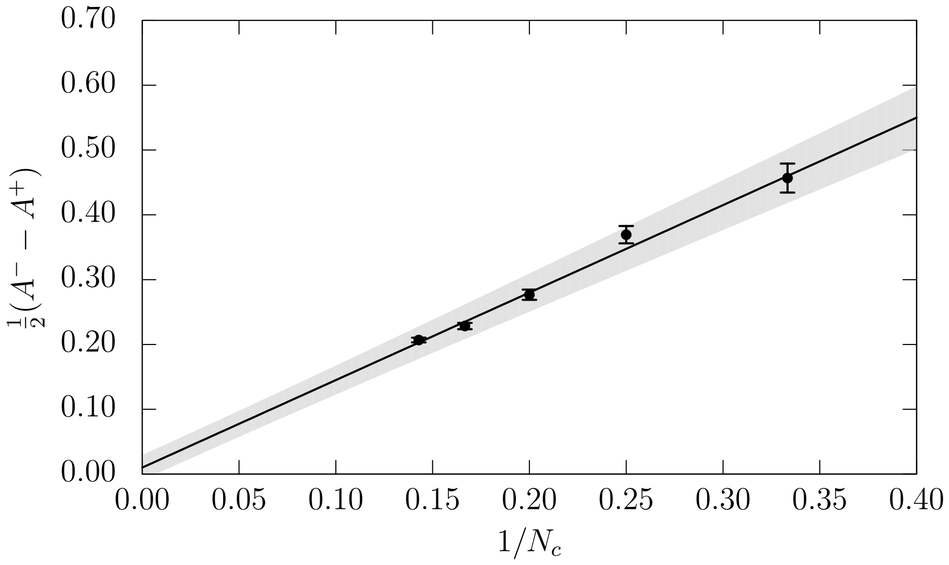

perturbative two-loop RG running in RI to connect to RGIs

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ΔI=1/2 @ large N_c: numerical study

dynamical simulations at varying PS mass

(+ extra quenched points)

Ensemble	N_c	$L \times T$	β	m_0	aM	M (MeV)
A301	3	20×36	1.778	-0.4040	0.2191(36)	570
A302		24×48		-0.4060	0.1831(17)	480
A303		24×48		-0.4070	0.1612(24)	420
A304		32×60		-0.4080	0.1384(15)	360
A401	4	20×36	3.570	-0.3725	0.2035(14)	530
A402		24×48		-0.3752	0.1804(7)	470
A403		24×48		-0.3760	0.1714(8)	440
A404		32×60		-0.3780	0.1397(8)	360
A501	5	20×36	5.969	-0.3458	0.2128(9)	560
A502		24×48		-0.3490	0.1802(6)	470
A503		24×48		-0.3500	0.1712(6)	450
A504		32×60		-0.3530	0.1328(8)	350
A601	6	20×36	8.974	-0.3260	0.2150(7)	570
A602		24×48		-0.3300	0.1801(5)	470
A603		24×48		-0.3311	0.1690(7)	450
A604		32×60		-0.3340	0.1354(7)	360

other technicalities as before

ΔI=1/2 @ large N_c: numerical study

dynamical simulations at varying PS mass

(+ extra quenched points)

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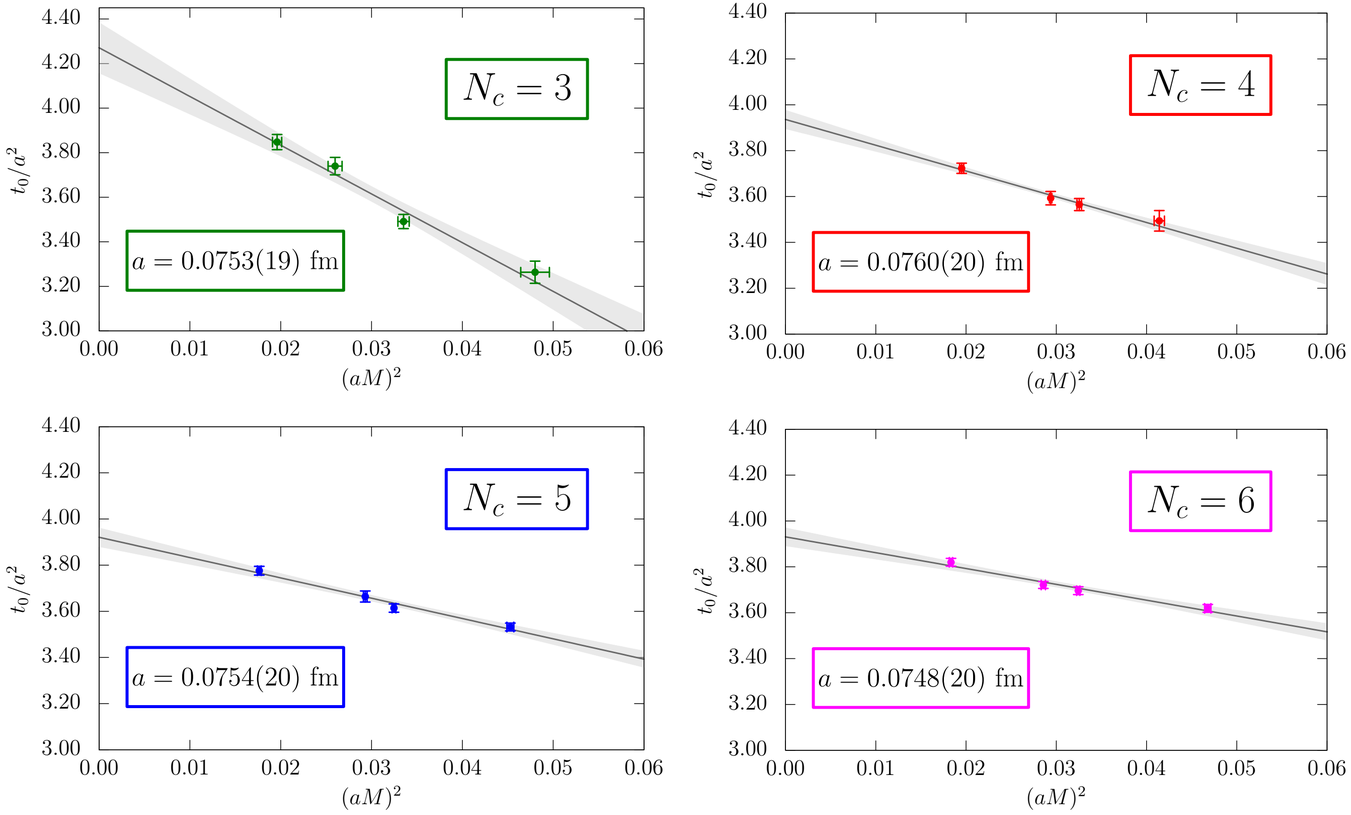
other technicalities as before



$$\langle t^2 E(t) \rangle_{t=t_0} = 0.1125 \frac{N_c^2 - 1}{N_c}$$

$$(M\sqrt{t_0})_{M=420\ \mathrm{MeV}} = 0.3090(83)$$

ΔI=1/2 @ large N_c: numerical study

dynamical simulations at varying PS mass
(+ extra quenched points)

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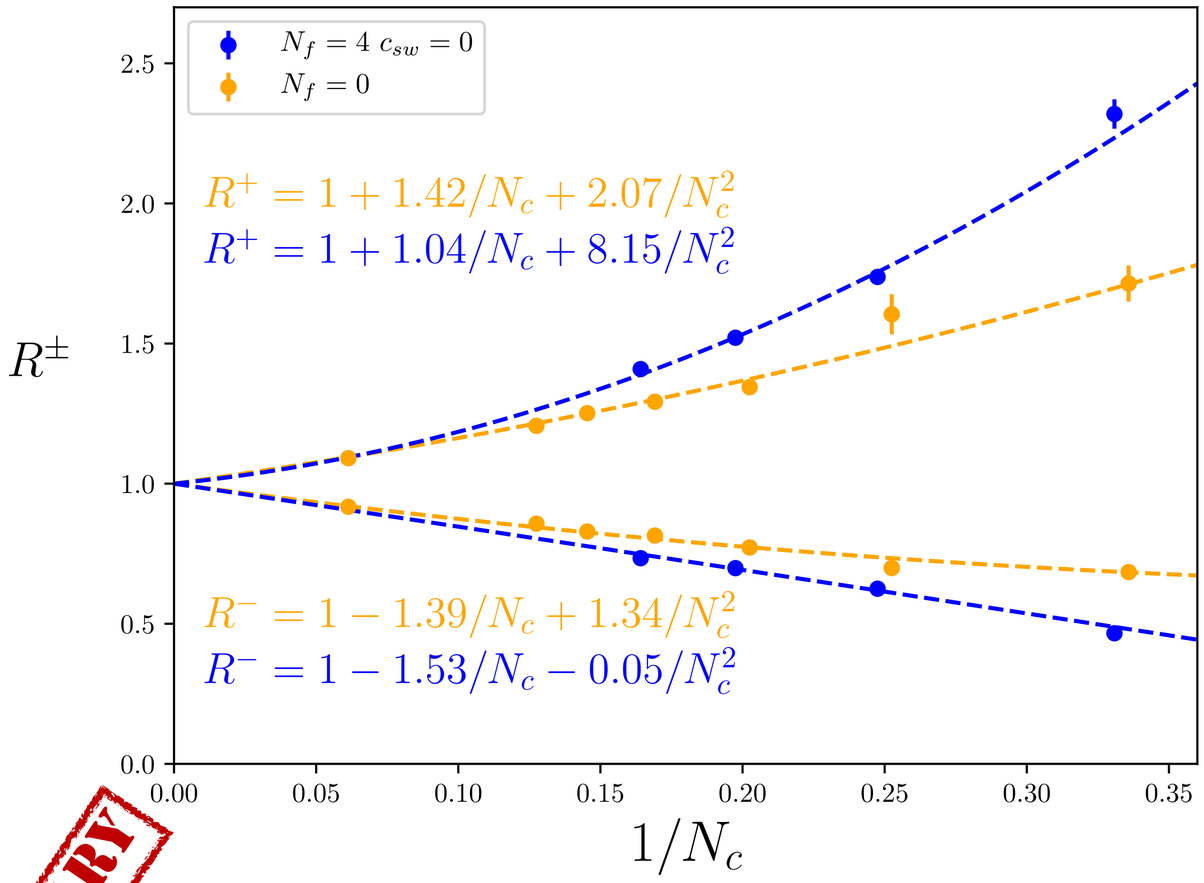


PRELIMINARY

- quenching effects $O(1/N_c^2)$
- $$\left.\frac{A_0}{A_2}\right|_{N_c=3, N_f=4} = 5.7(3)^{\text{stat}}$$

ΔI=1/2 anatomy: a summary

several possible sources for ΔI=1/2 enhancement (on top of short-distance's x2):

- physics at charm scale (penguins)
- physics at "intrinsic" QCD scale $\sim \Lambda_{\rm QCD}$
- final state interactions
- all of the above (no dominating "mechanism")

ΔI=1/2 anatomy: a summary

several possible sources for ΔI=1/2 enhancement (on top of short-distance's x2):

- physics at charm scale (penguins) $\longrightarrow$ x [>1.3]
- physics at "intrinsic" QCD scale $\sim \Lambda_{\rm QCD}$ $\longrightarrow$ x [1.5-2.0 (glue) x 1.0-1.5 (quarks)]
- final state interactions $\longrightarrow$?
- all of the above (no dominating "mechanism") likely, if I were to put my money...

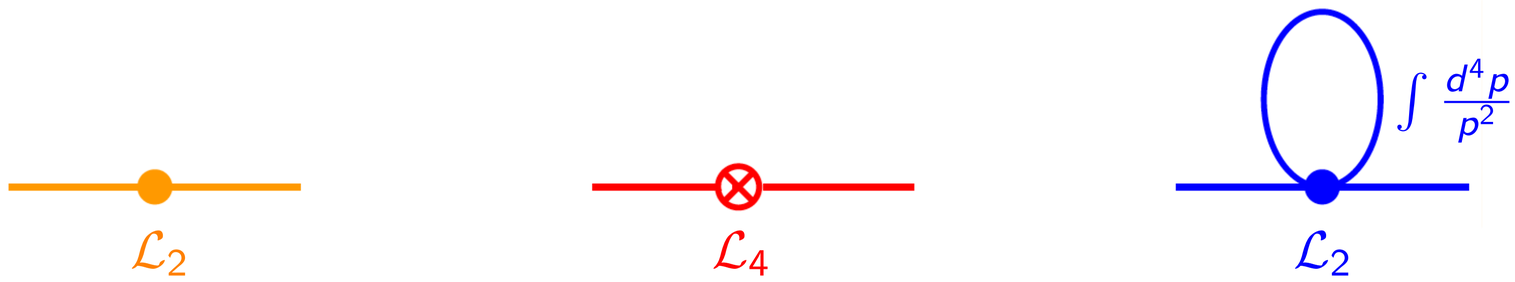
$1/N_c$ corrections are very large, consistent with the enhancement (and RBC/UKQCD's findings), and however still consistent with $1/N_c$ scaling

several interesting byproducts

meson interactions @ large N_c: χPT

Goldstone boson physics is well-parametrized by Chiral Perturbation Theory

$$F_\pi = F\left\{1 + \frac{M_\pi^2}{F_\pi^2}\left[4L_5(\mu) + 4N_\mathrm{f}L_4(\mu)\right] + \frac{N_\mathrm{f}}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\left(\frac{M_\pi^2}{\mu}\right)\right\}$$



$$F_\pi^2 = \mathcal{O}(N_c)$$
$$L_5 = \mathcal{O}(N_c)$$
$$L_4 = \mathcal{O}(1)$$

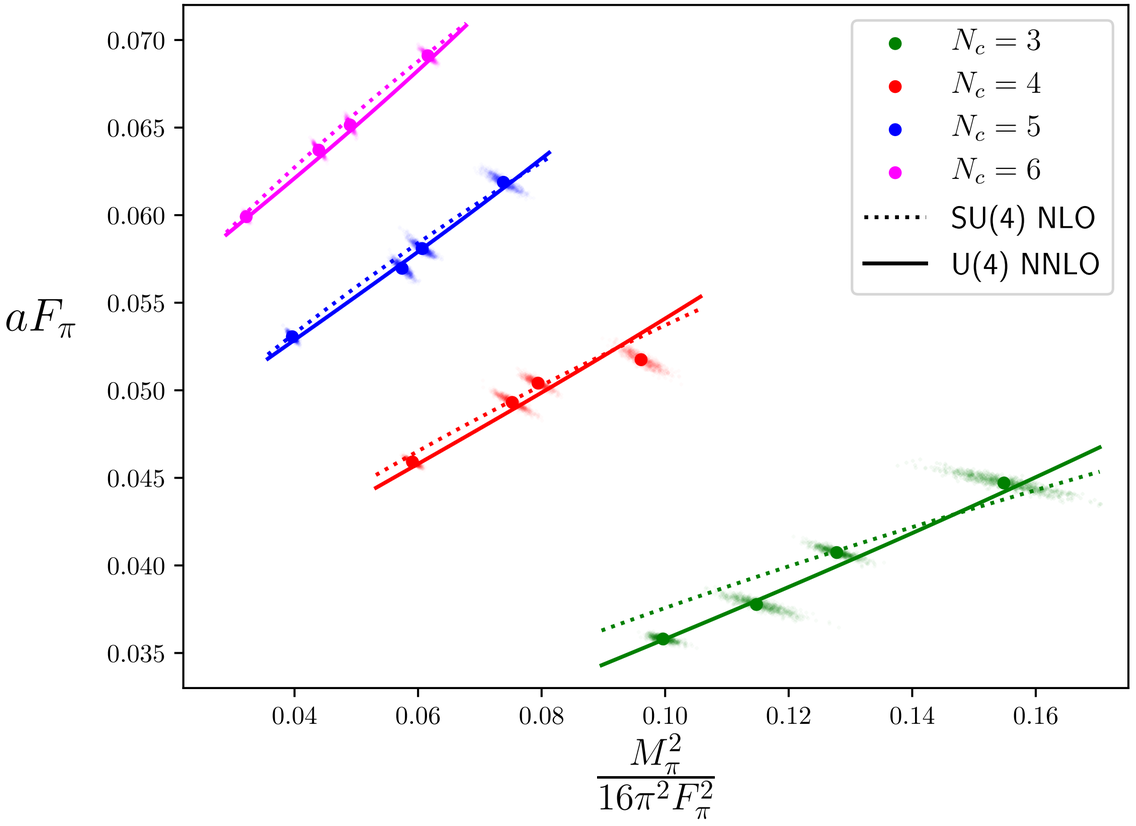
$$\longrightarrow \qquad F_\pi \underset{N_c\to\infty}{\approx} F\left\{1 + 4\frac{M_\pi^2}{F_\pi^2}L_5 + \mathrm{logs}\right\}$$

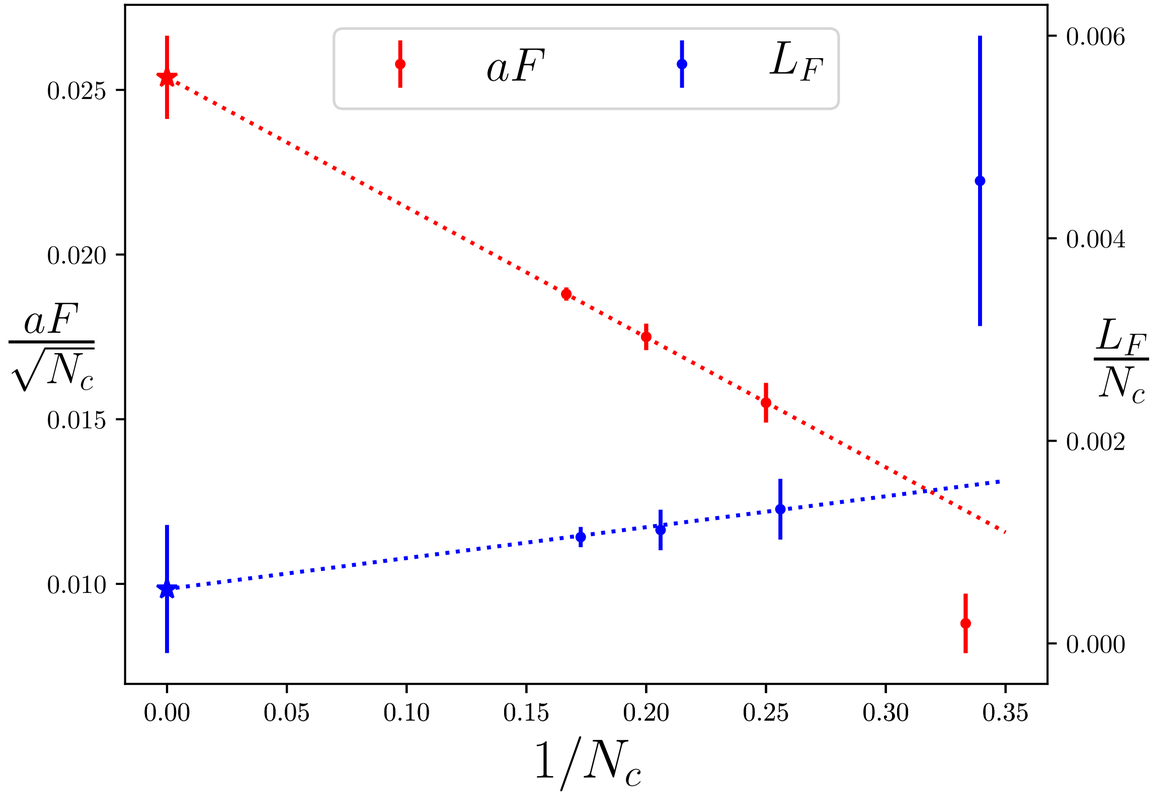
[Gasser, Leutwyler 1985]

meson interactions @ large N_c: χPT

$$F_\pi = F\left\{1 + \frac{M_\pi^2}{F_\pi^2}4L_F + \frac{N_\mathrm{f}}{2}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right]$$

$$F = \sqrt{N_c}\left(F^{(0)} + \frac{F^{(1)}}{N_c}\right) \qquad L_F = N_c L_F^{(0)} + L_F^{(1)}$$

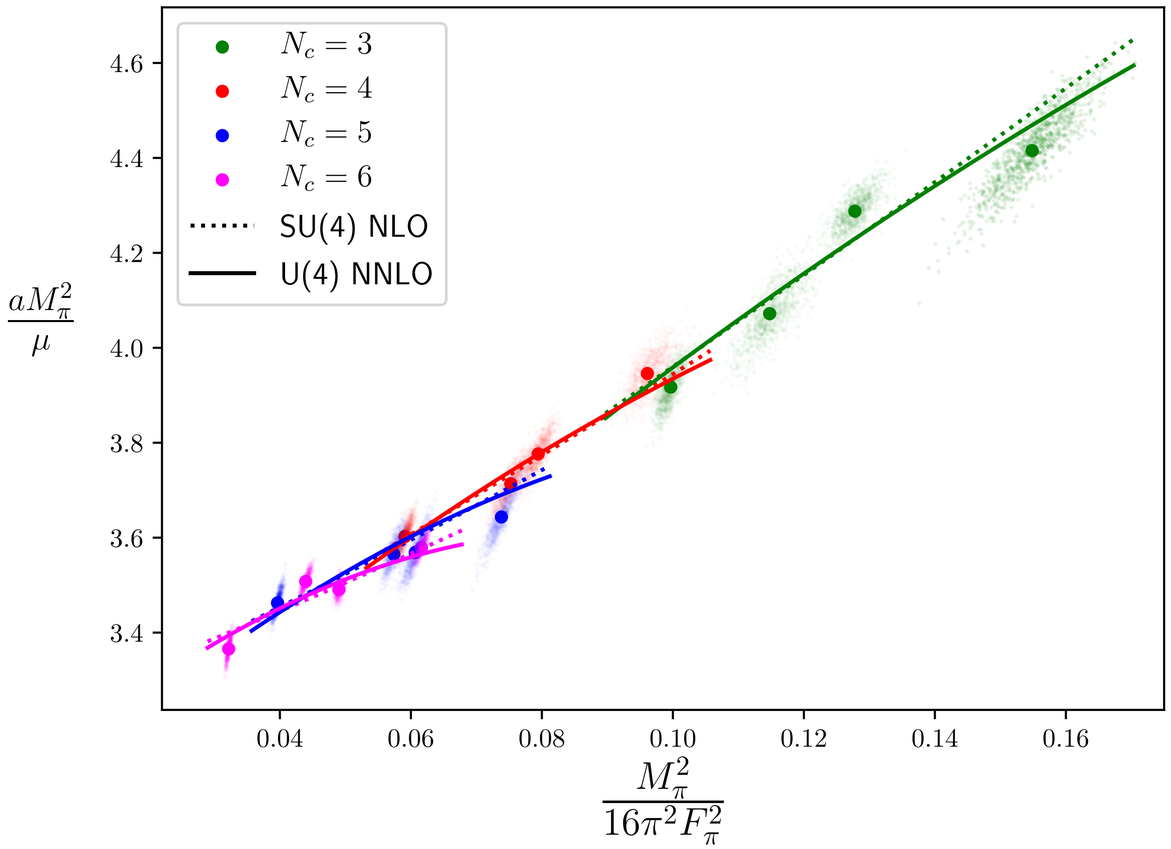


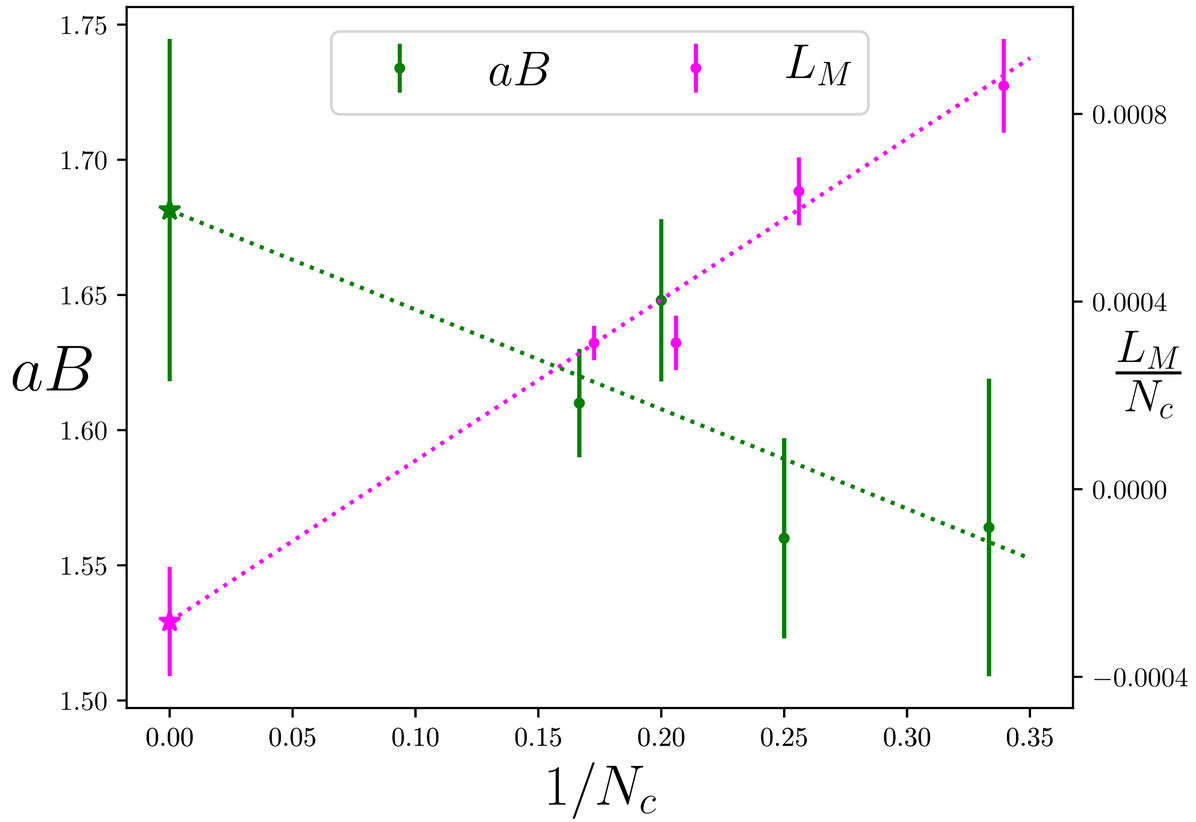


meson interactions @ large N_c: χPT

$$M_\pi^2 = 2Bm\left\{1 + \frac{M_\pi^2}{F_\pi^2}8L_M + \frac{1}{N_\mathrm{f}}\frac{M_\pi^2}{(4\pi F_\pi)^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right\}$$

$$B = B^{(0)} + \frac{B^{(1)}}{N_c} \qquad L_M = N_c L_M^{(0)} + L_M^{(1)}$$





meson interactions @ large N_c: χPT

selected results:

meson interactions @ large N_c: χPT

selected results:

- LO LECs:

  - $$\frac{F}{\sqrt{N_c}} = \left[67(3) - 26(4)\frac{N_f}{N_c}\right] \text{ MeV} \quad \Rightarrow \quad F_{N_f=2} = 86(3) \text{ MeV} \qquad F_{N_f=3} = 71(3) \text{ MeV}$$

ETM 15A	[386]	86.3(2.8)
Engel 14	[50]	85.8(0.7)(2.0)
Brandt 13	[49]	84(8)(2)
QCDSF 13	[402]	86(1)
TWQCD 11	[394]	83.39(35)(38)
ETM 09C	[48]	$85.91(07)\left(^{+78}_{-07}\right)$
ETM 08	[53]	86.6(7)(7)
Hasenfratz 08	[397]	90(4)
JLQCD/TWQCD 08A	[376]	$79.0(2.5)(0.7)\left(^{+4.2}_{-0.0}\right)$
JLQCD/TWQCD 07	[398]	87.3(5.6)
Colangelo 03	[403]	86.2(5)

JLQCD/TWQCD 10A	[389]	71(3)(8)
MILC 10	[36]	80.3(2.5)(5.4)
MILC 09A	[17]	78.3(1.4)(2.9)
MILC 09	[129]	
PACS-CS 08	[162]	83.8(6.4)
RBC/UKQCD 08	[163]	66.1(5.2)

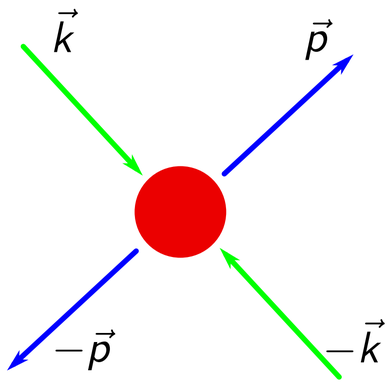
meson interactions @ large N_c: χPT

selected results:

- LO LECs:
  - $\frac{F}{\sqrt{N_c}} = \left[67(3) - 26(4)\frac{N_f}{N_c}\right]\ \text{MeV} \quad \Rightarrow \quad F_{N_f=2} = 86(3)\ \text{MeV} \qquad F_{N_f=3} = 71(3)\ \text{MeV}$
  - $\Sigma_{N_f=3} = 223(9)\ \text{MeV} \qquad \text{vs} \qquad \Sigma^{1/3}_{N_f=3} = 214(6)(24)\ \text{MeV}$ [Fukaya et al. 2010]
  - $\frac{\Sigma_{N_f=3}}{\Sigma_{N_f=2}} = 1.49(10) \qquad \text{vs} \qquad \frac{\Sigma_{N_f=3}}{\Sigma_{N_f=2}} = 1.51(11)$ [Bernard, Descotes-Genon, Toucas 2012]

- NLO LECs:
  - $\bar{\ell}_4 = 5.1(3) \qquad \text{vs} \qquad \bar{\ell}_4 = 4.40(28)$ [FLAG 2019]
  - n.b. subleading corrections to LECs are sizable: $\frac{L_M^{N_f=4}}{N_c} \times 10^3 = -0.2(2) + \frac{2.9(6)}{N_c} + \mathcal{O}\left(\frac{1}{N_c^2}\right)$

meson interactions @ large N_c: 2→2 scattering



infinite volume: phase shifts parametrize S-matrix

$$\langle \mathbf{k}\ell m|\hat{S}|\mathbf{p}\ell m\rangle = S_\ell = e^{2\delta_\ell(k)}\delta(|\mathbf{k}| - |\mathbf{p}|)$$

finite volume: use Lüscher method to derive phase shifts from volume-scaling of energies

$$\det[\cot\delta_\ell + \mathcal{M}] = 0$$

[Lüscher 1986]

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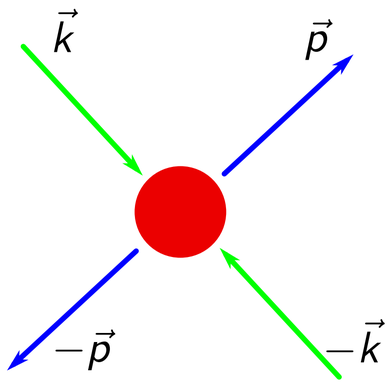

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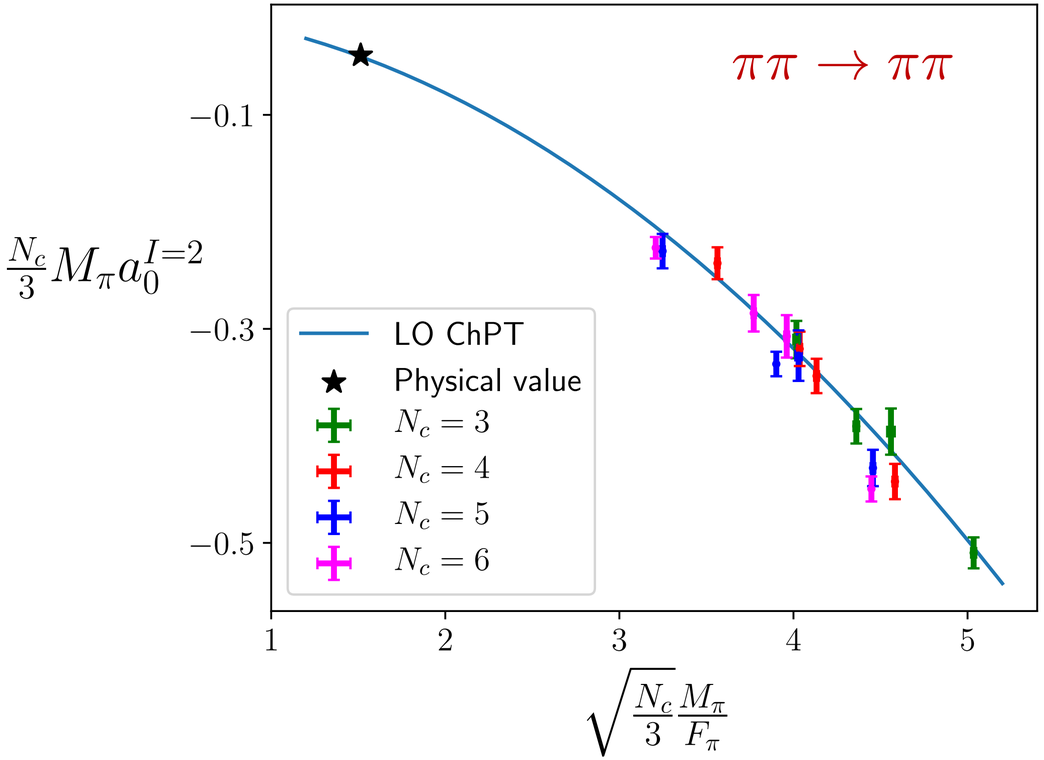

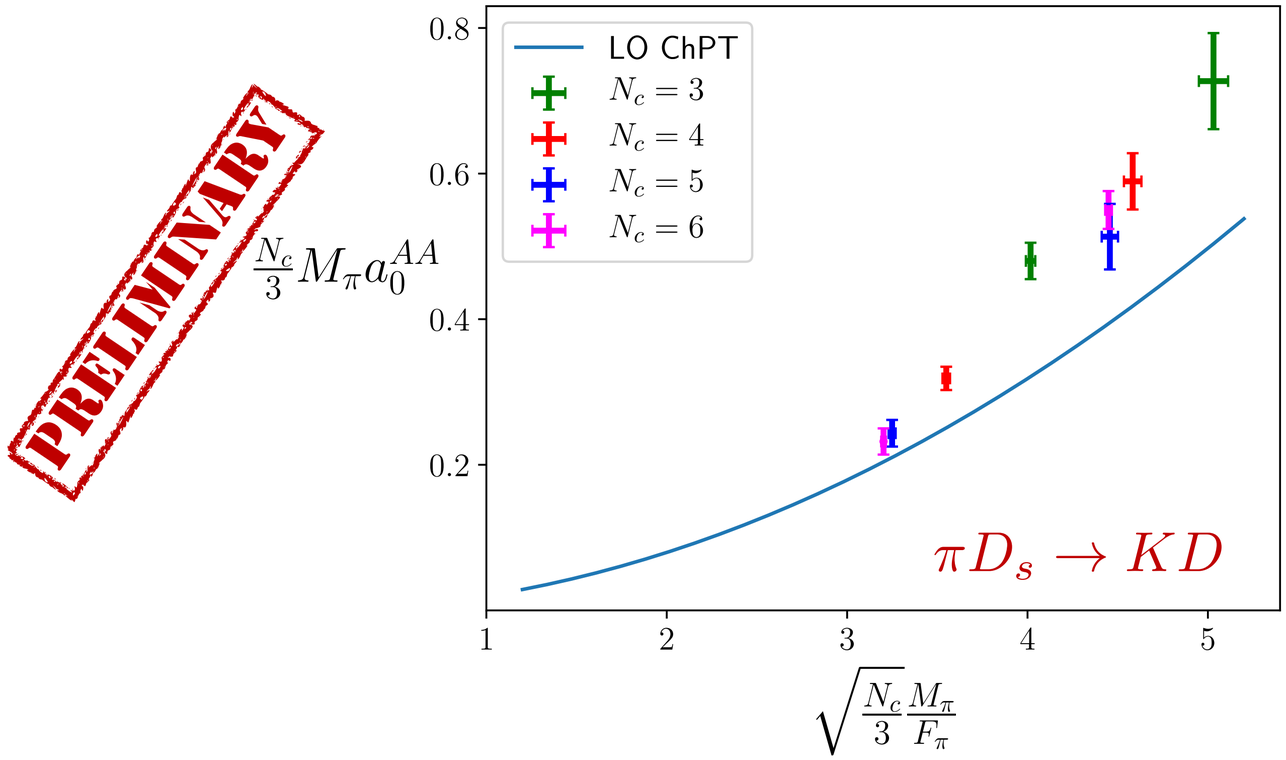

plan

- phenomenology of neutral kaon decay

- understanding non-leptonic kaon decay within the SM
  - electroweak effective Hamiltonian analysis
  - exact (lattice) vs. approximate (effective theory/large N/models)
  - why is it so hard?
  - state-of-the-art quantitative results

- understanding the anatomy of ΔI=1/2
  - the strategy
  - (old) results for QCD amplitudes
  - large N_c
  - insight into light meson physics

- outlook

conclusions and outlook

- non-leptonic kaon decay remains an open problem... and a fertile ground to learn about strong interaction physics
  - indirect CP violation well under control
  - direct CP violation, isospin enhancement still witness claims of new physics

- lattice toolbox making steady progress
  - controlled quantitative predictions for amplitudes are at hand
  - the anatomy of the effect is ever better understood, complex interplay of accumulated enhancements seems to emerge

- a theorist's paradise: field-theory, phenomenology, and computational physics all simultaneously at play!