Vacuum stability and scalar masses in the superweak extension of the standard model[1]

Z. Péli, Z. Trócsányi

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[1] Zoltán Péli and Zoltán Trócsányi. "Vacuum stability and scalar masses in the superweak extension of the standard model". In: *Phys. Rev. D* 106 (5 Sept. 2022), p. 055045. DOI: 10.1103/PhysRevD.106.055045. URL: https://link.aps.org/doi/10.1103/PhysRevD.106.055045.

Outline

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Introduction

How to obtain the VEV in QFT?

- Is our vacuum state stable against quantum fluctuations?
- The potential V in $\mathcal{L}$ contains quantum fields.
- Average out the quantum fluctuations to obtain a classical object for investigating the VEV.

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- Average out the quantum fluctuations to obtain a classical object for investigating the VEV.
- Analogy between QFT and stat. phys $\Rightarrow$ construct something similar to the Gibbs free energy
- This is the effective potential
- At LO it coincides with V, but with a classical field variable. Quantum corrections can be computed loop-by-loop.

BEH VEV and Higgs mass in the SM

Tree level

The tree level effective potential of the SM is

$$V(h) = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda h^4. \tag{1}$$

Its nontrivial minimum is located

$$\left.\frac{\partial V}{\partial h}\right|_{h=v} = 0 = (-\mu^2 + \lambda v^2)v. \tag{2}$$

This equation is also referred to as the tadpole equation. The mass of the Higgs particle at tree level is then

$$m_h^2 = \left.\frac{\partial^2 V}{\partial h^2}\right|_{h=v} = -\mu^2 + 3\lambda v^2 = 2\lambda v^2 = \left[-\frac{1}{v}\frac{\partial V}{\partial h} + \frac{\partial^2 V}{\partial h^2}\right]_{h=v}, \tag{3}$$

Vacuum stability in the SM

Loop corrections

The potential is bounded from below if

$$\lambda > 0 \tag{4}$$

- In the renormalized theory the couplings are functions of the RG scale Q. $\Longrightarrow \lambda(Q)$
- $V^{\text{1-loop}}$ is well known. (Coleman-Weinberg potential)
- In the SM for $h \to \infty$ one may obtain a λ_{eff} by setting $Q = h$:

$$\begin{aligned}\lambda_{\text{eff.}} = \lambda \quad + \quad & \frac{1}{(4\pi)^2}\Bigg[12\lambda^2\left(\log(3|\lambda|) - \frac{3}{2}\right) - 3y_t^4\left(\log(y_t^2) - \frac{3}{2}\right) \\ & + \frac{3}{8}g_L^4\left(\log(g_L^2/4) - \frac{5}{6}\right) + \frac{3}{16}G_L^4\left(\log(G_L^2/4) - \frac{5}{6}\right)\Bigg] \end{aligned} \tag{5}$$

Vacuum stability in the SM

RG running

- The potential should be stable for sensible values of Q: in practice up to M_{Pl}.
- The RG equations determine $\lambda(Q)$: coupled, first order, autonomous system of DE-s.
- So far so good, but we need initial conditions for the couplings: match on-shell measured values to running couplings. (usually $Q = M_t$ for convenience)

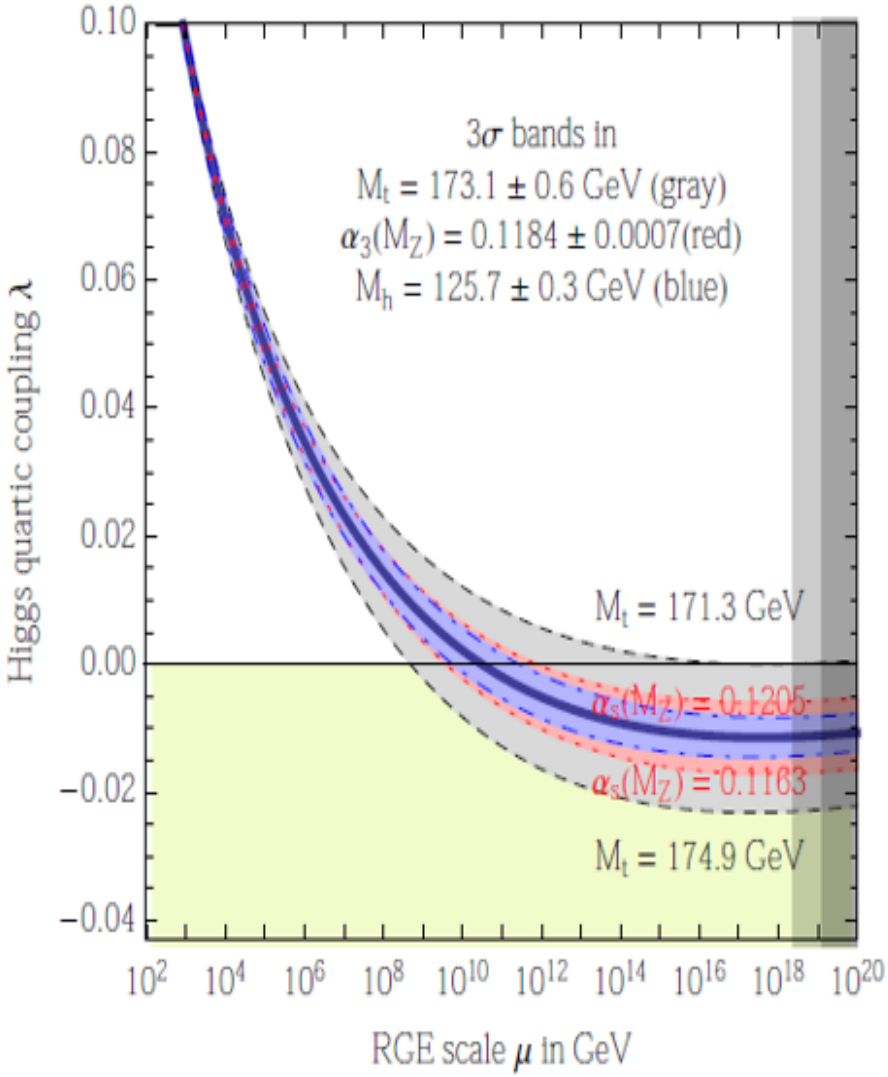
$$M_Z, \quad G_f, \quad \alpha_{\mathrm{em.}}(M_Z), \qquad \alpha_S^{(5)}(M_Z), \quad M_h, \quad M_t$$

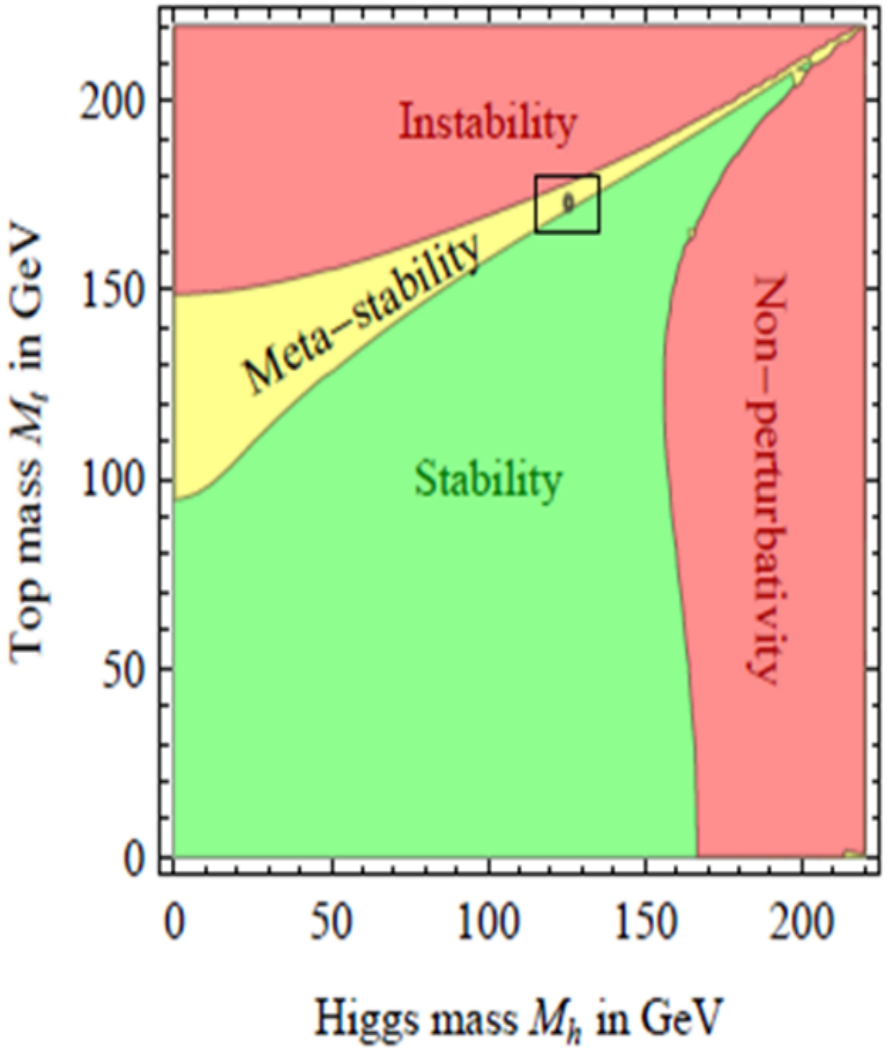
$$\Updownarrow$$

$$g_L(M_t), \quad v(M_t), \quad g_Y(M_t), \qquad g_s(M_t), \quad \lambda(M_t), \quad y_t(M_t)$$

Vacuum stability in the SM

RG running[2]





[2] Giuseppe Degrassi et al. "Higgs mass and vacuum stability in the Standard Model at NNLO". In: *JHEP* 08 (2012), p. 098. DOI: 10.1007/JHEP08(2012)098. arXiv: 1205.6497 [hep-ph].

Outline

Scalar sector @ tree level[3]

$$V(H,S) = V_0 - \frac{1}{2}\mu_\phi^2 H^2 + \frac{1}{4}\lambda_\phi H^4 - \frac{1}{2}\mu_\chi^2 S^2 + \frac{1}{4}\lambda_\chi S^4 + \frac{1}{4}\lambda H^2 S^2.$$

- VEVs:

$$v\left(-\mu_\phi^2 + \frac{1}{2}\lambda w^2 + \lambda_\phi v^2\right) = w\left(-\mu_\chi^2 + \frac{1}{2}\lambda v^2 + \lambda_\chi w^2\right) = 0 \tag{6}$$

- Mass matrix (after applying the tadpole eqns.):

$$\begin{pmatrix} 2\lambda_\phi v^2 & \lambda v w \\ \lambda v w & 2\lambda_\chi w^2 \end{pmatrix} \tag{7}$$

- Rotation to mass eigenstates:

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} H \\ S \end{pmatrix} \tag{8}$$

[3] Zoltán Trócsányi. "Super-weak force and neutrino masses". In: *Symmetry* 12.1 (2020), p. 107. DOI: 10.3390/sym12010107. arXiv: 1812.11189 [hep-ph].

Conditions

- Stability:

$$\begin{aligned} \lambda_\phi(Q) > 0, \qquad & \text{and} \qquad & \lambda_\chi(Q) > 0, \\ 4\lambda_\phi(Q)\lambda_\chi(Q) - \lambda(Q)^2 > 0 \qquad & \text{if} \qquad & \lambda(Q) < 0. \end{aligned} \tag{9}$$

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- Perturbativity: $|g(Q)| < 4\pi$ for and coupling g in the theory.
- We need w to exist for non-trivial phenomenology. (otherwise no mixing)
- It is numerically demanding to check the stability of the one-loop eff. potential for the SWSM.
- Investigate the stability of the tree level eff. pot and compute w precisely from the scalar pole masses.

The scalar pole masses

The scalar inverse-propagator matrix with quantum corrections:

$$\begin{pmatrix} p^2 & 0 \\ 0 & p^2 \end{pmatrix} - \begin{pmatrix} 2\lambda_\phi v^2 & \lambda v w \\ \lambda v w & 2\lambda_\chi w^2 \end{pmatrix} - \begin{pmatrix} \Pi_{HH}(p^2) - T_H/v & \Pi_{SH}(p^2) \\ \Pi_{HS}(p^2) & \Pi_{SS}(p^2) - T_S/w \end{pmatrix}, \tag{10}$$

Two eigenvalues with $M_1(p^2 = M_h^2) = M_h^2$ and $M_2(p^2 = M_s^2) = M_s^2$.

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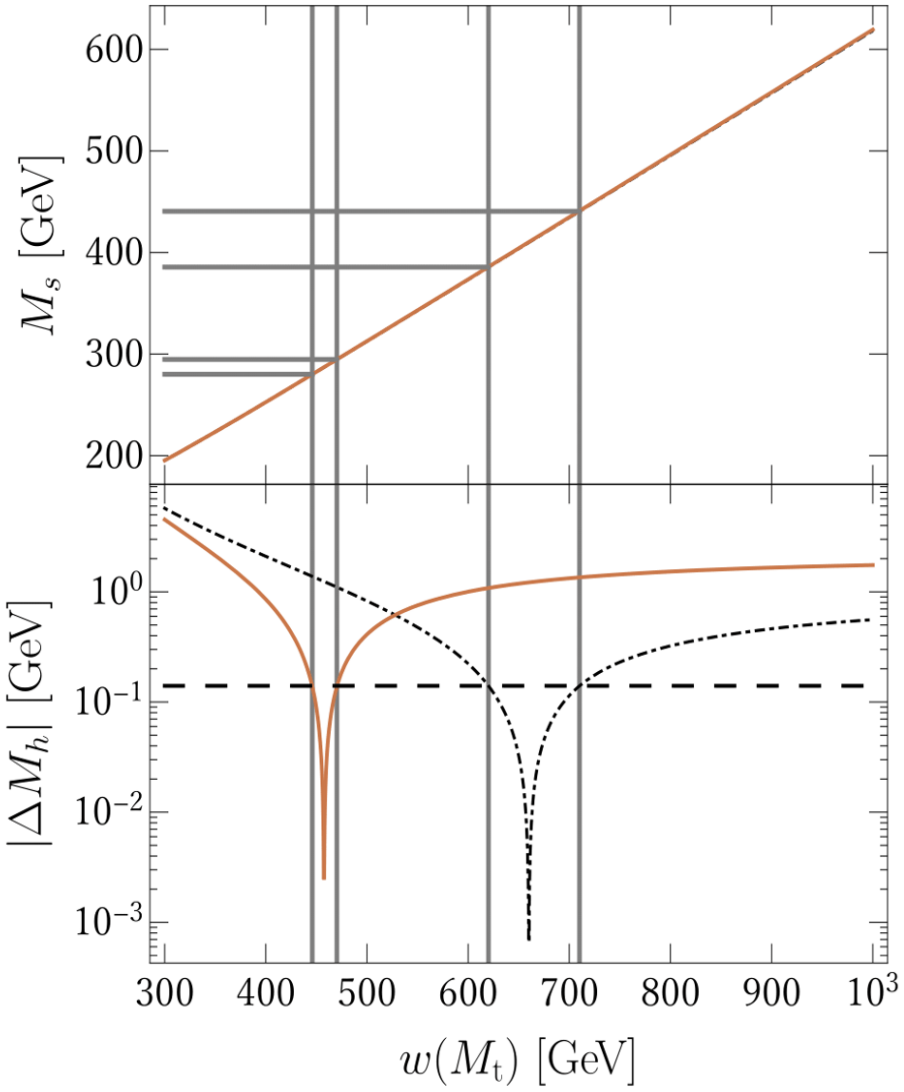
- Tree-level masses M_h^2, M_s^2 are functions of $v, w, \lambda_\phi, \lambda_\chi, \lambda$ (2 eqns, 5 parameters $\Rightarrow$ 3 free)
- M_h^2 **and** $v(M_\mathrm{t})$ **are known** and $\lambda_\phi(M_\mathrm{t}), \lambda_\chi(M_\mathrm{t}), \lambda(M_\mathrm{t})$ are provided $\Rightarrow$ w(M_t) can be extracted.
- There are sterile neutrinos in the theory (with Yukawa coupling y_x), which contribute at the loop level. **4 free parameters in total:**

$$(y_x(M_\mathrm{t}), \lambda_\phi(M_\mathrm{t}), \lambda_\chi(M_\mathrm{t}), \lambda(M_\mathrm{t})) \Leftrightarrow (y_x(M_\mathrm{t}), M_s, \sin\theta_\mathrm{s}, \lambda(M_\mathrm{t}))$$

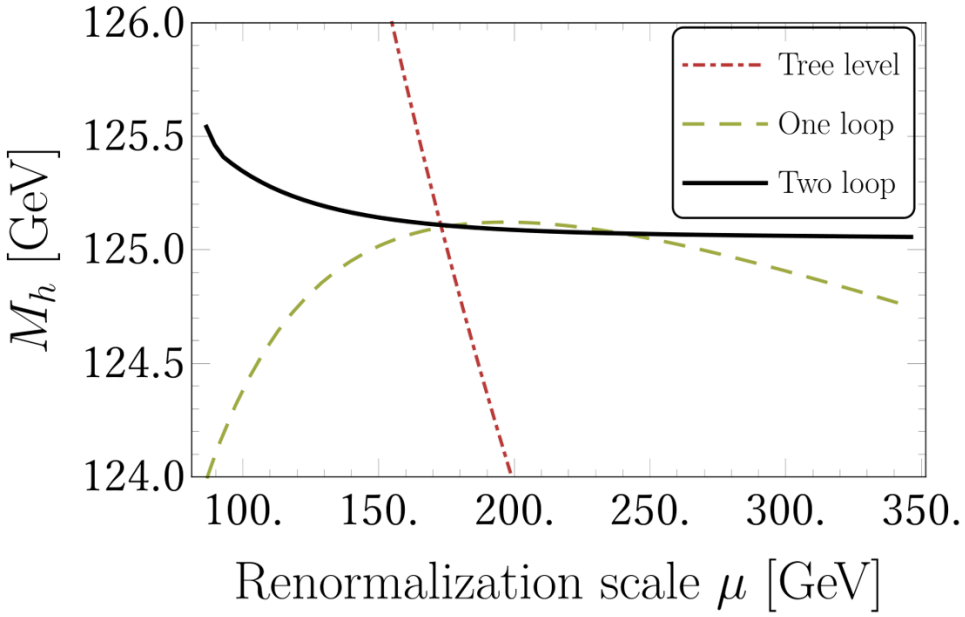
The scalar pole masses



- Horizontal dashed line = 0.14 GeV, experimental uncertainty of the Higgs mass
- Black dot-dashed curve: Abs(tree level prediction for M_h as function of w minus 125.1 GeV)
- Colored curve: Abs(one-loop prediction for M_h as function of w minus 125.1 GeV)

Precision

2-loops with SARAH[4] and SPheno[5]



- Superweak model file (SARAH) $\Rightarrow$ 2-loop RGE
- Analytical checks up to one-loop.
- Match the precision of the RGEs with the precision of w $\Rightarrow$ numerical 2-loop pole masses from SPheno
- Compute couplings shifts due to superweak contributions: $g(M_t) = g^{SM}(M_t) + \delta g^{SW}(M_t)$

[4] Florian Staub and Werner Porod. "Improved predictions for intermediate and heavy Supersymmetry in the MSSM and beyond". In: *Eur. Phys. J. C* 77.5 (2017), p. 338. DOI: 10.1140/epjc/s10052-017-4893-7. arXiv: 1703.03267 [hep-ph].

[5] Werner Porod. "SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+ e- colliders". In: *Comput. Phys. Commun.* 153 (2003), pp. 275–315. DOI: 10.1016/S0010-4655(03)00222-4. arXiv: hep-ph/0301101.

$M_s < M_h$ region

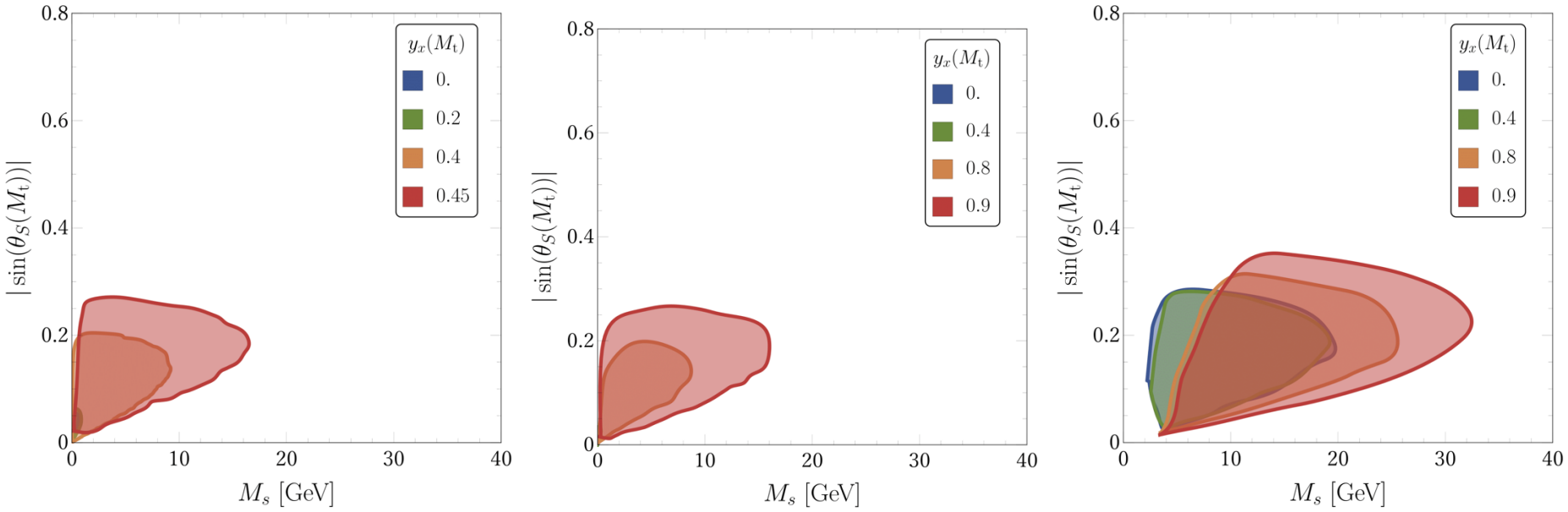
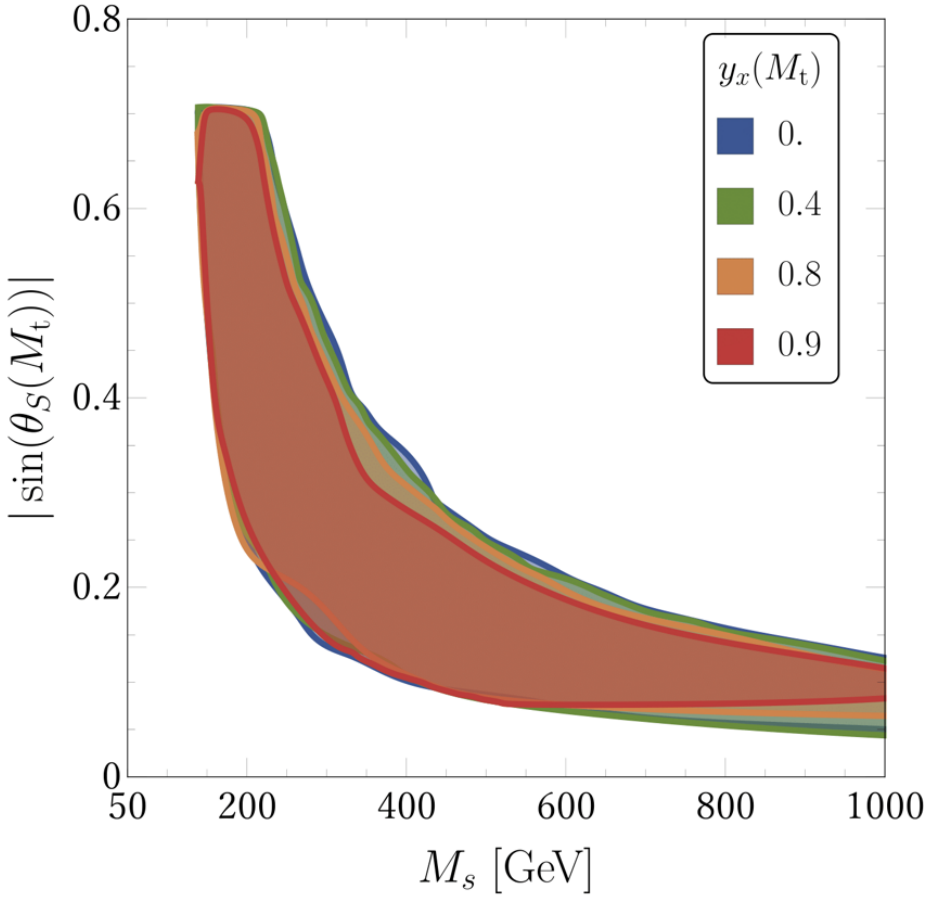


Figure: $y_x(M_t)$ fixed and projection to mixing-mass plane. Left: 1-loop RGE, w from M_h^{tree}. Center: 1-loop RGE, w from $M_h^{(1)}$. Right: 2-loop RGE, w from $M_h^{(2)}$.

$M_h < M_s$ region and 3d slice



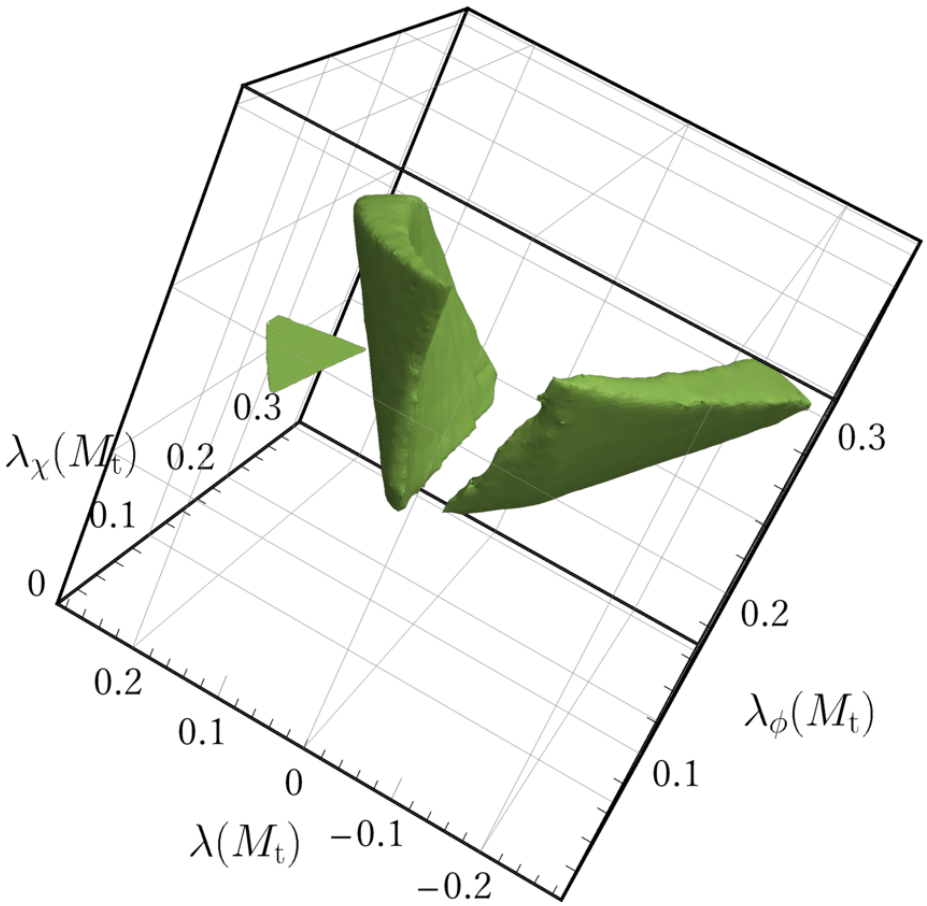


Figure: Left: $M_s > M_h$ region, same setting as previos slide. Right: $y_x(M_t) = 0.4$ fixed, full 3D slice for the quartic couplings

Outline

Higgs boson width

- The total width of the Higgs particle is measured[6]: $\Gamma_h = 3.2^{+2.8}_{-2.2}$ MeV
- The superweak prediction is

$$\Gamma_h = \left(\cos\theta_{\rm s}\right)^2 \times \Gamma_h^{\rm SM} + \Gamma\left(h \to ss\right) + \ldots \tag{11}$$

- How dominant is the decay

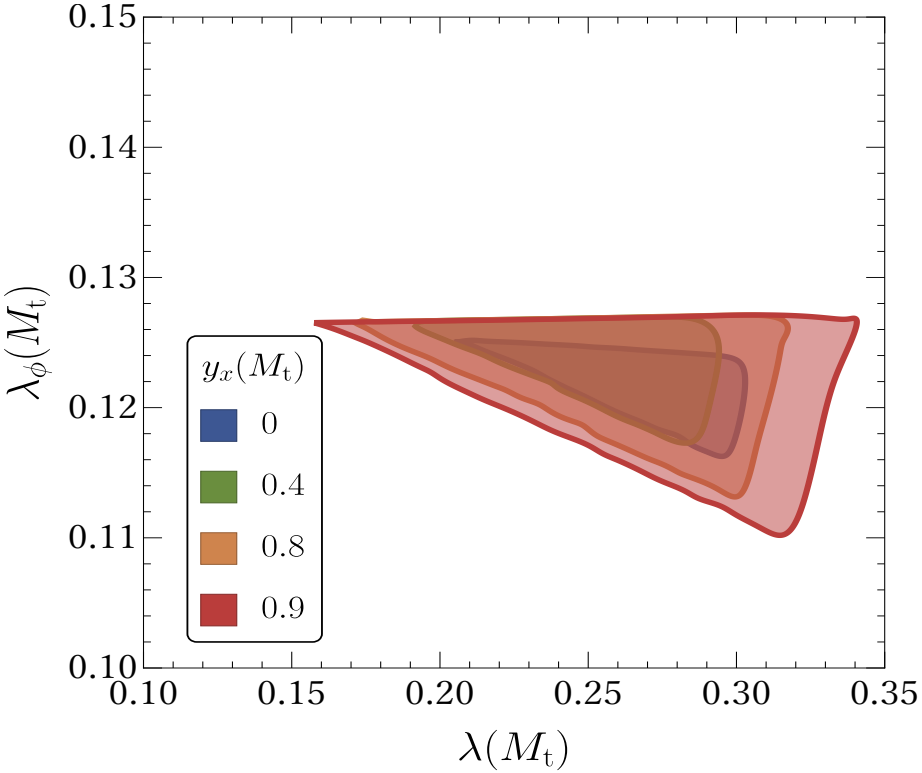
$$\Gamma\left(h \to ss\right) \quad = \quad \frac{\Gamma_{hss}^2}{32\pi M_h}\sqrt{1 - 4\frac{M_s^2}{M_h^2}} \tag{12}$$

when kinematically allowed with

$$\Gamma_{hss} \quad = \quad \frac{1}{2}\sin(2\theta_S)\left(M_h^2 + 2M_s^2\right)\left(\frac{\cos\theta_{\rm s}}{w} - \frac{\sin\theta_{\rm s}}{v}\right). \tag{13}$$

[6] P. A. Zyla et al. "Review of Particle Physics". In: *PTEP* 2020.8 (2020), p. 083C01. DOI: 10.1093/ptep/ptaa104.

Higgs boson width



- w depends on λ implicitly

$$w = \frac{1}{2v}\left|\frac{\sin(2\theta_s)(M_h^2 - M_s^2)}{\lambda}\right|$$

- The lower bound for the partial width is estimated

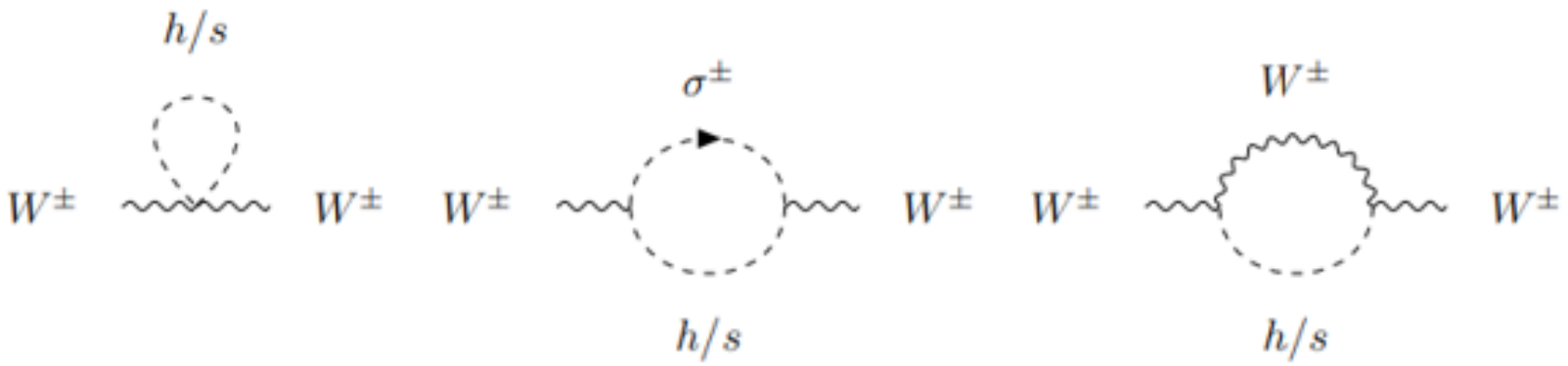
$$\Gamma\big(h \to ss\big) = \lambda^2 \times \big(4.8\ \text{GeV}\big)$$
$$\text{or} \quad \lambda^2 \times \big(3.4\ \text{GeV}\big)\sqrt{\epsilon},$$

with $M_s = (1-\epsilon) \times M_h/2$.

W boson mass

- Measured value $M_W^{\text{exp.}} = 80.379 \pm 0.012$ GeV
- SM theory prediction $M_W^{\text{theo.}} = 80.360 \pm 0.011$ GeV
- 19 MeV difference, ~ 17 MeV combined uncertainty
- 95% CL exclusion if δM_W^{BSM} is not in the range

$$-15 \text{ MeV} < \delta M_W^{\text{BSM}} < +53 \text{ MeV} \tag{14}$$



W boson mass

- Z' contribution is negligible, sterile neutrinos contribute at tree level ⇒ later work, we consider pure scalar corr.
- The superweak contribution (in MS-bar scheme):

$$\delta M_W^{\mathrm{SW}} = M_W \frac{\delta v^{\mathrm{SW}}}{v} + \frac{1}{2}\delta g_L^{\mathrm{SW}} v + \frac{1}{2}\frac{\Pi_{WW}^{\mathrm{SW}}(M_W^2)}{M_W} \tag{15}$$

$$\boxed{\delta M_W^{\mathrm{SW}} < 0 \quad \text{for} \quad M_s > M_h}$$

$$\boxed{\delta M_W^{\mathrm{SW}} > 0 \quad \text{for} \quad M_s < M_h}$$

- If the CDF II measurement[7] $M_W^{\mathrm{CDFII}} = 80.433$ GeV is not refuted ⇒ the singlet scalar extension is completely excluded.

[7] T. Aaltonen et al. "High-precision measurement of the W boson mass with the CDF II detector". In: *Science* 376.6589 (2022), pp. 170–176. DOI: 10.1126/science.abk1781.

Collider and signal strength measurements

- Already existing programs to compare model predictions to scalar sector measurements: **HiggsBounds**[8]**, HiggsSignals**[9]
- Need theory predictions for every scalar decay channel, including

$$\Gamma(\phi_i \to NN),\ \Gamma(\phi_i \to Z'Z'),\ \Gamma(\phi_i \to Z'Z),\ \Gamma(\phi_i \to \phi_j\phi_j)$$

  with $\phi_i = (h, s)_i$.
- Ongoing project is a global scan in the full parameter space of the SWSM with Josu Hernández-García and Zoltán Trócsányi.
- Here we show exclusion limits taken from the literature for the singlet scalar extension.

[8] Philip Bechtle et al. "HiggsBounds-5: Testing Higgs Sectors in the LHC 13 TeV Era". In: *Eur. Phys. J. C* 80.12 (2020), p. 1211. DOI: 10.1140/epjc/s10052-020-08557-9. arXiv: 2006.06007 [hep-ph].

[9] Philip Bechtle et al. "HiggsSignals-2: Probing new physics with precision Higgs measurements in the LHC 13 TeV era". In: *Eur. Phys. J. C* 81.2 (2021), p. 145. DOI: 10.1140/epjc/s10052-021-08942-y. arXiv: 2012.09197 [hep-ph].

Combined exclusions

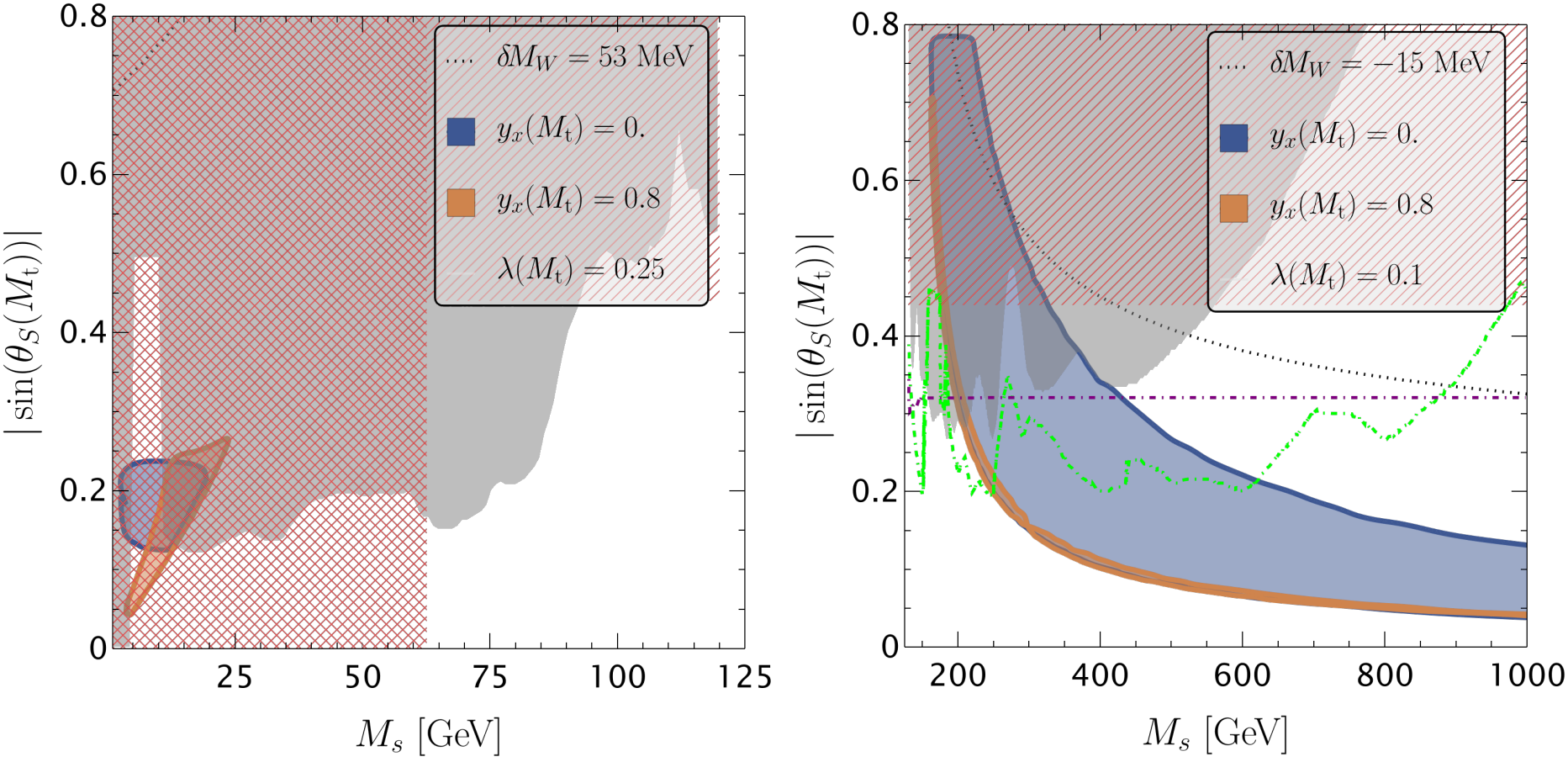


Figure: Slices of parameter space, and exclusion bands. Red grid: Higgs width, Red dashed and purple: Higgs signal strength measurements Gray and green dashed: collider searches (null results to exclusions)

Summary

- SM vacuum is metastable $\Rightarrow$ new physics to stabilize (or at least don't make things worse)
- Due to the Higgs portal $\lambda H^2 S^2$, absolute stability can be achieved on a well defined region of the parameter space of the SWSM.
- The width of the Higgs boson ($\Gamma^{\text{theo.}} < 8.8$ MeV with 95% CL) restricts $|\lambda| = 0.042$ if $h \to ss$ is allowed kinematically. To lift this, M_s needs to be very close to the kinematic threshold.
- The restriction from the W-boson mass becomes important for large M_s
- Limits from direct searches and signal strength measurements are taken from the literature[10,11]

[10] Adam Falkowski, Christian Gross, and Oleg Lebedev. "A second Higgs from the Higgs portal". In: *JHEP* 05 (2015), p. 057. DOI: 10.1007/JHEP05(2015)057. arXiv: 1502.01361 [hep-ph].

[11] Tania Robens. "Constraining extended scalar sectors at current and future colliders". In: *21st Hellenic School and Workshops on Elementary Particle Physics and Gravity*. Mar. 2022. arXiv: 2203.17016 [hep-ph].