Stabilisation of semilocal strings by dark scalar condensates

Based on 1612.03151 (Phys. Rev. D95 (2017) 035003), 1603.03291 (Phys. Lett. B762 (2016) 271-275), 1608.00021 (Phys. Rev. D94 (2016), 125018)

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Outline

What is a vortex?

- A complex, spontaneously breaking scalar field in 2D

$$\phi(\mathbf{x}) = \phi(r, \vartheta)\,, \quad |\phi(r \to \infty)| = \eta$$

- Winding number (no. flux quanta)

$$\int_0^{2\pi} \phi^{-1} \partial_\vartheta \phi \mathrm{d}\vartheta = 2\pi i n$$

- Consequence: a zero in the middle

What is a vortex?

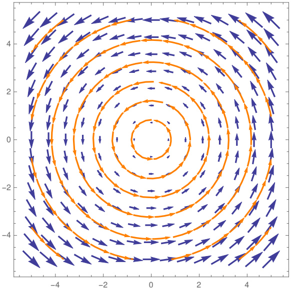
- A complex, spontaneously breaking scalar field in 2D

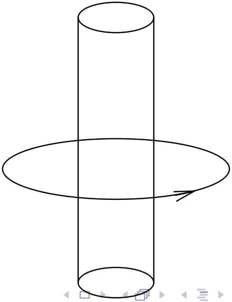
$$\phi(\mathbf{x}) = \phi(r, \vartheta)\,, \quad |\phi(r \to \infty)| = \eta$$

- Winding number (no. flux quanta)

$$\int_0^{2\pi} \phi^{-1} \partial_\vartheta \phi \mathrm{d}\vartheta = 2\pi i n$$

- Consequence: a zero in the middle
- In three dimension: vortex line (vortex string) or flux tube





Nielsen–Olesen strings

Abelian Higgs model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - \frac{\beta}{2}(\phi^*\phi - 1)^2 \,,$$

where $D_\mu\phi = (\partial_\mu - iA_\mu)\phi$, β: Ginzburg-Landau parameter

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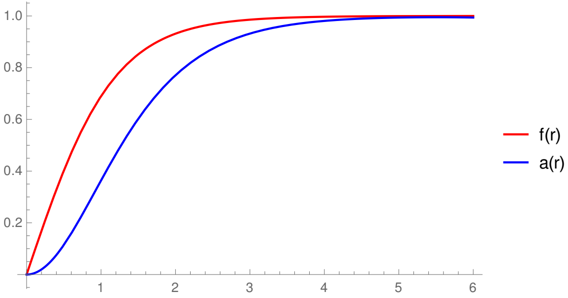
where $D_\mu\phi = (\partial_\mu - iA_\mu)\phi$, β: Ginzburg-Landau parameter
Ansatz:

$$\phi(t,\vartheta) = f(r)\exp(in\vartheta)\,, \quad A_\vartheta = na(r)$$



Abrikosov (1957), Nielsen and Olesen (1973), Kibble (1976)

Physics of vortices and strings

Cosmic strings

- Strings (flux tubes) in the Higgs field(s) of particle physics

  Abrikosov (1957), Nielsen and Olesen (1973), Kibble (1976)
- Most relevant: energy scale (GUT, electroweak, ...)
- Tension $\mu = E/L$ characterises gravitational effects
- Possible accelerator signatures Nambu (1977), Huang, Tipton (1981)

See also Vilenkin, Shellard (1994), Hindmarsh, Kibble (1995), Vachaspati etal. (2015)

Vortices in condensed matter

- vortex lines in superfluids, BECS
- flux tubes in superconductors Abrikosov (1957)
- multiple order parameter

  Babaev (2002), Babaev, Speight (2005), Catelani, Yuzbashyan (2010), Forgács, Lukács (2016)

See also Pismen (1999)

Analogies: "Cosmology in the laboratory"

Scalar phantom dark matter

26.8 % of the matter content of the Universe is dark
Kapteyn (1922), Oort (1932), Zwicky (1933)

The simplest model of dark matter: a scalar field
Silveira, Zee (1985), Patt, Wilczek (2006)

Scalar field χ and its interaction with SM Higgs

$$\mathcal{L}_{DM} = \partial_\mu \chi^* \partial^\mu \chi - \frac{\beta_2}{2}|\chi|^4 - \alpha|\chi|^2\,,$$

$$\mathcal{L}_{\mathrm{int}} = \beta'(\Phi^\dagger \Phi)|\chi|^2$$

Terminology: Higgs portal coupling

Observational constraints

- Higgs decays into the dark sector: contrains β'
- Quantity of DM: constraints m_χ

Cheung, Tsai, Tseng, Yuan, Zee (2012), Beniwal, Rajec, Savage, Scott, Weniger, White, Williams (2016)

A theory of Dark Matter

The Standard Model (SM) extended with a Dark Sector (DS)

Glashow-Salam-Weinberg theory

$$\mathcal{L}_{GSW} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + D_\mu \Phi^\dagger D^\mu \Phi - \lambda_1 (\Phi^\dagger \Phi - \eta_1^2)^2 \,,$$

where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \epsilon_{abc} W^b_\mu W^c_\nu$, $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$, $D_\mu \Phi = (\partial_\mu - igW^a_\mu \tau^a/2 - ig' Y_\mu)\Phi$

Dark sector

$$\mathcal{L}_{DS} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \tilde{D}_\mu \chi^* \tilde{D}^\mu \chi + \lambda_2 (\chi^* \chi - \eta_2^2)^2 \,,$$

$$\mathcal{L}_{int} = -\lambda' (\Phi^\dagger \Phi - \eta_1^2)(\chi^* \chi - \eta_2^2) + \frac{\sin \epsilon}{2} C_{\mu\nu} Y_{\mu\nu} \,,$$

where $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, $\tilde{D}_\mu \chi = (\partial_\mu - i\hat{g} C_\mu/2)\chi$

Couplings: λ' Higgs portal, $\sin \epsilon$ gauge kinetic mixing Holdom (1986)

Arkani-Hamed, Finkbeiner, Slatyer, Weiner (2009), Arkani-Hamed, Weiner (2008)

Dark strings

Cosmic strings in the DS Vachaspati (2009)

Ansatz

$$\phi_1 = 0\,, \quad \phi_2 = f(r)\mathrm{e}^{in_1\vartheta}\,, \quad \chi = g(r)\mathrm{e}^{in_2\vartheta}$$

$$Z_\vartheta = z(r)\,, \quad X_\vartheta = x(r)$$

where (r, ϑ, z): cylindrical coordinates, (n_1, n_2) windings

Properties of dark strings

- Coupled to visible sector (VS) fields (GKM)

  Vachaspati (2009), Gomez-Sanchez, Holdom (2011), Hyde, Long, Vachaspati (2014), Long, Hyde, Vachaspati (2014), Long, Vachaspati (2014)

- Nonzero flux in the DS $n_2 \neq 0$
- Stable, lower energy than electroweak

  Hartmann, Arbabzadah (2009), Brihaye, Hartmann (2009), Babeanu, Hartmann (2012)

- $n_1 \neq 0$, $n_2 = 0$ not considered: known instability of semilocal and electroweak strings Hindmarsh (1992), Hindmarsh (1993), James, Perivolaropoulos, Vachaspati (1992, 1993), Goodband, Hindmarsh (1995), Achúcarro, Vachaspati (2000)

The semilocal+dark model

Weinberg angle: $\tan\theta_W = g'/g$
$\theta_W \to \pi/2$ limit of GSW: semilocal model; non-Abelian field decouples, symmetry: $SU(2)_{\text{global}} \times U(1)_{\text{local}}$

Semilocal+dark model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{\epsilon}{2}F_{\mu\nu}H^{\mu\nu} + D_\mu\Phi^\dagger D^\mu\Phi + \tilde{D}_\mu\chi^*\tilde{D}^\mu\chi + \frac{\beta_1}{2}(\Phi^\dagger\Phi - 1)^2 + \frac{\beta_2}{2}|\chi|^4 - \alpha|\chi|^2 + \beta'\Phi^\dagger\Phi|\chi|^2$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, $D_\mu\Phi = (\partial_\mu - iA_\mu)\Phi$, $\tilde{D}_\mu\chi = (\partial_\mu - iqC_\mu)\chi$

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Symmetry breaking pattern

$$\text{1VEV: } \|\langle\Phi\rangle\| = \eta_1\,,\ \langle\chi\rangle = 0 \ \longleftrightarrow\ \text{2VEV : } \|\langle\Phi\rangle\| = \eta_1\,,\ |\langle\chi\rangle| = \eta_2$$

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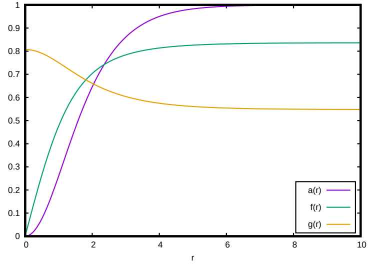
Scalar phantom as a special case: $\epsilon = q = 0,\ C_\mu = 0$

Dark core (DC) strings

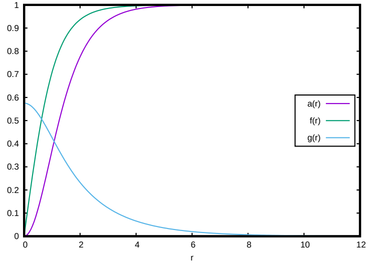
Ansatz

$$\phi_1 = f(r)\mathrm{e}^{in\vartheta}\,, \quad \chi = g(r)\,, \quad A_\vartheta = na(r)\,, \quad C_\vartheta = c(r)\,,$$

and the other field components (ϕ_2, A_r, A_z, C_r, C_z) vanish.



2VEV $\epsilon = 0$
$\beta_1 = 2,\ \beta_2 = 3,\ \beta' = 2,\ \alpha = 2.3$



1VEV,
$\beta_1 = \beta_2 = \alpha = 2$ and $\beta' = 2.1$

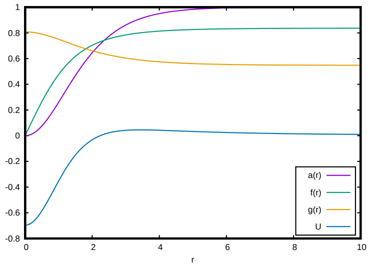
2VEV strings

Properties:

- $\epsilon = 0$: n flux quanta in A_μ, 0 for C_μ
- $E = E_0 + \epsilon^2 E_2 + \dots,\ E_2 < 0$

Stability: $\delta\phi_2 = s_{2,\ell}(r) \exp(i\ell\vartheta) \exp(i\Omega t)$

$$-\frac{1}{r}(r s_{2\ell}')' + U s_{2\ell} = \Omega^2 s_{2\ell}\,, \qquad U = \frac{(na-\ell)^2}{r^2} + \beta_1(f^2-1) + \beta' g^2$$



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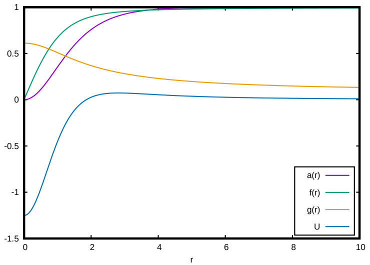
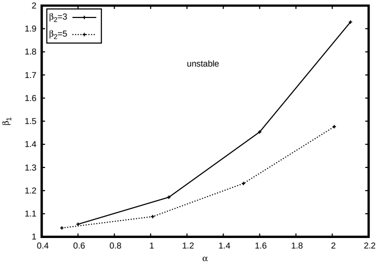
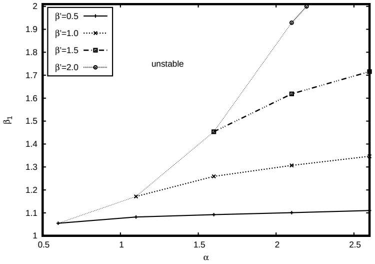


2VEV $\epsilon = 0$
$\beta_1 = 2,\ \beta_2 = 3,\ \beta' = 2,\ \alpha = 2.011$

2VEV string domain of stability

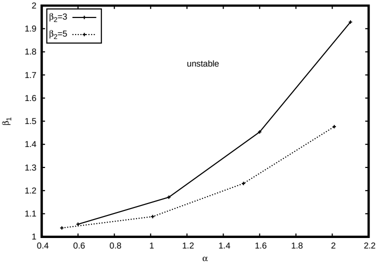


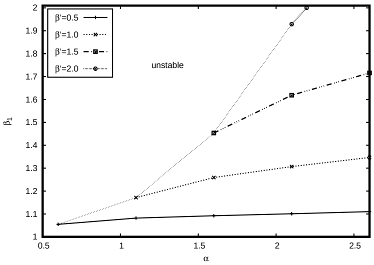


Properties of the potential U:

- $r \to \infty$: $U \sim (n-\ell)^2/r^2$ repulsive
- attractive valley close to the origin

2VEV string domain of stability





Properties of the potential U:

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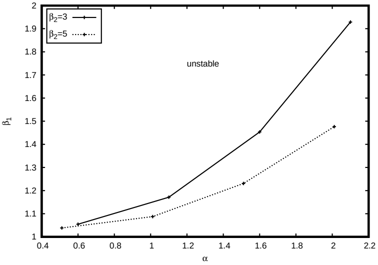
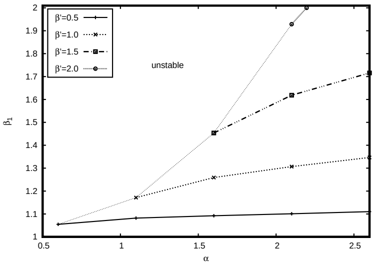
- Instability (bound state) depends on the parameters
- Stabilisation by increasing α at $\alpha = \alpha_s$
- Large α: χ heavy, allowed experimentally

2VEV string domain of stability





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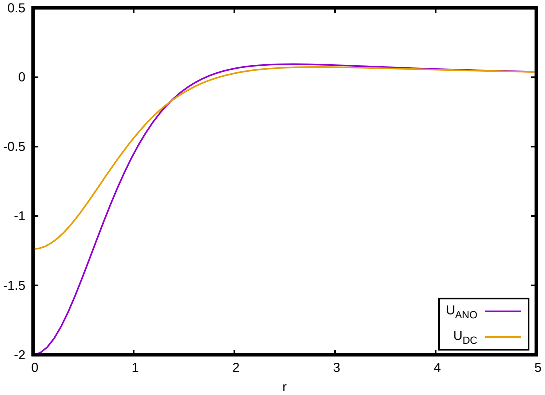
- Explanation: $U = \frac{(na-\ell)^2}{r^2} + \beta_1(f^2-1) + \beta' g^2$
  lowest energy false vacuum with $\Phi = 0$: $|\chi| = \sqrt{\alpha/\beta_2}$

1VEV strings

- Embedded Abrikosov-Nielsen-Olesen strings: $\phi_2 = \chi = 0$
  - Unstable against perturbations in ϕ_2 for $\beta_1 > 1$ Hindmarsh (1992, 1993)
  - Always unstable against perturbations in χ Haws, Hindmarsh, Turok (1988)
- DC strings have a larger domain of stability

$$\beta_{1,2} = 2,\ \beta' = 2.3, \text{ and } \alpha = 2.05$$



Conclusions

- Semilocal model + Higgs portal coupling (+gauge kinetic mixing)
- Both Higgs portal coupling and GKM act to stabilise strings
  HP mechanism: fill flux tube with a lower energy false vacuum
- 2VEV: stable semilocal-dark strings
- unfortunately difficult to obtain bounds on parameters
- 1VEV: stable string solutions found
- unfortunately $\beta_{1s} < 1.92$ and β' large

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Outlook: electroweak-dark string stability?

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Outlook: electroweak-dark string stability?

THANK YOU FOR YOUR ATTENTION

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References I

- A.A. Abrikosov, Sov. Phys. JETP **5**, 1174–1182 (1957).
- H.B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45–61 (1973).
- T.W.B. Kibble, J. Phys. A: Math. Gen. **9**, 1387 (1976).
- Y. Nambu, Nucl. Phys. **B 130**, 505-516 (1977).
- K. Huang and R. Tipton, Phys. Rev. **D23**, 3050 (1981).
- A. Vilenkin and E.P.S. Shellard, *Cosmic strings and other topological defects*, CUP, 1994.
- T.W.B. Kibble and M. Hindmarsh, Rep. Prog. Phys. **58**, 477 (1995).
- T. Vachaspati, L. Pogosian, and D. Steer, Scholarpedia **10**, 3168 (2015).
- E. Babaev, Phys. Rev. Lett. *89*, 067001 (2002).
- E. Babaev, M. Speight, Phys. Rev. **B 72**, 180502 (2005).
- G. Catelani and E.A. Yuzbashyan, Phys. Rev. **B81**, 033629 (2010)
- L.M. Pismen, *Vortices in nonlinear fields* , Oxford, 1999.
- J.C. Kapteyn, ApJ. **55**, 302 (1922).

References II

- J.H. Oort, Bulletin of the Astronomical Institutes of the Netherlands, **6**, 249 (1932).
- F. Zwicky, Helvetica Physica Acta **6**, 110 (1933).
- V. Silveira and A. Zee, Phys. Lett. **161B**, 136-140 (1985).
- B. Patt and F. Wilczek, arXiv:hep-ph/0605188 (2006).
- K. Cheung, Y.-L.S. Tsai, P.-Y. Tseng, T.-C. Yuan, and A. Zee, JCAP **1210**, 042 (2012).
- A. Beniwal, F. Rajec, C. Savage, P. Scott, C. Weniger, M. White, and A.G. Williams, Phys. Rev. **D93**, 115016 (2016).
- B. Holdom, Phys. Lett. **166B**, 196-198 (1986).
- N. Arkani-Hamed *etal*, Phys. Rev. **D79**, 015014 (2009).
- N. Arkani-Hamed and N. Weiner, JHEP **0812**, 104 (2008).
- T. Vachaspati, Phys. Rev. **D80**, 063502 (2009).
- C. Gomez-Sanchez and B. Holdom, Phys. Rev. **D83**, 123524 (2011).
- J.M. Hyde, A.J. Long, and T. Vachaspati, Phys. Rev. **D89**, 065031 (2014).
- A.J. Long, J.M. Hyde, and T. Vachaspati, JCAP **09**, 030 (2014).

References III

- A.J. Long and T. Vachaspati, JCAP **12**, 040 (2014).
- B. Hartmann and F. Arbabzadah, JHEP **07**, 068 (2009)
- Y. Brihaye and B. Hartmann, Phys. Rev. **D80**, 123502 (2009)
- A. Babeanu and B. Hartmann, Phys. Rev. **D85**, 023518 (2012)
- M. Hindmarsh, Phys. Rev. Lett. **68**, 1263 (1992)
- M. Hindmarsh, Nucl. Phys. **B392**, 461-492 (1993)
- M. James, L. Perivolaropoulos, and T. Vachaspati, Phys. Rev. **D46**, R5232 (1992); Nucl. Phys. **B395**, 534-546 (1993).
- A. Achúcarro and T. Vachaspati, Phys. Rept. **327**, 427 (2000).
- M. Goodband and M. Hindmarsh, Phys. Lett. **B363** 58-64 (1995).
- D. Haws, M. Hindmarsh, and N. Turok, Phys. Lett. **B209**, 255-261 (1988).

For more details, see:

- P. Forgács and Á. Lukács, Phys. Rev. **D95**, 035003 (2017).
- P. Forgács and Á. Lukács, Phys. Rev. **D94**, 125018 (2016).
- P. Forgács and Á. Lukács, Phys. Lett. **B762**, 271-275 (2016).
- Á. Lukács, In: A. Zichichi (ed.): *What is known and unexpected at the LHC?*, Proceedings of the 2010 ISSP, Erice, World Scientific, 2013.

Extended Abelian Higgs models

Multi-component scalar fields

- $SU(2)_{\text{global}} \times U(1)_{\text{locals}}$ semilocal model
  The $\theta_{\mathrm{W}} \to \pi/2$ limit of the electroweak theory
- Two-component Ginzburg–Landau theory, one or two charged field
  - 1VEV: $\langle \phi_1 \rangle = 1$, $\langle \phi_2 \rangle = 0$
  - 2VEV: $\langle \phi_a \rangle = \eta_a$
  - 2band superconductors (MgB_2) Suhl etal (1959), Moshchalkov etal (2009)
  - high-pressure liquid metallic hydrogen Ashcroft (2000), Babaev etal (2005)
  - neutron star interiors Jones (2006)
- Witten model
- Semilocal-dark model:

$$\mathcal{L} = -\frac{1}{4}F^2 + (D\Phi)^\dagger D\Phi + \partial\chi^*\partial\chi - V$$

$$V = \frac{\beta_1}{2}(\Phi^\dagger\Phi - 1)^2 + \frac{\beta_2}{2}(\chi^*\chi)^2 - \alpha\chi^*\chi + \beta'\Phi^\dagger\Phi\chi^*\chi$$

- A model of dark matter

Silveira and Zee (1985), Pratt and Wilczek (2006), Arkani-Hamed etal. (2009)

EAH strings

Semilocal model:

- Embedded Nielsen-Olesen: unstable for $\beta > 1$ Hindmarsh (1992)
- One-parameter family for $\beta = 1$ Hindmarsh (1992)
- One-parameter family for $\beta > 1$ Forgács, Reuillon, Volkov (2006)
  unstable Garaud and Volkov (2008), Forgács and Á.L (2009), Hartmann and Peter (2012)

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ANO Instability: nonzero ϕ_2 in the core, $\delta\phi_2 = \exp(i\Omega t)\exp(i\ell\vartheta)s_2(r)$

$$-\frac{1}{r}(rs_{2\ell}')' + U_{2\mathrm{ANO}}s_{2\ell} = \Omega^2 s_{2\ell}\,, \qquad U_{2\mathrm{ANO}} = \frac{n^2a^2}{r^2} + \beta_1(f^2-1)\,,$$

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Two-component extended Abelian Higgs models/Ginzburg-Landau

- 2VEV: fractional flux Babaev (2002), Babaev and Speight (2005)
  $n > 1$ stable Forgács and Á.L. (2016)
- 1VEV: endpoint of the family stable if $\beta_1\beta_2 \neq \beta'^2$
  $n > 1$ stable
  $\phi_2(0) \neq 0$: condensate core vortices Forgács and Á.L. (2016)

The vacuum manifold

2VEV:

$$\alpha > \beta' \,, \quad \beta_1\beta_2 > \alpha\beta'$$

and

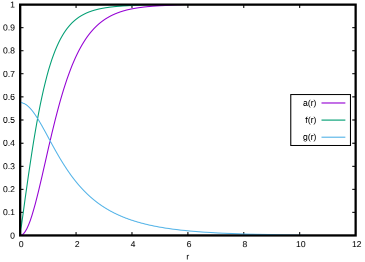
$$\eta_1^2 = \frac{\beta_1\beta_2 - \alpha\beta'}{\beta_1\beta_2 - (\beta')^2} \,, \quad \eta_2^2 = \frac{\beta_1(\alpha - \beta')}{\beta_1\beta_2 - (\beta')^2}$$

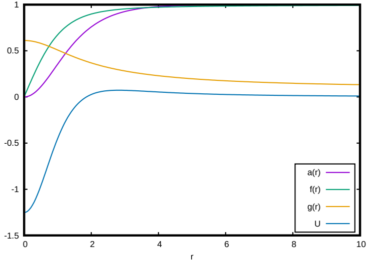
Both need to be positive

1VEV: otherwise

	$\beta_1\beta_2 > \beta'^2$	$\beta_1\beta_2 < \beta'^2$
upper	$\beta' > \alpha$	$\sqrt{\beta_1\beta_2} > \alpha$
lower	$\beta' < \alpha$	$\sqrt{\beta_1\beta_2} < \alpha$

Numerical solutions





$\beta_1 = \beta_2 = \alpha = 2$ and $\beta' = 2.1$

$\beta_1 = 2,\ \beta_2 = 3,\ \beta' = 2$ and $\alpha = 2.011$

Stability domains

1VEV

β_2	β'	α	β_{1s}	$E/(2\pi)$
3	2.3	2.05	1.615	1.0846
4	2.3	2.05	1.459	1.0630
5	2.3	2.05	1.367	1.0504
6	2.3	2.05	1.247	1.0299
2	2	1.85	1.805	1.1022

ANO Stable: $\beta < 1$

$$E_{\mathrm{ANO}}(\beta = 2) = 2\pi \times 1.1568,$$
$$E_{\mathrm{ANO}}(\beta = 1) = 2\pi$$

2VEV

β_1	β_2	β'	α_s	$E/(2\pi)$	α_s	$E/(2\pi)$	α_s	$E/(2\pi)$
			$\epsilon = 0$		$\epsilon = 0.1$		$\epsilon = 0.2$	
2	5	2	4.571	0.149	4.567	0.150	4.559	0.153
2	3	2	2.196	0.792	2.193	0.795	2.180	0.808
2	1.5	1.25	2.025	0.329	2.020	0.332	2.011	0.341

Additional references

- H. Suhl, B.T. Matthias, and L.R. Walker, Phys. Rev. Lett. **3**, 552 (1959).
- V. Moshchalkov *etal.*, Phys. Rev. Lett. **112**, 117001 (2009).
- N.W. Ashcroft, J. Phys.: Condens. Matter **12**, A129 (2000).
- E. Babaev, A. Sudbø, and N.W. Ashcroft, Phys. Rev. Lett. **95**, 105301 (2005).
- P.B. Jones, Mon. Not. R. Astron. Soc. **371**, 1327 (2006).
- P. Forgács, S. Reuillon, and M.S. Volkov, Phys. Rev. Lett. **96**, 041601 (2006); Nucl. Phys. **B751**, 390-418 (2006)
- J. Garaud and M.S. Volkov, Nucl. Phys. **B799**, 430-455 (2008)
- P. Forgács and Á.L., JHEP **0912**, 064 (2009)
- B. Hartmann and P. Peter, Phys. Rev. **D86**, 103516 (2012)