Integrability for higher-point functions in $\mathcal{N} = 4$ SYM

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Motivation

Quantum field theories (QFT): mathematical framework for elementary particles and interactions

Goal: develop non-perturbative methods using **integrability**

$\Rightarrow$ Consider toy models: **CFT** characterised by $\{\Delta_i, C_{ijk}\}$

Two-point function:

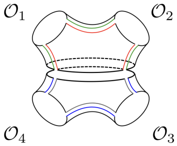
$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle = \frac{\delta_{12}}{|x-y|^{2\Delta}}$$

Three-point function:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}|x_{31}|^{\Delta_3+\Delta_1-\Delta_2}}$$

Operator product expansion (OPE) for four-point functions:

(depends on conformal cross ratios)

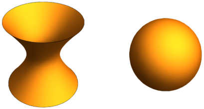


AdS/CFT correspondence

[Maldacena '97]

AdS$_d$ String theory $\longleftrightarrow$ $(d-1)$-dim CFT

strings on AdS$_5$ $\times$ S^5 $\quad$ $\mathcal{N}=4$ SYM in $d=4$



$\longleftrightarrow$

Φ^{IJ}, Ψ^I_α, $\bar{\Psi}_{I\dot{\alpha}}$, $\mathcal{A}^\mu$

coupling constant g_{YM}

gauge group $\mathrm{SU}(N)$

$$\frac{R^4}{\alpha'^2} = \lambda = g_{\text{YM}}^2 N$$

consider planar limit $g_{\text{YM}} \to 0$, $N \to \infty$ and λ finite ['t Hooft '74]

strong coupling α' $\longleftrightarrow$ weak coupling λ

in the following: $g^2 = \frac{\lambda}{16\pi^2}$

The spectral problem and integrability in AdS/CFT

Anomalous dimension in $\mathcal{N} = 4$ SYM $\longleftrightarrow$ Energy of string states
$\Delta = \Delta_0 + g^2\gamma_1 + \mathcal{O}(g^4)$

Spectral problem can be mapped to an **integrable** spin chain [Minahan, Zarembo '02]

Example: $\mathrm{SU}(2)$ **sector**

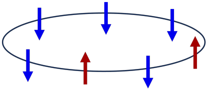
Choose vacuum Z ($\downarrow$) and excitations X ($\uparrow$)

$\rightarrow$ **BMN-operator** with two scalar excitations $\mathrm{Tr}(Z^{L-k-2}XZ^kX)$

$\rightarrow$ planar one-loop **dilatation operator** $\leftrightarrow$ Spin chain Hamiltonian $H_0 = 1 - \mathbb{P}$

Bethe Ansatz leads to **energy** and **S matrix** in terms of **rapidity** u:

$$E = \sum_{j=1}^{M} \frac{1}{u_j^2 + \frac{1}{4}} \quad \text{and} \quad S(u_j, u_k) = \frac{u_j - u_k - i}{u_j - u_k + i}\,.$$



For M excitations, the **Bethe equations** are given by

$$\left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}}\right)^L \prod_{j\neq k} S(u_j, u_k) = 1 \quad \text{and} \quad \prod_{j=1}^{M}\left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}}\right) = 1\,.$$

Hexagon-like formula from the spin chain

Bethe state:

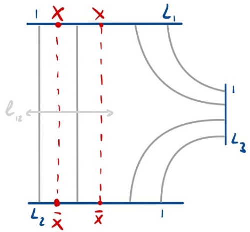
$$|\Psi(p_1,p_2)\rangle = \sum_{1\le n<m\le L} \underbrace{\left(e^{ip_1 n+ip_2 m} + S(p_1,p_2)e^{ip_2 n+ip_1 m}\right)}_{\psi(n,m)} |n,m\rangle$$

Normalized cyclic state given by [Gaudin '76][Korepin '82]

$$\mathcal{O}_L = \frac{|\Psi(p_1,p_2)\rangle}{\sqrt{\mathcal{G}\, L\, S_{12} \prod_j (u_j^2+\frac{1}{4})}}$$

Overlap:

$$c_{123} \propto \sum_{1\le n<m\le \ell_{12}} \psi_1(n,m) \quad \psi_2(L_2-m+1, L_2-n+1)$$

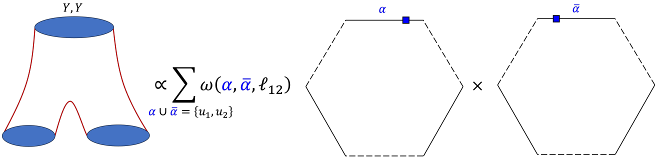


→ **Tailoring tools** for three-point functions

[Escobedo, Gromov, Sever, Vieira '10]

Three-point functions from integrability

Three-point functions by **hexagon operators** [Basso, Komatsu, Vieira '15]



$$\propto \sum_{\alpha \cup \bar{\alpha} = \{u_1, u_2\}} \omega(\alpha, \bar{\alpha}, \ell_{12})$$

$$\mathcal{A} = \sum_{v1, v2, \ldots} \int d\mu_{\{v\}} \sum_{\alpha \cup \bar{\alpha}} \omega(\alpha, \bar{\alpha}, \ell) \; \langle \mathbf{h} | \alpha, v \rangle \; \langle \mathbf{h} | \bar{v}, \bar{\alpha} \rangle$$

The **splitting factor** $\omega(\alpha, \bar{\alpha}, \ell)$ is given by

$$\omega(\alpha, \bar{\alpha}, \ell) = (-1)^{|\bar{\alpha}|} \prod_{j \in \bar{\alpha}} e^{i p_j \ell} \prod_{\substack{k \in \alpha \\ j < k}} S(p_j, p_k) \,.$$

Mirror corrections are hard to evaluate

$$\mathbf{1} = |0\rangle \langle 0| + \sum_i \int d\mu_p |p, i\rangle \langle p, i| + \ldots$$

Symmetries of the three-point function

Choosing Z as the vacuum

Take 1/2-BPS operator $\mathcal{O}(0)$ at $x = 0$

→ want to construct *three* translated operators $\mathcal{O}(x)$

→ should preserve as much (super)symmetry as possible

Introduce the **supertranslation generator** [Basso, Komatsu, Vieira '15]

$$\mathcal{T} = -i\epsilon_{\alpha\dot{\alpha}} P^{\alpha\dot{\alpha}} + \epsilon_{\dot{a}a} R^{a\dot{a}} \,,$$

Use $\mathcal{T}$ to construct one parameter family of operators starting from $\mathcal{O}(0)$

$$\mathcal{O}_t = e^{t\,\mathcal{T}}\,\mathcal{O}(0)\,e^{-t\,\mathcal{T}} \,.$$

Constraining the hexagon form factor by symmetry

Charges commuting with $\mathcal{T}$ form diagonal subalgebra $\mathfrak{psu}(2|2)_D$

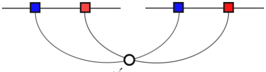
Write $\mathfrak{psu}(2|2)^2$ excitations as $\chi^{a\dot{a}} = \xi^a \otimes \dot{\xi}^{\dot{a}}$

Use **bootstrap principle** $\langle \mathbf{h}|g|\Psi\rangle = 0$, $g \in \mathfrak{psu}(2|2)_D$

→ non-vanishing one-particle form factors for Y, $\bar{Y}$, $\mathcal{D}^{3\dot{4}}$, $\mathcal{D}^{4\dot{3}}$

→ two-particle form factors given by Beisert S matrix elements [Beisert '06]

$$\begin{aligned}\langle \mathbf{h}|\chi^{a_1\dot{a}_1}\chi^{a_2\dot{a}_2}\rangle &= (-1)^f \langle \xi^{a_2}\xi^{a_1}|\mathcal{S}|\dot{\xi}^{\dot{a}_1}\dot{\xi}^{\dot{a}_2}\rangle \\ &= (-1)^f \dot{S}^{\dot{b}_1\dot{b}_2}_{\dot{a}_1\dot{a}_2}\, h_{\chi^{a_1\dot{b}_1}}\, h_{\chi^{a_2\dot{b}_2}}\ .\end{aligned}$$



→ Multi-particle form factor:

$$\langle \mathbf{h}|\chi^{a_1\dot{a}_1}\chi^{a_2\dot{a}_2}\dots\chi^{a_N\dot{a}_N}\rangle = (-1)^f \langle \xi^{a_N}\dots\xi^{a_2}\xi^{a_1}|\mathcal{S}|\dot{\xi}^{\dot{a}_1}\dot{\xi}^{\dot{a}_2}\dots\dot{\xi}^{\dot{a}_N}\rangle\ .$$

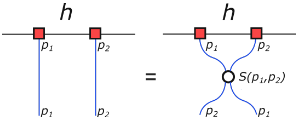
[Basso, Komatsu, Vieira '15]

Constraining the scalar h-factor

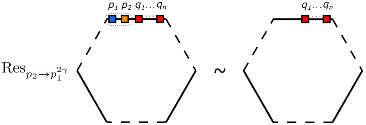
Scalar factor h in the hexagon $\longleftrightarrow$ dressing phase S_0 in the S matrix

- **Watson equation**
  Scattering with the full S matrix

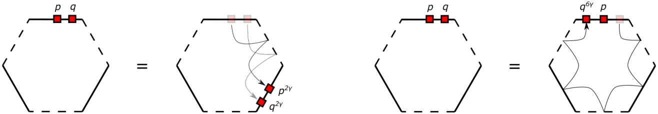
$$\langle \mathbf{h}|\,\mathbf{S}\,|\chi^{A\dot{A}}(p_1)\chi^{B\dot{B}}(p_2)\rangle = \langle \mathbf{h}|\chi^{A\dot{A}}(p_1)\chi^{B\dot{B}}(p_2)\rangle$$



- **Decoupling condition** for a singlet



- **Cyclicity**

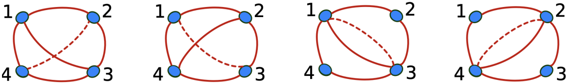


$\Rightarrow$ Fixes the h-factor!
$\Rightarrow$ Similar construction in AdS_3

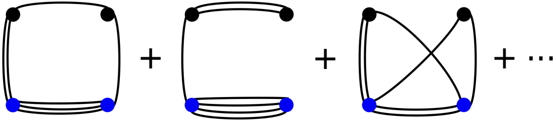
[Basso, Komatsu, Vieira '15]
[Eden, DℓP, Sonfdrini '21]

Higher-point functions

Natural way to tesselate four-point functions into hexagons [Eden, Sfondrini '16] [Fleury, Komatsu '16]



→ Need to include **conformal cross-ratio** dependence $v_{i;jk}$



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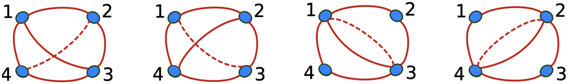

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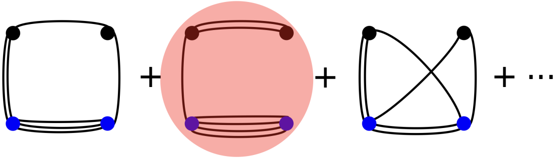
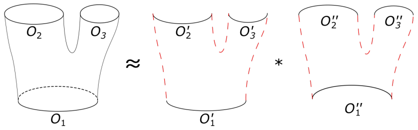


→ Need to include **colour factors** [Eden, Jiang DℓP, Sfondrini '17]

Hexagon program

- **Spectrum** is fairly well-understood
- Three-point functions by **hexagon operators**
  for AdS_5 [Basso, Komatsu, Vieira '15]
  for AdS_3 [Eden, DℓP, Sonfdrini '21]



- In principle:
  - → **higher-point functions** [Eden, Sfondrini '17] [Fleury, Komatsu '17]
  - → **non-planar correlators** [Eden, Jiang, DℓP, Sfondrini '17] [Bargheer, Caetano, Fleury, Komatsu, Vieira '17] [Bargheer, Coronado, Vieira '19] ...
  - → **gluing corrections** [Basso, Komatsu, Vieira '15] [Eden, Sfondrini '15] [Fleury, Komatsu '17] ...

So far: Operators in rank-one sectors
→ How to generalise formalism to **higher-rank** sectors?
→ replace hexagon by nested wave function [Basso, Coronado, Komatsu, Lam, Vieira, Zhong '17]

Plan

Higher-rank models

Consider the $\mathrm{SU}(3)$ sector at tree level with excitations X and Y

Consider the wave function $|\Psi(X_1, Y_2)\rangle$, with the scattering

$$|X_1\, Y_2\rangle \quad \rightarrow \quad T_{12}\, |Y_2\, X_1\rangle + R_{12}\, |X_2\, Y_1\rangle \ ,$$

with **transmission** and **reflection** amplitudes

$$T_{12} = \frac{A_{12} - B_{12}}{2} \qquad \text{and} \qquad R_{12} = \frac{A_{12} + B_{12}}{2} \ .$$

Introduce a second wave function $|\Psi(Y_1, X_2)\rangle$ with initial ordering X, Y, scattering to

$$|Y_1\, X_2\rangle \quad \rightarrow \quad T_{12}\, |X_2\, Y_1\rangle + R_{12}\, |Y_2\, X_1\rangle \ ,$$

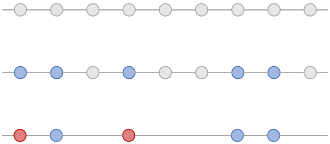
and consider the sum

$$|\Psi_{XY}(p_1, p_2)\rangle = g_{XY}\, |\Psi(X_1, Y_2)\rangle + g_{YX}\, |\Psi(Y_1, X_2)\rangle \ ,$$

with yet to be determined **coefficients** g_{XY} and g_{YX}.

Extracting the coefficients from nesting

- **Level-0** vacuum of length L
- M **level-1** excitations move on level-0 vacuum with $S^{10} = e^{ip}$ and $S^{11}_{jk} = S(u_j, u_k)$
- k **level-2** excitations move on level-1 vacuum of length M with S^{21}, are scattered by S^{22} and have a creation amplitude f^{21}



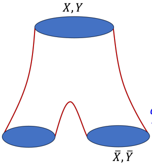
$$|Y(v)\rangle^2 = f^{21}(v, u_1)\,|Y_1\, X_2\rangle + f^{21}(v, u_2)S^{21}(v, u_1)\,|X_1\, Y_2\rangle\ .$$

Scattering leads to

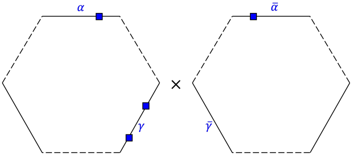
$$\begin{aligned} g_{XY}\, T_{12} + g_{YX} R_{12} &= f^{21}(v, u_2)S^{11}(u_1, u_2)\,, \\ g_{XY}\, R_{12} + g_{YX}\, T_{12} &= f^{21}(v, u_1)S^{21}(v, u_2)S^{11}(u_1, u_2)\,. \end{aligned}$$

$\Rightarrow$ **Coefficients** g_{XY} and g_{YX} inherit dependence on the auxiliary Bethe roots v.

The nested hexagon



$$\propto \sum_{\substack{\alpha \cup \bar{\alpha} = \{u_1, u_2\} \\ \gamma \cup \bar{\gamma} = \{u_3, u_4\}}} \omega(\alpha, \bar{\alpha}, \ell_{12})\, \omega(\gamma, \bar{\gamma}, \ell_{23})$$



Cutting the $\mathrm{SU}(3)$ state

$$\omega(\alpha, \bar{\alpha}, \ell)\psi_{\{\alpha\}}\psi_{\{\bar{\alpha}\}} = \begin{cases} g_{XY}\, \psi_{\{X_{u_1}, Y_{u_2}\}}\psi_{\{\}} + g_{YX}\, \psi_{\{Y_{u_1}, X_{u_2}\}}\psi_{\{\}} + \\ e^{ip_2\ell}\left(g_{XY}\, \psi_{\{X_{u_1}\}}\psi_{\{Y_{u_2}\}} + g_{YX}\, \psi_{\{Y_{u_1}\}}\psi_{\{X_{u_2}\}}\right) + \\ e^{ip_1\ell}\left(g_{YX}\ T_{12} + g_{XY}\ R_{12}\right)\psi_{\{X_{u_2}\}}\psi_{\{Y_{u_1}\}} + \\ e^{ip_1\ell}\left(g_{XY}\ T_{12} + g_{YX}\ R_{12}\right)\psi_{\{Y_{u_2}\}}\psi_{\{X_{u_1}\}}\ , + \\ e^{i(p_1+p_2)\ell}\left(g_{XY}\psi_{\{\}}\psi_{\{X_{u_1}, Y_{u_2}\}} + g_{YX}\psi_{\{\}}\psi_{\{Y_{u_1}, X_{u_2}\}}\right) . \end{cases}$$

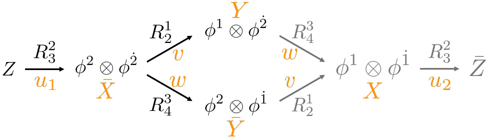
$\Rightarrow$ **Agreement** with free field theory [Eden, DℓP, Spiering '22]

Double excitations

Consider the Konishi operator $\mathcal{K} = \frac{1}{\sqrt{3}}\mathrm{Tr}(X\bar{X} + Y\bar{Y} + Z\bar{Z})$

How can we describe $\bar{Z}$?

$\longrightarrow$ **double** excitations!



$$|Y\rangle = \mathfrak{R}^1{}_2\, \mathfrak{R}^2{}_{\dot{3}}\, |Z\rangle = \mathbf{c}^{1\dagger}\mathbf{c}_{\dot{3}}\, |Z\rangle \;, \qquad \text{and} \qquad |\bar{Y}\rangle = \mathfrak{R}^{\dot{3}}{}_{\dot{4}}\, \mathfrak{R}^2{}_{\dot{3}}\, |Z\rangle = \mathbf{c}_{\dot{4}}\mathbf{c}^{2\dagger}\, |Z\rangle \;.$$

$$|\, {}^{Y}_{\bar{Y}} \rangle = \mathfrak{R}^1{}_2\, \mathfrak{R}^2{}_{\dot{3}}\, \mathfrak{R}^{\dot{3}}{}_{\dot{4}}\, \mathfrak{R}^2{}_{\dot{3}}\, |Z\rangle = \mathbf{c}^{1\dagger}\mathbf{c}^{2\dagger}\, |0\rangle = |\bar{Z}\rangle \;,$$

Can introduce **double excitations** with creation amplitude $\hat{f}(u_1, u_2, v, w)$ in the nested picture and $\hat{e}(u_1, u_2)$ in the matrix ansatz

Computations makes no further reference to the local structure of the state
$\rightarrow$ cut the wave function in the usual way [Eden, DℓP, Spiering '22]

Konishi example

Let us evaluate $\langle \mathcal{K}\, \mathcal{O}^{L_2}\, \mathcal{O}^{L_3} \rangle$ with $\mathcal{K} = \frac{1}{\sqrt{3}} \mathrm{Tr}(X\bar{X} + Y\bar{Y} + Z\bar{Z})$.

This yields at tree-level

$$\mathcal{A}_{\mathrm{QFT}} = \frac{1}{\sqrt{3}} \sqrt{L_2 L_3}\,.$$

Using $g_{X\bar{X}} = g_{\bar{X}X} = -g_{Y\bar{Y}} = -g_{\bar{Y}Y}$ and $u_2 = -u_1 = \frac{1}{\sqrt{12}}$, $v = 0$, $w = 0$

$$\mathcal{A}^{\ell_{12}=1}_{\mathrm{hexagon}}(-u, u) = \frac{8\, g_{X\bar{X}}\, u}{(u - \frac{i}{2})(u + \frac{i}{2})^2} = \frac{\sqrt{3}}{2}\,.$$

We find agreement

$$\mathcal{A}_{\mathrm{QFT}} = \left(u^2 + \frac{1}{4}\right) L_1 \sqrt{L_2 L_3}\, \mathcal{A}_{\mathrm{hexagon}}\,.$$

$\rightarrow$ Analogous results for $L_1 = 3, 4, \ldots$ with $u = \frac{1}{2}, \frac{1}{2}\sqrt{1 \pm \frac{2}{\sqrt{5}}}, \ldots$

Plan

Lagrangian insertion method

Consider n-point function [Eden, Howe, West '99] [Eden, Petkou, Schubert, Sokatchev '01]...

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \int D\phi \, DA \, D\psi \; e^{\frac{i}{g^2} \int d^4 x_0 \mathcal{L}(x_0)} \; \mathcal{O}_1 \ldots \mathcal{O}_n \, .$$

It follows that

$$g^2 \frac{\partial}{\partial g^2} \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = -\frac{i}{g^2} \int d^4 x_0 \; \langle \mathcal{L}_0 \; \mathcal{O}_1 \ldots \mathcal{O}_n \rangle \; .$$

$\rightarrow$ Introduce **Lagrange operator** as $L = 2$ vacuum descendant

Integrability picture:

- Introduce **double excitations**, *eg.* $|\,{}^{Y}_{Y}\rangle = |\bar{Z}\rangle$, $|\,{}^{\Psi^{\alpha\dot{4}}}_{\Psi^{\beta\dot{3}}}\rangle = |F^{\alpha\beta}\rangle$, . . .
- Yang-Mills Lagrangian $\mathrm{Tr}(F^2)$ build from four fermions

$$\Psi_1^{4\dot{2}} \, , \quad \Psi_2^{4\dot{1}} \, , \quad \Psi_3^{3\dot{2}} \, , \quad \Psi_4^{3\dot{1}} \, ,$$

with (infinite) rapidities $u_1, \ldots, u_4$ and auxiliary rapidities

Idea: Cut correlators into hexagons

On-shell Lagrangian

[Eden, Heslop, Korchemsky, Sokatchev '11]

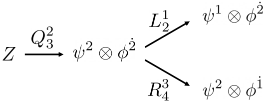
$$\mathcal{L} = \mathrm{tr}\left(-\frac{1}{2}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \sqrt{2}g\,\Psi^{\alpha I}[\Phi_{IJ},\Psi^{J}_{\alpha}] - \frac{1}{8}g^2\,[\Phi^{IJ},\Phi^{KL}][\Phi_{IJ},\Phi_{KL}]\right)\ .$$

We aim to recover the **Yang-Mills term**

$$|\mathcal{F}^{11}\mathcal{F}^{22}\rangle - 2\,|\mathcal{F}^{12}\mathcal{F}^{12}\rangle + |\mathcal{F}^{22}\mathcal{F}^{11}\rangle\ .$$

We can build the field strength as double excitations

$$\begin{aligned}
|\,{}^{\Psi^{1\dot{4}}}_{\Psi^{2\dot{3}}}\rangle &= \mathfrak{L}^{1}{}_{2}\ \mathfrak{Q}^{2}{}_{\dot{3}}\ \mathfrak{R}^{\dot{3}}{}_{\dot{4}}\ \mathfrak{Q}^{2}{}_{\dot{3}}\,|Z\rangle &&= \mathbf{a}^{1\dagger}\mathbf{a}^{2\dagger} = |\mathcal{F}^{12}\rangle\ ,\\
|\,{}^{\Psi^{1\dot{4}}}_{\Psi^{1\dot{3}}}\rangle &= \mathfrak{L}^{1}{}_{2}\ \mathfrak{Q}^{2}{}_{\dot{3}}\ \mathfrak{L}^{1}{}_{2}\ \mathfrak{R}^{\dot{3}}{}_{\dot{4}}\ \mathfrak{Q}^{2}{}_{\dot{3}}\,|Z\rangle &&= \mathbf{a}^{1\dagger}\mathbf{a}^{1\dagger} = |\mathcal{F}^{11}\rangle\ ,\\
|\,{}^{\Psi^{2\dot{4}}}_{\Psi^{2\dot{3}}}\rangle &= \mathfrak{Q}^{2}{}_{\dot{3}}\ \mathfrak{R}^{\dot{3}}{}_{\dot{4}}\ \mathfrak{Q}^{2}{}_{\dot{3}}\,|Z\rangle &&= \mathbf{a}^{2\dagger}\mathbf{a}^{2\dagger} = |\mathcal{F}^{22}\rangle\ .
\end{aligned}$$



Lagrangian insertion: A first test

⇒ First test: **protected** two-point function $\langle \mathcal{L}_0 \, \mathcal{O}_1^L \, \mathcal{O}_2^L \rangle = 0$ [Eden, DℓP, Spiering '23]

$$
\begin{aligned}
\langle \mathcal{L}_0 \, \mathcal{O}_1^L \, \mathcal{O}_2^L \rangle \quad = \quad & 2 \Big[\langle \mathbf{h} | \Psi_1^{4\dot{2}} \, \Psi_2^{4\dot{1}} \, \Psi_3^{3\dot{2}} \, \Psi_4^{3\dot{1}} \rangle + \langle \mathbf{h} | \Psi_1^{4\dot{2}} \, \Psi_4^{3\dot{1}} \rangle \, \langle \mathbf{h} | \Psi_2^{4\dot{1}} \, \Psi_3^{3\dot{2}} \rangle \Big] + \\
& \tilde{g} \Big[\langle \mathbf{h} | D_1^{4\dot{3}} \rangle \, \langle \mathbf{h} | \Psi_2^{4\dot{1}} \, \Psi_3^{3\dot{2}} \, D_4^{3\dot{4}} \rangle + \langle \mathbf{h} | D_2^{4\dot{3}} \rangle \, \langle \mathbf{h} | \Psi_1^{4\dot{2}} \, D_3^{3\dot{4}} \, \Psi_4^{3\dot{1}} \rangle + \\
& \quad \langle \mathbf{h} | D_3^{3\dot{4}} \rangle \, \langle \mathbf{h} | \Psi_1^{4\dot{2}} \, D_2^{4\dot{3}} \, \Psi_4^{3\dot{1}} \rangle + \langle \mathbf{h} | D_4^{3\dot{4}} \rangle \, \langle \mathbf{h} | D_1^{4\dot{3}} \, \Psi_2^{4\dot{1}} \, \Psi_3^{3\dot{2}} \rangle + \\
& \quad \langle \mathbf{h} | Y_1 \rangle \, \langle \mathbf{h} | \Psi_2^{4\dot{1}} \, \Psi_3^{3\dot{2}} \, \bar{Y}_4 \rangle + \langle \mathbf{h} | \bar{Y}_2 \rangle \, \langle \mathbf{h} | \Psi_1^{4\dot{2}} \, Y_3 \, \Psi_4^{3\dot{1}} \rangle + \\
& \quad \langle \mathbf{h} | Y_3 \rangle \, \langle \mathbf{h} | \Psi_1^{4\dot{2}} \, \bar{Y}_2 \, \Psi_4^{3\dot{1}} \rangle + \langle \mathbf{h} | \bar{Y}_4 \rangle \, \langle \mathbf{h} | Y_1 \, \Psi_2^{4\dot{1}} \, \Psi_3^{3\dot{2}} \rangle \Big] + \\
& \tilde{g}^2 \Big[\langle \mathbf{h} | D_1^{4\dot{3}} \, D_2^{4\dot{3}} \rangle \, \langle \mathbf{h} | D_3^{3\dot{4}} \, D_4^{3\dot{4}} \rangle + \langle \mathbf{h} | D_1^{4\dot{3}} \, \bar{Y}_2 \rangle \, \langle \mathbf{h} | Y_3 \, D_4^{3\dot{4}} \rangle + \\
& \quad \langle \mathbf{h} | Y_1 \, D_2^{4\dot{3}} \rangle \, \langle \mathbf{h} | D_3^{3\dot{4}} \, \bar{Y}_4 \rangle + \langle \mathbf{h} | Y_1 \, \bar{Y}_2 \rangle \, \langle \mathbf{h} | Y_3 \, \bar{Y}_4 \rangle \Big]
\end{aligned}
$$

$$
\langle \mathcal{L}_0 \, \mathcal{O}_1^L \, \mathcal{O}_2^L \rangle \to 4 \, \tilde{g}^2 \, (1 - 2 + 1) = 0
$$

Anomalous dimension

Consider two-point function

$$\langle \mathcal{B}_1^L \mathcal{B}_2^L \rangle = \frac{1}{(a_1 - a_2)^{2(\Delta_0 + g^2 \gamma_1 + \dots)}} = \frac{1}{(a_1 - a_2)^{2\Delta_0}} - 2\gamma_1 \frac{\log(a_1 - a_2)}{(a_1 - a_2)^{2\Delta_0}} g^2 + \dots ,$$

$\Rightarrow$ Reproduce **anomalous dimension** $\langle \mathcal{L}_0 \, \mathcal{B}_1^L \, \mathcal{B}_2^L \rangle \propto \gamma_1$ [Eden, Gottwald, DℓP, Scherdin '23]

Finite momenta of physical excitations
$\rightarrow$ length changing effects become apparent: introduce **Z markers**
$\rightarrow$ distribution of Z is not important here!
$\rightarrow$ likely to be important for more complicated tessellations

$\rightarrow$ cancellation of **particle creation poles**

Finally, we obtain

$$\langle \mathcal{L}_0 \, \mathcal{B}_1^L \, \mathcal{B}_2^L \rangle = -\frac{\gamma_1}{\sqrt{4!}} ,$$

for lengths $L = 4, \dots, 9$.

Plan

Conclusions and Outlook

- Powerful tool to calculate correlation functions in $\mathcal{N} = 4$ SYM

- Maintain the hexagon operator for **higher-rank** sectors
  - import the g-coefficients from the nested Bethe ansatz
  - local details of the wave functions are eclipsed

- **Marginal deformations** for certain classes of correlators
  $\rightarrow$ Is there a hexagon operator for deformed theories?

- Lagrangeoperator in integrability formalism using **double excitations**
  - four fermions build the Yang-Mills term

- First tests of **Lagrangian insertion** method for hexagon tessellations
  - protected two-point functions
  - anomalous dimension from two-point functions

  $\rightarrow$ Loop corrections for more general two- and three-point functions?
  $\rightarrow$ Non-planar corrections?