The fate of chiral symmetry in the quark-gluon plasma

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Symmetries of QCD and their realization

- partition function

$$Z = \int \mathscr{D}U \prod_f \det\left(D[U] + m_\mathrm{f}\right) \cdot \mathrm{e}^{-S_\mathrm{g}[U]}$$

- $m_\mathrm{u} \approx m_\mathrm{d} \approx 0$

- Symmetries: $SU(2)_\mathrm{V} \times SU(2)_\mathrm{A} \times U(1)_\mathrm{V} \times U(1)_\mathrm{A}$
  - Axial: left-right opposite phase
  - $U(1)$ same for two flavors
  - $SU(2)$ mixes two flavors

- $U(1)_\mathrm{A}$ anomalous

- $SU(2)_\mathrm{A}$ spontaneously broken below T_c

The $SU(2)_A$ symmetry

- Order parameter of $SU(2)_A$

$$\langle \bar{\psi}\psi \rangle \;\propto\; \frac{1}{V}\sum_i \frac{1}{\lambda_i + m} \;\propto\; \int_{-\Lambda}^{\Lambda} d\lambda \, \frac{m}{\lambda^2 + m^2} \cdot \rho(\lambda)$$

  - λ_i — eigenvalues of the Dirac operator
  - — imaginary opposite sign pairs or zeros
  - $\rho(\lambda)$ — spectral density of Dirac operator

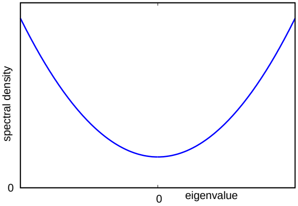
- $\langle \bar{\psi}\psi \rangle$ dominated by spectral part $|\lambda| \lesssim m$

- $\lim_{m \to 0} \langle \bar{\psi}\psi \rangle \propto \rho(0)$ Banks-Casher formula

The finite temperature transition
Standard picture

Below T_c

- Chiral symmetry broken
- Order parameter: $\rho(0) \neq 0$



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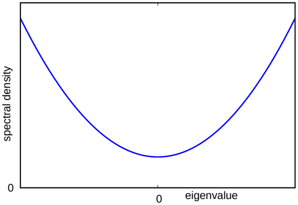
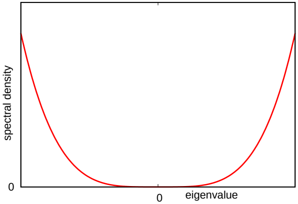


Above T_c

- Chiral symmetry restored
- Order parameter $\rho(0) = 0$
- (Pseudo)gap (lowest Matsubara mode)

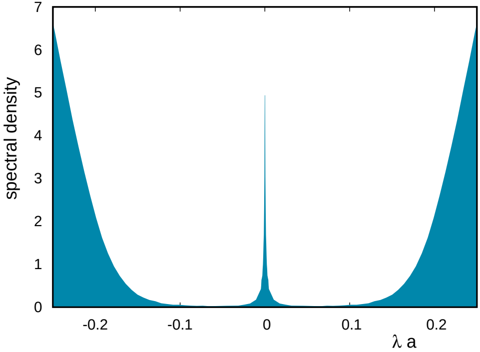


spectral density at 0 $\Longleftrightarrow$ realization of chiral symmetry

Spectral density at $T = 1.1\,T_c$ from the lattice

quenched: quark back reaction omitted

$$Z = \int \mathscr{D}U \, \prod_f \cancel{\det(D[U] + m_f)} \cdot e^{-S_g[U]}$$



Spike at zero in the spectral density!

Edwards et al. 2000; Alexandru & Horvath 2015; Kaczmarek, Mazur, Sharma 2021

Questions

- Why is there a spike at zero?
- Suppression by the quark determinant?
  (dynamical quarks)
- How does the peak influence the realization of chiral symmetry as $m \to 0$?

Spectral peak $\leftarrow$ instantons

- Instanton: special gauge field configuration
  with topological winding

- A “lump” of action localized in space-time

- (Anti)instanton
  $\rightarrow$ zero eigenvalue of $D(A)$ with $(-)+$ chirality eigenmode

- Topological charge $Q = n_i - n_a$
  $\rightarrow$ $|Q|$ exact zero eigenvalues (Atiyah-Singer index theorem)

- Its fluctuations $\chi = \frac{1}{V}\langle Q^2 \rangle$ (topological susceptibility)

Two instantons

- Two instantons
  - Topological charge $Q = 2$
  - $\rightarrow$ two zero modes
- An instanton and an antiinstanton
  - Topological charge $Q = 0$
  - No exact zero eigenvalues
  - Symmetric splitting from zero controlled by distance of I and A

Instantons above T_c $\rightarrow$ small Dirac eigenvalues

- $T = 1/L_t \quad \rightarrow \quad$ High T: small temporal size
  large instantons "squeezed out" in the temporal direction
- Dilute gas of instantons and antiinstantons
- Zero modes exponentially localized
- Mixing (splitting) of I-A near zero modes small
- n_i instantons n_a antiinstantons
  $\rightarrow$ $|n_i - n_a|$ exact zero modes + mixing near zero modes

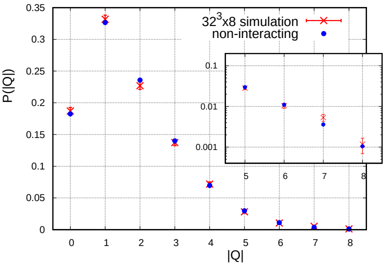
Quenched approximation → free instanton gas

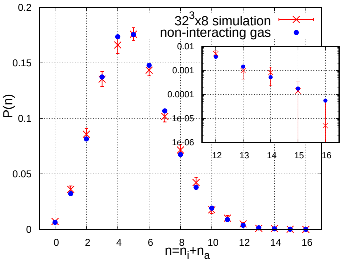
lattice result

- Free instanton gas:
  (anti)instanton number distribution iid Poisson:

$$p(n_\mathrm{i}, n_\mathrm{a}) = \mathrm{e}^{-\chi V/2} \frac{(\chi V/2)^{n_\mathrm{i}}}{n_\mathrm{i}!} \cdot \mathrm{e}^{-\chi V/2} \frac{(\chi V/2)^{n_\mathrm{a}}}{n_\mathrm{a}!}$$

- Number distribution of exact and near zero modes in the peak consistent with free instanton gas   Vig R. & TGK 2021





Including dynamical quarks (quark determinant)

$$Z = \int \mathscr{D}U \prod_f \det\left(D[U] + m_f\right) \cdot e^{-S_g[U]}$$

- Light dynamical quarks are expected to
  - Suppress small Dirac eigenvalues ($\rightarrow$ fewer instantons)
  - Introduce I-A interaction ($\rightarrow$ pull pairs closer)
- What happens to the spectral peak and χsymm if $m \rightarrow 0$?
- Dirac operator with exact chiral symmetry needed
  - Exact zero eigenvalues
  - Resolve peak in spectral density
- Direct lattice simulation with det presently not possible

Instanton-based random matrix model (quenched)

work by Shuryak, Diakonov, Verbaarschot et al.

- Model of Dirac operator in the subspace of zero modes
- Quenched – ideal instanton gas:
  - Choose n_i and n_a from independent Poisson distributions of mean $\chi_0 V/2$.
  - Place (anti)instantons randomly in 3d box of size L^3 ($V = L^3/T$).
  - Construct $(n_\text{i}+n_\text{a}) \times (n_\text{i}+n_\text{a})$ random matrix:

$$D = \begin{pmatrix} \overbrace{0}^{n_\text{i}} & \overbrace{iW}^{n_\text{a}} \\ iW^\dagger & 0 \end{pmatrix}$$

  - $w_{ij} = A \cdot \exp(-\pi T \cdot r_{ij})$

    r_{ij} is the distance of instanton i and antiinstanton j.

Properties of the random matrix Dirac operator

- $\mathrm{rank}(D) = 2\min(n_i, n_a) \quad \Rightarrow \quad |n_i - n_a| \quad$ zero modes

- $D\gamma_5 + \gamma_5 D = 0 \quad \Leftrightarrow \quad$ chiral symmetry

- Generator of chiral symmetry

$$\gamma_5 = \left(\begin{array}{c|c} \overbrace{\mathbb{1}}^{n_i} & \overbrace{0}^{n_a} \\ \hline 0 & -\mathbb{1} \end{array}\right)$$

- Structure of D: tight-binding Hamiltonian on bipartite lattice

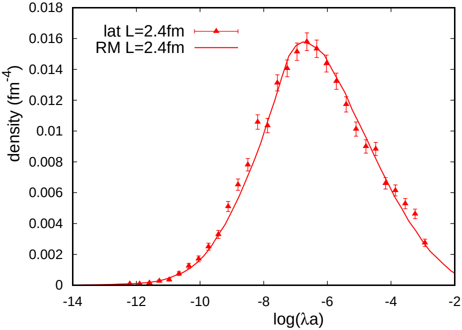
  (zero on-site energy, off-diagonal disorder)

Fit parameters to quenched lattice Dirac spectrum

$T = 1.1\,T_c$ overlap Dirac spectrum

- Two parameters:
  - χ_0 – instanton density: from exact zero modes $\rightarrow \chi_0 = \langle Q^2 \rangle / V$
  - A – prefactor of the exponential mixing between zero modes
- Fit A to distribution of Dirac eigenvalues

lowest eigenvalue: $L = 2.4$fm fit



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  - χ_0 – instanton density: from exact zero modes $\rightarrow$ $\chi_0 = \langle Q^2 \rangle / V$
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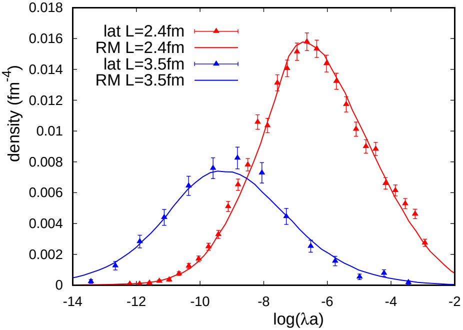
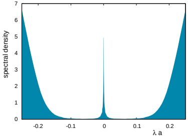
lowest eigenvalue: $L = 2.4$fm fit $L = 3.5$fm prediction



Random matrix model with dynamical quarks

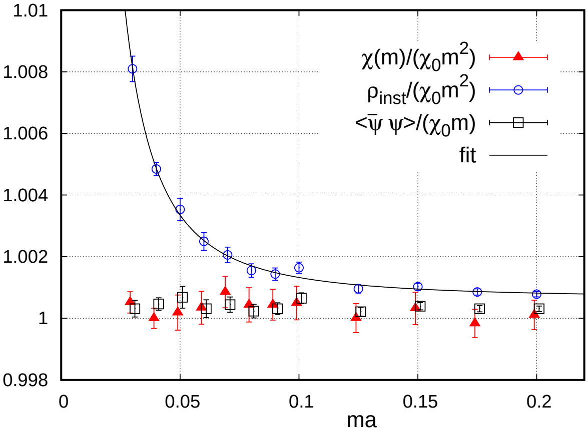
- On the lattice additional weight $\det(D+m)^{N_f}$
- $\det(D+m) = \prod_{\text{zmz}} (\lambda_i + m) \times \prod_{\text{bulk}} (\lambda_i + m)$
- Bulk weakly correlated with zero mode zone



- Approximate det with $\prod_{\text{zmz}} (\lambda_i + m)$
- Consistently included in RM model:

$$P(n_i, n_a) = \underbrace{e^{-\chi_0 V} \frac{1}{n_i!} \frac{1}{n_a!} \left(\frac{\chi_0 V}{2} \right)^{n_i + n_a}}_{\text{free instanton gas}} \cdot \det(D+m)^{N_f}$$

Random matrix model simulation results for $N_f = 2$



$$\chi(m) \approx m^2\chi_0 \qquad \langle\bar{\psi}\psi\rangle \approx m\chi_0 \qquad \rho_{inst} \approx m^2\chi_0 + c$$

Explanation – free instanton gas?

- If $|\lambda_i| \ll m \quad \Longrightarrow \quad \prod_i (\lambda_i + m) \approx m^{n_\mathrm{i}+n_\mathrm{a}}$

  det depends on number of topological objects, not on their type or location

- $$\left(\frac{\chi_0 V}{2}\right)^{n_\mathrm{i}+n_\mathrm{a}} \cdot \det(D+m)^{N_\mathrm{f}} \approx \left(\frac{m^{N_\mathrm{f}}\chi_0 V}{2}\right)^{n_\mathrm{i}+n_\mathrm{a}}$$

- Distribution of number of (anti)instantons still Poisson

- Free gas, but susceptibility suppressed as $\chi_0 \to m^{N_\mathrm{f}}\chi_0$

- What happens in the chiral limit, if m becomes smaller?

$|\lambda| \ll m$ even in the chiral limit

- Quark determinant for n_i instantons and n_a antiinstantons:

$$\det(D+m)^{N_f} = \prod_{n_\mathrm{i},n_\mathrm{a}} (\lambda_i + m)^{N_f} = m^{N_f(n_\mathrm{i}+n_\mathrm{a})} \cdot \prod_{i=1}^{\#pairs} \left(1 + \frac{\lambda_i^2}{m^2}\right)^{N_f}$$

- smaller m

$\rightarrow$ $m^{N_f(n_\mathrm{i}+n_\mathrm{a})}$ suppresses instantons

$\rightarrow$ instanton gas more dilute

$\rightarrow$ smaller matrix elements

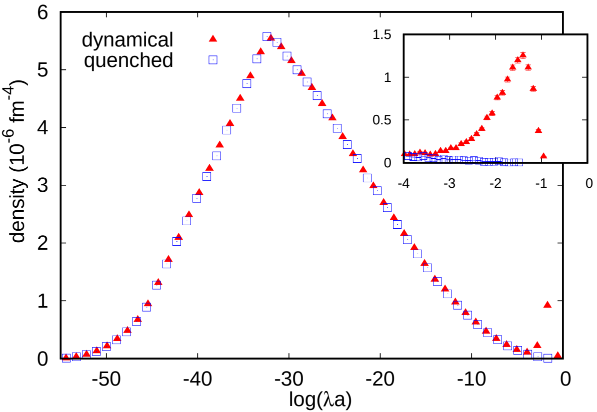
$\rightarrow$ smaller eigenvalues $\Longrightarrow$ $|\lambda| \ll m$ even in the chiral limit

$\rightarrow$ free instanton gas

Spectral density – full QCD vs. ideal instanton gas

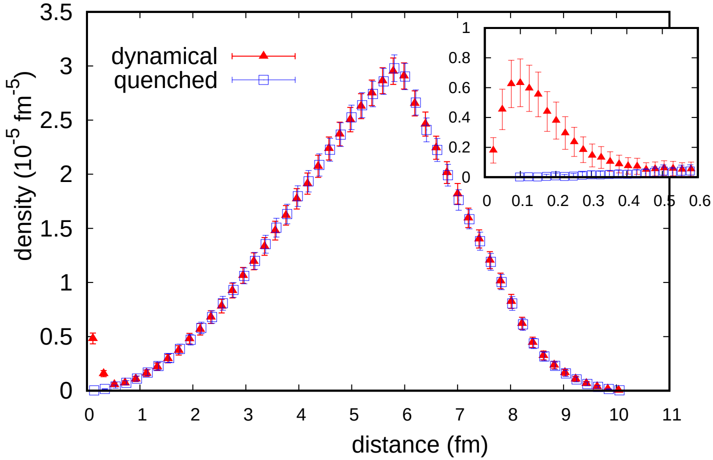
dynamical — determinant included

quenched — no determinant, but top. susceptibility scaled with m^{N_f}



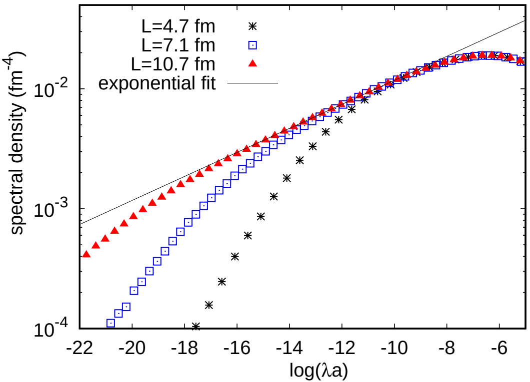
Instanton-antiinstanton molecules

density of closest opposite charge pairs at given distance



Spectral density singular at the origin for $V \to \infty$

RM model simulation, parameters from quenched $T = 1.1\, T_c$ overlap spectrum



If $\rho(\lambda) \propto \lambda^{\alpha}$ then $\tilde{\rho}(\log(\lambda)) \propto e^{(1+\alpha)\log(\lambda)}$ fit: $\alpha = -0.775(5)$ $\Rightarrow$ Matteo's talk

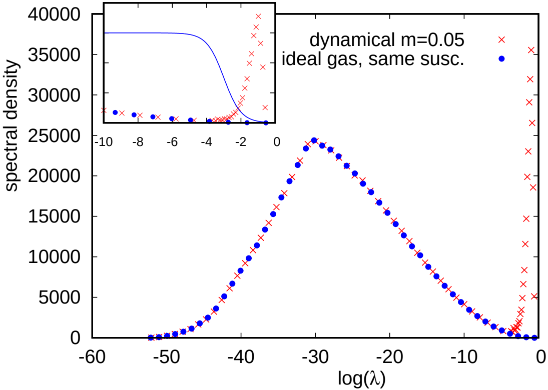
Banks-Casher for a singular spectral density?

“Banks-Casher” for singular spectral density

free instanton gas contribution dominates condensate

$$\langle\bar{\psi}\psi\rangle \approx \langle\sum_i \frac{m}{m^2+\lambda_i^2}\rangle \approx \underbrace{\begin{pmatrix}\text{avg. number of in-}\\ \text{stantons in free gas}\end{pmatrix}}_{m^{N_f}\chi_0 V} \cdot \frac{1}{m} = m^{N_f-1}\chi_0 V$$

$$|\lambda_i| \ll m$$



Fate of $U(1)_A$ symmetry as $m \to 0$

- Free IA gas eigenvalues $|\lambda_i| \ll m$ for any quark mass

- $U(1)_A$ breaking susceptibility $\chi_\pi - \chi_\delta = ?$
  - HotQCD: $\chi_\pi - \chi_\delta \neq 0$ (staggered) $\rightarrow$ no restoration above T_c
  - JLQCD: $\chi_\pi - \chi_\delta = 0$ (domain wall and overlap) $\rightarrow$ restored above T_c

- $$\chi_\pi - \chi_\delta \approx \langle \sum_i \frac{m^2}{(m^2+\lambda_i^2)^2} \rangle \approx \underbrace{\begin{pmatrix}\text{avg. number of in-}\\ \text{stantons in free gas}\end{pmatrix}}_{m^{N_f}\chi_0 V} \cdot \frac{1}{m^2} = m^{N_f-2}\chi_0 V$$

- Contribution of IA molecules in $m \to 0$ limit: $|\lambda_i| \gg m$
  - Contribution to $\langle \bar{\psi}\psi \rangle \propto m$
  - Contribution to $\chi_\pi - \chi_\delta \propto m^2$

Conclusions

- At high T non-interacting degrees of freedom:
  free instantons (+ IA molecules)

- Dirac spectral density has singular peak at zero
  at any finite T, for any nonzero quark mass

- Chiral symmetry restoration nontrivial (anomaly remains)

- Even though $SU(N_f)_A$ restored,
  order of the $m \to 0$ and $V \to \infty$ limit can be important

- Chiral limit with N_f degenerate light quarks:
  - $\langle \bar{\psi}\psi \rangle \propto m^{N_f - 1}$ agrees with small m expansion of the free energy
    Kanazawa and Yamamoto (2015)
  - $\chi_\pi - \chi_\delta \propto m^{N_f - 2}$