Topological String Theory and Generalized Geometry

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Outline

Introduction

- Topological string theory
- T-duality
- Generalized geometry

AKSZ sigma-models

- Courant-algebroids and Courant sigma-models
- Double field theory and the corresponding algebraic structures
- AKSZ sigma-models

A/B-models and generalized geometry

[1805.11485]

- Double field construction
- Relation to generalized geometry
- Topological S-duality from generalized complex structure

Topological string theory

- Where does it come from?

$$\left.\begin{array}{r} \mathcal{N}=2 \text{ sigma-model} \\ \& \quad \text{coupled to gravity} \end{array}\right\} \longrightarrow \quad \text{bosonic string theory}$$

$$\left.\begin{array}{r} \text{topological sigma-model} \\ \& \quad \text{coupled to gravity} \end{array}\right\} \longrightarrow \quad \text{topological string theory}$$

- Procedure to get the topological sigma-model: $\rightsquigarrow$ topological twisting

$$\mathcal{N}=2 \text{ sigma-model} \quad \xrightarrow{\text{twist}} \quad \text{topological sigma-model}$$

- Two non-equivalent twists:

$\rightsquigarrow$ A- and B-models

- Where does it appear in 'physical' string theory?

a) IIA or IIB compactifications $\rightsquigarrow$ $\mathcal{N}=2$ superpotential in 4 dim

b) IIA compactification $\rightsquigarrow$ entropy of BPS black hole

Topological A-model

- A-twist:

$$\mathcal{Q} = Q_{++} + Q_{--}$$

  is a cohomological charge $\rightsquigarrow$ topological theory.

- A-model action

$$S_{\mathrm{A}} = 2t \int_{\Sigma_2} \mathrm{d}^2 z \, \Big(g_{a\bar{b}} \, \partial_{\bar{z}} X^a \, \partial_z X^{\bar{b}} + \mathrm{i} \, g_{a\bar{b}} \, (\chi^a_{\bar{z}} \, \nabla_z \psi^{\bar{b}} + \chi^{\bar{b}}_z \, \nabla_{\bar{z}} \psi^a) - R_{a\bar{b}c\bar{d}} \, \chi^a_{\bar{z}} \, \chi^{\bar{b}}_z \, \psi^c \, \psi^{\bar{d}} \Big) + t \int_{\Sigma_2} \mathrm{d}^2 z \, X^*(k)$$

  where

  $X^a, X^{\bar{a}} : \Sigma_2 \rightarrow X$ Calabi-Yau,

  $(\psi^a, \psi^{\bar{a}})$ fermionic scalars with ghost number 1,

  $(\chi^a_{\bar{z}}, \chi^{\bar{a}}_z)$ fermionic 1-forms with ghost number -1,

- Complex structure deformation $\rightarrow$ BRST exact term

  $\rightsquigarrow$ Quantities depend only on the Kähler structure of X ($\leftrightarrow$ Poisson str.)

Topological B-model

- B-twist:

$$\mathcal{Q} = Q_{+-} + Q_{--}$$

is a cohomological charge $\rightsquigarrow$ topological theory.

- B-model action

$$S_{\mathrm{B}} = t \int_{\Sigma_2} \mathrm{d}^2 z \, \Big(g_{a\bar{b}} \, (\partial_z X^a \, \partial_{\bar{z}} X^{\bar{b}} + \partial_{\bar{z}} X^a \, \partial_z X^{\bar{b}}) - g_{a\bar{b}} \, (\rho_z^a \, \nabla_{\bar{z}} \eta^{\bar{b}} + \rho_{\bar{z}}^a \, \nabla_z \eta^{\bar{b}})$$

$$+ \rho_z^a \, \nabla_{\bar{z}} \chi_a - \rho_{\bar{z}}^a \, \nabla_z \chi_a - R^a{}_{b\bar{c}d} \, \rho_z^b \, \rho_{\bar{z}}^d \, \eta^{\bar{c}} \, \chi_a \Big)$$

where

$X^a, X^{\bar{a}} : \Sigma_2 \rightarrow X$ Calabi-Yau,

$(\eta^{\bar{a}}, \chi_a)$ fermionic scalars with ghost number 1,

$(\rho_z^a, \rho_{\bar{z}}^a)$ fermionic 1-forms with ghost number -1,

- Holomoprhic and antiholomorphic fields are explicitly distinguished

$\rightsquigarrow$ Quantities depend only on the complex structure of X

T-duality

- Compactification on a 6 dim torus X

$$\rightsquigarrow \text{ 4 dim effective theory on } M_4$$

- In this case there exists a symmetry transformation on the inner space X

$$\text{IIA string theory} \quad \overset{\mathrm{T}}{\longleftrightarrow} \quad \text{IIB string theory}$$

$\Rightarrow$ The closed string spectrum

$$M^2 = (N + \widetilde{N} - 2) + p^2 \frac{l_s^2}{R^2} + w^2 \frac{R^2}{l_s^2}$$

is invariant under T-duality:

$$R \quad \overset{\mathrm{T}}{\longleftrightarrow} \quad \frac{l_s^2}{R}$$

$$\text{string momenta}\,(p_\mu) \quad \overset{\mathrm{T}}{\longleftrightarrow} \quad \text{winding numbers}\,(w_\mu)$$

- Generalized tangent bundle:

$$\begin{pmatrix} p_\mu\, \mathrm{d}x^\mu \\ w^\mu\, \partial_\mu \end{pmatrix} \in T^*X \oplus TX \implies \text{Generalized geometry}$$

Generalized geometry

- 'Generalization' is based on two premises:

  1) TX is replaced by $TX \oplus T^*X$

  2) Lie bracket on TX is replaced by the Courant-bracket

$$[A+\alpha, B+\beta]_{\mathrm{C}} \;=\; [A,B]_{TX} \;+\; \mathcal{L}_A\beta \;-\; \mathcal{L}_B\alpha \;-\; \frac{1}{2}\,\mathrm{d}(\iota_A\beta - \iota_B\alpha)$$

  for $A, B \in TX$ and $\alpha, \beta \in T^*X$.

- T-duality is a covariant $O(d,d)$ symmetry group ($d = \mathrm{dim}X$), which leaves the symmetric pairing on $TX \oplus T^*X$ invariant

$$\langle A+\alpha, B+\beta\rangle \;=\; \frac{1}{2}\,(\iota_A\beta + \iota_B\alpha)$$

- Unified framework for symplectic geometry and complex geometry:

  $\rightsquigarrow$ within generalized complex geometry

  Different choice of gen. complex str. $\nearrow$ symplectic str. / $\searrow$ complex str.

Fluxes in string theory

- After comapctification, the theory is left with massless scalar fields with no potential (called moduli) $\rightsquigarrow$ instability
- Solution: introducing fluxes ($H = \mathrm{d}B$ or RR-fields)

$$\Rightarrow \quad \text{stabilizes the moduli} \quad \& \quad \text{breaks supersymmetry}$$

- We have a chain of fluxes acting with T-duality

$$\underbrace{H_{\mu\nu\rho}}_{H=\mathrm{d}B} \overset{T}{\longleftrightarrow} \underbrace{F^{\mu}{}_{\nu\rho}}_{\text{torsion of the metric}} \overset{T}{\longleftrightarrow} \underbrace{Q^{\mu\nu}{}_{\rho}}_{\text{non-comm. space}} \overset{T}{\longleftrightarrow} \underbrace{R^{\mu\nu\rho}}_{\text{non-assoc. space}}$$

We need all fluxes to stabilize the moduli!

- The fluxes back-react on the compactified geometry

$$\text{Compact. without fluxes} \longrightarrow \begin{cases} M_4 : \mathcal{N} = 2 \text{ SuSy} \\ X : \text{Calabi-Yau} \end{cases} \leftsquigarrow \text{topological strings}$$

$$\text{Compact. with fluxes} \longrightarrow \begin{cases} M_4 : \mathcal{N} = 1 \text{ SuSy (!)} \\ X : \text{Generalized Calabi-Yau} \end{cases} \leftsquigarrow \quad ?$$

Courant algebroid

- Algebraic construction to the Courant bracket is called Courant algebroid
  Vector bundle $E \to X$ equiped with

$$\langle\,.\,,\,.\,\rangle \;:\; \Gamma(E)\times\Gamma(E)\to\mathbb{R} \quad \text{non-degen. bilin. form}$$

$$\rho \;:\; E \to TX \quad \text{anchor map}$$

$$[\![\,.\,,\,.\,]\!] \;:\; \Gamma(E)\times\Gamma(E)\to\Gamma(E) \quad \text{Dorfman bracket}$$

  which together satisfy

$$[\![e_1, [\![e_2, e_3]\!]]\!] = [\![[\![e_1, e_2]\!], e_3]\!] + [\![e_2, [\![e_1, e_3]\!]]\!]$$

$$\rho(e_1)\langle e_2, e_3\rangle = \langle [\![e_1, e_2]\!], e_3\rangle + \langle e_2, [\![e_1, e_3]\!]\rangle$$

$$\rho(e_1)\langle e_2, e_3\rangle = \langle e_1, [\![e_2, e_3]\!] + [\![e_3, e_2]\!]\rangle$$

- Courant bracket is the antisymmetrization of a *particular* Dorfman bracket
- Fluxes appear as twist deformations of Courant algebroids

  e.g. for H-flux: $[\![A+\alpha, B+\beta]\!]_H = [A,B]_{TM} + \mathcal{L}_A\beta - \iota_B\,\mathrm{d}\alpha + \iota_A\iota_B H$

QP-manifolds

- Theorem by Roytenberg

$$\text{Courant algebroid} \quad \longleftrightarrow \quad \text{QP-manifold of degree 2}$$

- $(\mathcal{M}, Q_\gamma, \omega)$ is a QP-manifold of degree n ($\mathcal{M}$: dg-manifold)

ω: graded symplectic structure of degree $n+2$

$$\to \ \{.,.\} \quad \text{graded Poisson bracket of degree } -n$$

Q_γ: Hamiltonian cohomological vector field $Q_\gamma = \{\gamma, .\}$

$$Q_\gamma^2 = 0 \quad \Longleftrightarrow \quad \{\gamma,\gamma\} = 0 \quad \text{master equation}$$

- Courant algebroid operations ($n=2$) on degree 1 functions e_1, e_2 are

$$[\![e_1, e_2]\!] = \{\{e_1,\gamma\}, e_2\} \qquad \langle e_1, e_2\rangle = \{e_1, e_2\} \qquad \rho(e) = \{e, \{\gamma, \cdot\}\}$$

- Standard Courant algebroid ($E = TX \oplus T^*X$, $\mathcal{M} = T^*[2]T[1]X$)

$$\omega = \mathrm{d}\underset{0}{X^i} \wedge \mathrm{d}\underset{2}{F_i} + \mathrm{d}\underset{1}{\chi_i} \wedge \mathrm{d}\underset{1}{\psi^i} \quad \Rightarrow \quad \{X^i, F_j\} = \delta^i_j \quad \& \quad \{\chi_i, \psi^j\} = \delta^j_i$$

$$\gamma = F_i \psi^i \quad \rightsquigarrow \quad \text{Courant bracket}$$

Courant sigma-models

- General QP-manifold of degree 2

$$\omega = \mathrm{d}\underset{0}{X^i} \wedge \mathrm{d}\underset{2}{F_i} + \frac{1}{2} k_{ab}\, \mathrm{d}\underset{1}{\zeta^a} \wedge \mathrm{d}\underset{1}{\zeta^b}$$

$$\gamma = \underset{\rightsquigarrow \text{anchor}}{\rho^i{}_a(X)}\, F_i\, \zeta^a + \frac{1}{3!} \underset{\rightsquigarrow \text{Jacobiator}}{T_{abc}(X)}\, \zeta^a \zeta^b \zeta^c$$

- It corresponds to a 3 dim topological sigma-model: Courant sigma-model

A coordinate ϕ with degree $|\phi|$ $\longrightarrow$ field $\phi(\sigma)$ with ghost number $|\phi|$

$$S_{\mathrm{C}} = \int_{\Sigma_3} \mathrm{d}^3\sigma \Big(F_i \wedge \mathrm{d}X^i + \frac{1}{2} k_{ab} \zeta^a \wedge \mathrm{d}\zeta^b + \rho^i_a(X)\, F_i \wedge \zeta^a + \frac{1}{3!} T_{abc}(X)\, \zeta^a \wedge \zeta^b \wedge \zeta^c \Big)$$

Eq. of motion $\iff$ $\{\gamma, \gamma\} = 0$ $\iff$ Courant algebroid axioms

- BV-BRST quantization:

$\Longrightarrow$ AKSZ construction

AKSZ sigma-models

- AKSZ construction: a geometric method for constructing BV quantized topological sigma-models.

  [Alexandrov,Kontsevich,Schwartz,Zaboronsky]

  Well known examples are Poisson sigma-model, A/B-models, Chern-Simons theory, Courant sigma-model, etc.

- Construction:

  Source manifold: $T[1]\Sigma_d$ graded worldvolume with $\dim \Sigma_d = d$

  Target manifold: $(\mathcal{M}, Q_\gamma, \omega)$ QP-manifold of degree $d-1$

- The space of fields is the mapping space

$$\boldsymbol{\mathcal{M}} = \mathrm{Map}(T[1]\Sigma_d\,,\,\mathcal{M})$$

$\Longrightarrow$ An arbitrary coordinate $\phi \in \mathcal{M}$ of degree $|\phi|$ corresponds to a superfield $\boldsymbol{\phi} \in \boldsymbol{\mathcal{M}}$ of ghost number $|\phi|$

$$\boldsymbol{\phi}(\sigma,\theta) = \phi^{(0)}(\sigma) + \phi^{(1)}_\mu(\sigma)\theta^\mu + \ldots + \frac{1}{d!}\phi^{(d)}_{\mu_1\cdots\mu_d}\theta^{\mu_1}\ldots\theta^{\mu_d}$$

$\hat{z} = (\sigma,\theta) \in T[1]\Sigma_d$ are even and odd coordinates respectively.

AKSZ sigma-models

- $\mathcal{M} = \mathrm{Map}\big(T[1]\Sigma_d\,,\,\mathcal{M}\big)$ is also a QP-manifold with cohomological vector field $\boldsymbol{Q} = \boldsymbol{Q}_0 + \boldsymbol{Q}_\gamma$, which in local 'coordinates' $\phi^{\hat{\imath}}(\hat{z})$ of $\mathcal{M}$ is

$$\boldsymbol{Q}_0 \;=\; \int_{T[1]\Sigma_d} \mathrm{d}^d\hat{z}\; \boldsymbol{D}\phi^{\hat{\imath}}(\hat{z})\,\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}\phi^{\hat{\imath}}(\hat{z})}\;, \qquad \boldsymbol{Q}_\gamma \;=\; \int_{T[1]\Sigma_d} \mathrm{d}^d\hat{z}\; Q_\gamma^{\hat{\imath}}\big(\phi(\hat{z})\big)\,\frac{\boldsymbol{\delta}}{\boldsymbol{\delta}\phi^{\hat{\imath}}(\hat{z})}$$

where $\boldsymbol{D} := \theta^\mu\,\frac{\partial}{\partial\sigma^\mu}$, and the BV symplectic form is

$$\boldsymbol{\omega} \;=\; \int_{T[1]\Sigma_d} \mathrm{d}^d\hat{z}\;\mathrm{ev}^*(\omega)\;, \qquad \mathrm{ev} : (\hat{z},\phi) \longmapsto \phi(\hat{z})$$

- The AKSZ action is the Hamiltonian function of $\boldsymbol{Q} = (\boldsymbol{\mathcal{S}}\,,\,.\,)_{\mathrm{BV}}$

$$\boldsymbol{\mathcal{S}} = \underset{\substack{\uparrow\\ \vartheta}}{\boldsymbol{\mathcal{S}}_{\mathrm{kin}}} + \underset{\substack{\uparrow\\ \gamma}}{\boldsymbol{\mathcal{S}}_{\mathrm{int}}} \qquad\qquad (\omega = \mathrm{d}\vartheta)$$

It gives a solution to the master equation

$$(\boldsymbol{\mathcal{S}},\boldsymbol{\mathcal{S}})_{\mathrm{BV}} = 0 \qquad \longleftrightarrow \qquad \{\gamma,\gamma\} = 0$$

and defines the BV-BRST transformation

$$\boldsymbol{Q} = (\boldsymbol{\mathcal{S}}\,,\,.\,)_{\mathrm{BV}}$$

Example: Poisson sigma-model (or A-model)

- Target: $\mathcal{M} = T^*[1]X$ (degree $n = 1$); general Hamiltonian (with $|\gamma| = 2$):

$$\omega = \mathrm{d}\underset{1}{\chi_i} \wedge \mathrm{d}\underset{0}{X^i} \qquad \text{and} \qquad \gamma = \frac{1}{2}\,\pi^{ij}(X)\,\chi_i\,\chi_j$$

- The master equation makes π^{ij} to define a Poisson structure

$$\{\gamma,\gamma\} = 0 \qquad \longleftrightarrow \qquad (\pi^{[i|l}\partial_l \pi^{|jk]} = 0)$$

- In general, γ determines a derived bracket:

$$\{\{f,\gamma\},g\} = -\{f,g\}_\pi = -\pi(\mathrm{d}f \wedge \mathrm{d}g) \qquad \text{Poisson bracket on } C^\infty(X)$$

- BV bracket and AKSZ action are

$$(\,\cdot\,,\,\cdot\,)_{\mathrm{BV}} = \int_{T[1]\Sigma_2} \mathrm{d}^2\hat{z}\; \frac{\overleftarrow{\delta}}{\delta \boldsymbol{X}^i} \wedge \frac{\overrightarrow{\delta}}{\delta \boldsymbol{\chi}_i}$$

$$\boldsymbol{S}^{(2)}_\pi = \int_{T[1]\Sigma_2} \mathrm{d}^2\hat{z}\,\left(\boldsymbol{\chi}_i\,\boldsymbol{D}\boldsymbol{X}^i + \frac{1}{2}\,\boldsymbol{\pi}^{ij}\,\boldsymbol{\chi}_i\,\boldsymbol{\chi}_j\right) \quad \xleftarrow{\text{on-shell quant.}} \quad \int_{\Sigma_2} X^*(B)$$

Gauge fixing

- Gauge fixing:

  1) Assign 'fields' ϕ^a and 'antifields' ϕ_a^+:

$$\text{paired in} \quad \boldsymbol{\omega} \;=\; \int_{T[1]\Sigma_d} \mathrm{d}^d\hat{z}\;\; \boldsymbol{\delta}\phi_a^+(\hat{z})\,\boldsymbol{\delta}\phi^a(\hat{z})$$

  2) Gauge fix the antifields $\Longleftrightarrow$ Choose a Lagrangian submanifold

$$\boldsymbol{\mathcal{L}} \subset \boldsymbol{\mathcal{M}} \qquad (\boldsymbol{\omega}|_{\boldsymbol{\mathcal{L}}} = 0)$$

$$\text{e.g.} \quad \phi_a^+(\hat{z}) \;=\; (-1)^{|\boldsymbol{\Phi}^a|\,(d+1)}\,\frac{\vec{\delta}\,\boldsymbol{\Psi}}{\delta\phi^a(\hat{z})} \quad \Rightarrow \quad \boldsymbol{\omega}|_{\boldsymbol{\Psi}} = 0$$

- Example: A-model from AKSZ Poisson sigma-model ($\pi \rightsquigarrow$ Kähler form)

$$X^{(0)\,i} \;\rightarrow\; X^{a,\bar{a}} \qquad\qquad \chi_i^{(0)} \;\rightarrow\; \psi^{a,\bar{a}} \qquad\qquad X_\mu^{(1)} \;\rightarrow\; \chi_{\bar{z}}^{a}, \chi_z^{\bar{a}}$$

  Choosing an appropriate $\boldsymbol{\Psi}$ gauge fixing fermion

$$\Rightarrow \quad S_{\mathrm{A}} \;=\; 2t\int_{\Sigma_2} \mathrm{d}^2 z\,\Big(g_{a\bar{b}}\,\partial_{\bar{z}}X^a\,\partial_z X^{\bar{b}} \;+\; \mathrm{i}\,g_{a\bar{b}}\,(\chi_{\bar{z}}^{a}\,\nabla_z\psi^{\bar{b}} + \chi_z^{\bar{b}}\,\nabla_{\bar{z}}\psi^a) \;-\; R_{a\bar{b}c\bar{d}}\,\chi_{\bar{z}}^{a}\,\chi_z^{\bar{b}}\,\psi^c\,\psi^{\bar{d}}\Big)$$

Further examples

- Complex structure sigma-model or B-model ($\mathcal{M} = T^*[1]T^*X$ doubled)

$$\omega = \mathrm{d}\underset{1}{\chi_i} \wedge \mathrm{d}\underset{0}{X^i} + \mathrm{d}\underset{1}{\widetilde{\chi}^i} \wedge \mathrm{d}\underset{0}{\widetilde{X}_i} \qquad \text{and} \qquad \gamma = J^i{}_j\, \chi_i\, \widetilde{\chi}^j - \partial_j J^i{}_k\, \widetilde{X}_i\, \widetilde{\chi}^j\, \widetilde{\chi}^k$$

$$\{\gamma, \gamma\} = 0 \qquad \Longleftrightarrow \qquad \text{Integrability cond. for complex str. } J$$

$$\boldsymbol{\mathcal{S}}_J^{(2)} = \int_{T[1]\Sigma_2} \left(\chi_i\, \boldsymbol{D}\boldsymbol{X}^i - \widetilde{\boldsymbol{X}}_i\, \boldsymbol{D}\widetilde{\chi}^i + \boldsymbol{J}^i{}_j\, \chi_i\, \widetilde{\chi}^j + \boldsymbol{\partial}_j \boldsymbol{J}^i{}_k\, \widetilde{\boldsymbol{X}}_i\, \widetilde{\chi}^j\, \widetilde{\chi}^k\right)$$

[Ikeda, Tokunaga]

- Courant sigma-model; $\mathcal{M}$: general QP-manifold of degree 2

$$\omega = \mathrm{d}\underset{0}{X^i} \wedge \mathrm{d}\underset{2}{F_i} + \frac{1}{2}\, k_{ab}\, \mathrm{d}\underset{1}{\zeta^a} \wedge \mathrm{d}\underset{1}{\zeta^b}$$

$$\gamma = \underbrace{\rho^i{}_a(X)}_{\rightsquigarrow \text{anchor}}\, F_i\, \zeta^a + \frac{1}{3!}\, \underbrace{T_{abc}(X)}_{\rightsquigarrow \text{Jacobiator}}\, \zeta^a\, \zeta^b\, \zeta^c$$

$$\{\gamma, \gamma\} = 0 \qquad \Longleftrightarrow \qquad \text{Courant algebroid axioms}$$

$$\boldsymbol{\mathcal{S}}^{(3)} = \int_{\Sigma_3} \mathrm{d}^3\hat{z} \left(\boldsymbol{F}_i\, \boldsymbol{D}\boldsymbol{X}^i + \frac{1}{2}\, k_{ab}\, \zeta^a\, \boldsymbol{D}\zeta^b + \rho^i_a(\boldsymbol{X})\, \boldsymbol{F}_i\, \zeta^a + \frac{1}{3!}\, T_{abc}(\boldsymbol{X})\, \zeta^a\, \zeta^b\, \zeta^c\right)$$

Double field theory (DFT)

- In string compactification: Conjugate canonical coordinates $(x, \tilde{x})$ to momenta $(p, \tilde{p})$ on $TX \oplus T^*X$:

$$\Rightarrow \quad \text{Double field theory}$$

- A 'section condition' is needed to reduce a theory to a d dim slice
- Algebraic structure: $\rightsquigarrow$ DFT algebroid

[Chatzistavrakidis,Jonke,Khoo,Szabo]

- It reduces to ordinary Courant algebroid after section condition
- Construction:

1) Start with a doubled Courant algebroid ($X \rightsquigarrow T^*X$)

$$\omega = \mathrm{d}\underset{0}{X^I} \wedge \mathrm{d}\underset{2}{F_I} + \mathrm{d}\underset{1}{\chi_I} \wedge \mathrm{d}\underset{1}{\psi^I}$$

2) Halve the degree 1 coordinates with a so called DFT projection:

$$\chi_i, \widetilde{\psi}_i \longrightarrow p_i \quad \& \quad \psi_i, \widetilde{\chi}^i \longrightarrow q^i \qquad \Rightarrow \qquad \omega = \mathrm{d}\underset{1}{q^i} \wedge \mathrm{d}\underset{1}{p_i} + \dots$$

$$\rightsquigarrow \quad \text{DFT algebroid, C-bracket of DFT}$$

A/B-models and generalized geometry

[1805.11485]

Doubled Poisson sigma-model for A/B-models I.

- A/B-models with H-flux background and generalized complex structures: [Kapustin][Kapustin,Li][Pestun,Witten]
- AKSZ construction of A/B-model and generalized complex structure: [Zucchini][Pestun][Ikeda,Tokunaga][Stojevic]
- We give a DFT description for their AKSZ theory, which leads to a natural AKSZ theory corresponding to a generalized complex structure
- Poisson sigma-model on doubled targetspace $\mathcal{M} = T^*[1]T^*X$

$$\omega = \mathrm{d}\underset{1}{\chi_I} \wedge \mathrm{d}\underset{0}{X^I} \qquad \text{and} \qquad \gamma = \frac{1}{2}\Omega^{IJ}(X)\,\chi_I\,\chi_J$$

$$\text{with} \qquad X^I = \begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix} \qquad \text{and} \qquad \chi_I = \begin{pmatrix} \chi_i \\ \tilde{\chi}^i \end{pmatrix}$$

$$\boldsymbol{S}^{(2)}_{\Omega} = \int_{T[1]\Sigma_2} \mathrm{d}^2\hat{z}\left(\boldsymbol{\chi}_I\,\boldsymbol{D}\boldsymbol{X}^I + \frac{1}{2}\Omega^{IJ}(\boldsymbol{X})\,\boldsymbol{\chi}_I\,\boldsymbol{\chi}_J\right) .$$

Doubled Poisson sigma-model for A/B-models II.

$$\mathcal{S}^{(2)}_{\Omega} = \int_{T[1]\Sigma_2} \mathrm{d}^2\hat{z}\, \left(\boldsymbol{\chi}_I\, \boldsymbol{D}\boldsymbol{X}^I + \frac{1}{2}\boldsymbol{\Omega}^{IJ}\boldsymbol{\chi}_I\boldsymbol{\chi}_J \right) .$$

Observation I.:

- It gives the A-model with

$$\Omega^{IJ} = \begin{pmatrix} \pi^{ij} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{S}^{(2)}_{\pi} = \int_{T[1]\Sigma_2} \mathrm{d}^2\hat{z}\, \left(\boldsymbol{\chi}_i\, \boldsymbol{D}\boldsymbol{X}^i + \cancel{\widetilde{\boldsymbol{\chi}}^i\, \boldsymbol{D}\widetilde{\boldsymbol{X}}_i}^{\;\mathrm{GF}} + \frac{1}{2}\boldsymbol{\pi}^{ij}\boldsymbol{\chi}_i\boldsymbol{\chi}_j \right)$$

- It also gives the B-model with

$$\Omega^{IJ} = \begin{pmatrix} 0 & J^i{}_j \\ -J^j{}_i & -2\partial_{[i}J^k{}_{j]}\widetilde{X}_k \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{S}^{(2)}_{J} = \int_{T[1]\Sigma_2} \mathrm{d}^2\hat{z}\, \left(\boldsymbol{\chi}_i\, \boldsymbol{D}\boldsymbol{X}^i + \widetilde{\boldsymbol{X}}_i\, \boldsymbol{D}\widetilde{\boldsymbol{\chi}}^i + \boldsymbol{J}^i{}_j\,\boldsymbol{\chi}_i\,\widetilde{\boldsymbol{\chi}}^j - \boldsymbol{\partial}_j\boldsymbol{J}^i{}_k\,\widetilde{\boldsymbol{X}}_i\,\widetilde{\boldsymbol{\chi}}^j\,\widetilde{\boldsymbol{\chi}}^k \right)$$

Lift up to AKSZ membrane I.

Observation II.:

- Poisson sigma-model can be lifted up to the membrane level as a contravariant Courant sigma-model

[Bessho,Heller,Ikeda,Watamura]

- AKSZ construction: $\mathcal{M} = T^*[2]T[1]X$

$$\omega \;=\; \mathrm{d}\underset{2}{F_i} \wedge \mathrm{d}\underset{0}{X^i} + \mathrm{d}\underset{1}{\chi_i} \wedge \mathrm{d}\underset{1}{\psi^i}$$

$$\gamma \;=\; \pi^{ij}\, F_i\, \chi_j \;-\; \frac{1}{2}\partial_i \pi^{jk}\, \psi^i\, \chi^j\, \chi^k \;+\; \frac{1}{3!}\, R^{ijk}\, \chi_i\, \chi_j\, \chi_k$$

$$\{\gamma, \gamma\} = 0 \qquad \Longleftrightarrow \qquad \begin{cases} \pi \text{ Poisson str.} \\ [\pi, R]_{\mathrm{S}} = 0 \quad \text{Schouten bracket on } \wedge^{\bullet} TX \end{cases}$$

$$\boldsymbol{S}^{(3)}_{\pi,R} \;=\; \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z}\, \Big(\boldsymbol{F}_i\, \boldsymbol{D}\boldsymbol{X}^i \;-\; \boldsymbol{\chi}_i\, \boldsymbol{D}\boldsymbol{\psi}^i \;+\; \boldsymbol{\pi}^{ij}\, \boldsymbol{F}_i\, \boldsymbol{\chi}_j \;-\; \frac{1}{2}\, \boldsymbol{\partial}_i \boldsymbol{\pi}^{jk}\, \boldsymbol{\psi}^i\, \boldsymbol{\chi}_j\, \boldsymbol{\chi}_k \;+\; \frac{1}{3!}\, \boldsymbol{R}^{ijk}\, \boldsymbol{\chi}_i\, \boldsymbol{\chi}_j\, \boldsymbol{\chi}_k \Big)$$

Lift up to AKSZ membrane II.

$$\mathcal{S}^{(3)}_{\pi,R} = \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z} \Big(\boldsymbol{F}_i \, \boldsymbol{D}\boldsymbol{X}^i - \boldsymbol{\chi}_i \, \boldsymbol{D}\boldsymbol{\psi}^i + \boldsymbol{\pi}^{ij} \, \boldsymbol{F}_i \, \boldsymbol{\chi}_j - \frac{1}{2} \, \partial_i \boldsymbol{\pi}^{jk} \, \boldsymbol{\psi}^i \, \boldsymbol{\chi}_j \, \boldsymbol{\chi}_k + \frac{1}{3!} \, \boldsymbol{R}^{ijk} \, \boldsymbol{\chi}_i \, \boldsymbol{\chi}_j \, \boldsymbol{\chi}_k \Big)$$

- For $R = 0$ it gives back the Poisson sigma-model on the boundary $T[1]\partial\Sigma_3$:

Partial gauge: $\boldsymbol{F}_i = \boldsymbol{D}\boldsymbol{\chi}_i$ and $\boldsymbol{\psi}^i = -\boldsymbol{D}\boldsymbol{X}^i$

$$\Rightarrow \quad \boldsymbol{\omega}_{\mathrm{gf}} = \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z} \; \boldsymbol{\delta X}^i \boldsymbol{\delta \chi}_i$$

$\rightsquigarrow$ full gauge fix on the bulk,

$\rightsquigarrow$ AKSZ theory on the boundary

$$\mathcal{S}^{(3)}_{\pi,0} \quad \xrightarrow{\mathrm{gf}} \quad \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z} \, \Big(\boldsymbol{\chi}_i \, \boldsymbol{D}\boldsymbol{X}^i + \frac{1}{2} \, \boldsymbol{\pi}^{ij} \, \boldsymbol{\chi}_i \, \boldsymbol{\chi}_j \Big)$$

Lift up to AKSZ membrane III.

$$\boldsymbol{\mathcal{S}}^{(3)}_{\pi,R} = \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z}\, \Big(\boldsymbol{F}_i\, \boldsymbol{D}\boldsymbol{X}^i - \boldsymbol{\chi}_i\, \boldsymbol{D}\boldsymbol{\psi}^i + \boldsymbol{\pi}^{ij}\, \boldsymbol{F}_i\, \boldsymbol{\chi}_j - \frac{1}{2}\, \boldsymbol{\partial}_i \boldsymbol{\pi}^{jk}\, \boldsymbol{\psi}^i\, \boldsymbol{\chi}_j\, \boldsymbol{\chi}_k + \frac{1}{3!}\, \boldsymbol{R}^{ijk}\, \boldsymbol{\chi}_i\, \boldsymbol{\chi}_j\, \boldsymbol{\chi}_k \Big)$$

- For $R \neq 0$ but with $\pi = 0$;

Partial gauge: $\boldsymbol{F}_i = \boldsymbol{D}\boldsymbol{\chi}_i$ and $\boldsymbol{\psi}^i = -\boldsymbol{D}\boldsymbol{X}^i$

$$\boldsymbol{\mathcal{S}}^{(3)}_{0,R;\mathrm{gf}} = \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z}\, \boldsymbol{\chi}_i\, \boldsymbol{D}\boldsymbol{X}^i + \frac{1}{3!} \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z}\, \boldsymbol{R}^{ijk}\, \boldsymbol{\chi}_i\, \boldsymbol{\chi}_j\, \boldsymbol{\chi}_k$$

- It gives back the topological part of the string sigma-model on $T[1]\partial\Sigma_3$ that quantizes the non-geometric R-flux background

[Mylonas,Schupp,Szabo]

- Quantization $\rightsquigarrow$ non-associative star-product

Lift up to AKSZ membrane IV.

- We lift up our doubled Poisson sigma-model as a doubled contravariant Courant sigma-model on $\mathcal{M} = T^*[2]T[1]T^*X$:

$$\boldsymbol{S}^{(3)}_{\Omega,\mathcal{R}} = \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z} \Big(\boldsymbol{F}_I \, \boldsymbol{D}\boldsymbol{X}^I - \boldsymbol{\chi}_I \, \boldsymbol{D}\boldsymbol{\psi}^I + \boldsymbol{\Omega}^{IJ} \boldsymbol{F}_I \boldsymbol{\chi}_J - \frac{1}{2} \boldsymbol{\partial}_I \boldsymbol{\Omega}^{JK} \boldsymbol{\psi}^I \boldsymbol{\chi}_J \boldsymbol{\chi}_K + \frac{1}{3!} \boldsymbol{\mathcal{R}}^{IJK} \boldsymbol{\chi}_I \boldsymbol{\chi}_J \boldsymbol{\chi}_K \Big)$$

- On the boundary it gives the AKSZ constructions of both A- and B-models (partial gauge fix)
- It allows the definition of fluxes for A/B-models

$$\mathcal{R}^{IJK} \quad \longrightarrow \quad H_{ijk},\ F^i{}_{jk},\ Q^{ij}{}_k,\ R^{ijk}$$

- The master equation gives them Bianchi identities

$$\{\gamma,\gamma\} = 0 \qquad \Rightarrow \qquad [\Omega,\mathcal{R}]_{\mathrm{S}} = 0$$

Relation to generalized complex structure I.

- We apply the DFT projection

$$\chi_i, \widetilde{\psi}_i \longrightarrow p_i \qquad \& \qquad \psi^i, \widetilde{\chi}^i \longrightarrow q^i$$

and set the dual coordinate to $\widetilde{X}_i = 0$

$$\Rightarrow \quad \omega = \mathrm{d}\underset{2}{F_i} \wedge \mathrm{d}\underset{0}{X^i} + \mathrm{d}\underset{1}{q^i} \wedge \mathrm{d}\underset{1}{p_i}$$

- The resulting theory is not AKSZ (master equation does not hold)

$\rightsquigarrow$ We impose the master equation on Ω (section condition):

$$\Omega^{IJ} = \begin{pmatrix} \mathrm{P}^{ij} & \mathrm{J}^i{}_j \\ -\mathrm{J}^j{}_i & \mathrm{Q}_{ij} \end{pmatrix} \xrightarrow{\text{master}} \begin{pmatrix} \pi^{ij} & J^i{}_j \\ -J^j{}_i & \omega_{ij} \end{pmatrix}$$

- We set $\omega = 0$ (we do not need it in order to describe the A/B-models later)

$$\Rightarrow \quad \gamma_{\pi,J} = \pi^{ij} F_i\, p_j + J^i{}_j\, F_i\, q^j - \frac{1}{2}\, \partial_i \pi^{jk}\, q^i\, p_j\, p_k + \partial_i J^k{}_j\, q^i\, q^j\, p_k$$

Relation to generalized complex structure II.

$$\gamma_{\pi,J} = \pi^{ij} F_i p_j + J^i{}_j F_i q^j - \frac{1}{2} \partial_i \pi^{jk} q^i p_j p_k + \partial_i J^k{}_j q^i q^j p_k$$

- The reduced double Poisson str. is

$$\Omega^{IJ} \longrightarrow \mathbb{J}^I{}_J = \begin{pmatrix} J^i{}_j & \pi^{ij} \\ 0 & -J^j{}_i \end{pmatrix}$$

- The master equation gives the integrability conditions

$$\begin{aligned} \pi^{[i|l} \partial_l \pi^{jk]} &= 0 \\ J^l{}_i \partial_l \pi^{jk} + 2 \pi^{jl} \partial_{[i} J^k{}_{l]} + \pi^{kl} \partial_l J^j{}_i - J^j{}_l \partial_i \pi^{lk} &= 0 \\ J^l{}_{[i|} \partial_l J^k{}_{|j]} - J^k{}_l \partial_{[i} J^l{}_{j]} &= 0 \end{aligned}$$

for the generalized complex str. $\mathbb{J}^I{}_J$!

- It defines a novel Courant algebroid corresponding a gen. complex str. with the Dorfman bracket and anchor

$$[\![e_1, e_2]\!]_{\pi,J} = \{\{e_1, \gamma_{\pi,J}\}, e_2\} \qquad \text{and} \qquad \rho(e) = \{e, \{\gamma_{\pi,J}, \cdot\}\}$$

Relation to generalized complex structure III.

- It defines a AKSZ membrane model corresponding to the gen. complex str. $\mathbb{J}^I{}_J$:

$$\mathcal{S}^{(3)}_{\pi,J} = \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z} \Big(\boldsymbol{F}_i \boldsymbol{D}\boldsymbol{X}^i - \boldsymbol{p}_i \boldsymbol{D}\boldsymbol{q}^i + \pi^{ij} \boldsymbol{F}_i \boldsymbol{p}_j + \boldsymbol{J}^i{}_j \boldsymbol{F}_i \boldsymbol{q}^j - \frac{1}{2} \partial_i \pi^{jk} \boldsymbol{q}^i \boldsymbol{p}_j \boldsymbol{p}_k + \partial_i \boldsymbol{J}^k{}_j \boldsymbol{q}^i \boldsymbol{q}^j \boldsymbol{p}_k \Big)$$

- It can contain flux terms as well (twists of the Courant algebroid)

$$\int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z} \Big(\boldsymbol{H}_{ijk} \boldsymbol{q}^i \boldsymbol{q}^j \boldsymbol{q}^k + \boldsymbol{F}^i{}_{jk} \boldsymbol{p}_i \boldsymbol{q}^j \boldsymbol{q}^k + \boldsymbol{Q}^{ij}{}_k \boldsymbol{p}_i \boldsymbol{p}_j \boldsymbol{q}^k, + \boldsymbol{R}^{ijk} \boldsymbol{p}_i \boldsymbol{p}_j \boldsymbol{p}_k \Big)$$

- If $J = 0$ it gives the A-model on the boundary (as before)

Partial gauge: $\boldsymbol{F}_i = \boldsymbol{D}\chi_i$ and $\psi^i = -\boldsymbol{D}\boldsymbol{X}^i$

$$\mathcal{S}^{(3)}_{\pi,J} \xrightarrow{\mathrm{gf}} \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z} \Big(\chi_i \boldsymbol{D}\boldsymbol{X}^i + \frac{1}{2} \pi^{ij} \chi_i \chi_j \Big)$$

Relation to generalized complex structure III.

$$\mathcal{S}^{(3)}_{\pi,J} = \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z} \Big(\boldsymbol{F}_i \boldsymbol{D}\boldsymbol{X}^i - \boldsymbol{p}_i \boldsymbol{D}\boldsymbol{q}^i + \boldsymbol{\pi}^{ij} \boldsymbol{F}_i \boldsymbol{p}_j + \boldsymbol{J}^i{}_j \boldsymbol{F}_i \boldsymbol{q}^j - \frac{1}{2} \boldsymbol{\partial}_i \boldsymbol{\pi}^{jk} \boldsymbol{q}^i \boldsymbol{p}_j \boldsymbol{p}_k + \boldsymbol{\partial}_i \boldsymbol{J}^k{}_j \boldsymbol{q}^i \boldsymbol{q}^j \boldsymbol{p}_k \Big)$$

For $\pi = 0$ it also gives the B-model on the boundary:

- We cover Σ_3 as
$$\Sigma_3 = U \cup U'$$
where U contains the boundary, but U' does not
$$\Sigma_3\big|_U = \partial\Sigma_3 \times \mathbb{R}^+ \qquad \text{and} \qquad U' \subseteq \Sigma_3 \setminus \partial\Sigma_3$$
$\rightsquigarrow$ both $\boldsymbol{\omega}$ and $\mathcal{S}^{(3)}_{0,J}$ split to two parts on U and U'
- We use a gauge on U', which gives $\mathcal{S}^{(3)}_{0,J}|_{U'} = 0$ (partial gauge)
- We will integrate out $\mathbb{R}^+$ to reduce it to the B-model on the boundary

Relation to generalized complex structure IV.

- We use the notation $(t \in \mathbb{R}^+)$

$$\phi = \widehat{\phi} + \phi_t \, \theta^t$$

- We use further gauge fixing:

$$\boldsymbol{X}^i_t = 0 \qquad \text{and} \qquad \boldsymbol{q}^i_t = 0$$

- We integrate out $\widehat{\boldsymbol{F}}_i$ and $\widehat{\boldsymbol{p}}_i$ $\quad \Rightarrow \quad \partial_t \widehat{\boldsymbol{X}}^i = 0 \quad \text{and} \quad \partial_t \widehat{\boldsymbol{q}}^i = 0$

$$\mathcal{S}^{(3)}_{0,J|U;\mathrm{gf}} = \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z} \int_{\mathbb{R}^+} \mathrm{d}t \, \Big(\cancelto{0}{\widehat{\boldsymbol{F}}_i \partial_t \widehat{\boldsymbol{X}}^i} + \cancelto{0}{\widehat{\boldsymbol{p}}_i \partial_t \widehat{\boldsymbol{q}}^i} - (\boldsymbol{F}_t)_i \, \widehat{\boldsymbol{D}} \widehat{\boldsymbol{X}}^i - (\boldsymbol{p}_t)_i \, \widehat{\boldsymbol{D}} \widehat{\boldsymbol{q}}^i - \boldsymbol{J}^i{}_j \, (\boldsymbol{F}_t)_i \, \widehat{\boldsymbol{q}}^j + \boldsymbol{\partial}_i \boldsymbol{J}^k{}_j \, \widehat{\boldsymbol{q}}^i \, \widehat{\boldsymbol{q}}^j \, (\boldsymbol{p}_t)_k \Big) \, .$$

- In the notations

$$\chi_i = - \int_{\mathbb{R}^+} \mathrm{d}t \, (\boldsymbol{F}_t)_i \, , \qquad \boldsymbol{X}^i = \widehat{\boldsymbol{X}}^i$$

$$\widetilde{\boldsymbol{X}}_i = - \int_{\mathbb{R}^+} \mathrm{d}t \, (\boldsymbol{p}_t)_i \, , \qquad \widetilde{\chi}^i = \widehat{\boldsymbol{q}}^i$$

- We get the B-model on the boundary

$$\mathcal{S}^{(3)}_{0,J} \quad \to \quad \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z} \, \big(\chi_i \, \boldsymbol{D}\boldsymbol{X}^i + \widetilde{\boldsymbol{X}}_i \, \boldsymbol{D}\widetilde{\chi}^i + \boldsymbol{J}^i{}_j \, \chi_i \, \widetilde{\chi}^j - \boldsymbol{\partial}_j \boldsymbol{J}^i{}_k \, \widetilde{\boldsymbol{X}}_i \, \widetilde{\chi}^j \, \widetilde{\chi}^k \big)$$

Topological S-duality from generalized complex structure I.

- Topological S-duality originates from S-duality of type IIB strings
- Exchanges the weak/strong coupling sector of A- and B-models

$$g_{\rm A} = \frac{1}{g_{\rm B}} \qquad \text{and} \qquad k_{\rm A} = \frac{k_{\rm B}}{g_{\rm B}}$$

- We take the rescaling transformation ($\lambda \in \mathbb{R}$)

$$p_i \longmapsto \lambda\, p_i \qquad \text{and} \qquad q^i \longmapsto \frac{1}{\lambda}\, q^i\ ,$$

- It is a canonical transformation on the BV level (i.e. leaves ω invariant)

$$\gamma^{\lambda}_{\pi,J} = \lambda\, \pi^{ij}\, F_i\, p_j - \frac{\lambda}{2}\, \partial_i \pi^{jk}\, q^i\, p_j\, p_k + \frac{1}{\lambda}\, J^i{}_j\, F_i\, q^j + \frac{1}{\lambda}\, \partial_i J^k{}_j\, q^i\, q^j\, p_k$$

- Two limits give the Poisson or complex str. Courant algebroids:

$$\frac{1}{\lambda}\, \gamma_{0,J} \xleftarrow{\ \lambda \ll 1\ } \gamma^{\lambda}_{\pi,J} \xrightarrow{\ \lambda \gg 1\ } \lambda\, \gamma_{\pi,0}$$

- Gen. complex str. interpolates between the two Courant algebroids

Topological S-duality from generalized complex structure II.

- It can be lift up to the AKSZ level (Hamiltonian $\lambda\gamma_{\pi,J}$ and we introduce an overall coupling $1/\lambda$ then rescale with λ):

$$\mathcal{S}^{\lambda\,(3)}_{\mathrm{A/B}} = \int_{T[1]\Sigma_3} \mathrm{d}^3\hat{z} \left(\frac{1}{\lambda} \boldsymbol{F}_i \boldsymbol{D}\boldsymbol{X}^i - \frac{1}{\lambda} \boldsymbol{p}_i \boldsymbol{D}\boldsymbol{q}^i + \lambda \boldsymbol{\pi}^{ij} \boldsymbol{F}_i \boldsymbol{p}_j \right.$$
$$\left. - \frac{\lambda}{2} \partial_i \boldsymbol{\pi}^{jk} \boldsymbol{q}^i \boldsymbol{p}_j \boldsymbol{p}_k + \frac{1}{\lambda} \boldsymbol{J}^i{}_j \boldsymbol{F}_i \boldsymbol{q}^j + \frac{1}{\lambda} \partial_i \boldsymbol{J}^k{}_j \boldsymbol{q}^i \boldsymbol{q}^j \boldsymbol{p}_k \right)$$

- We get the A-model for $\lambda \gg 1$

$$\mathcal{S}^{\lambda\,(3)}_{\mathrm{A/B}} \xrightarrow{\lambda \gg 1} \frac{\lambda}{2} \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z}\; \boldsymbol{\pi}^{ij} \boldsymbol{p}_i \boldsymbol{p}_j \qquad \Rightarrow \qquad \lambda = \frac{1}{g_{\mathrm{A}}}$$

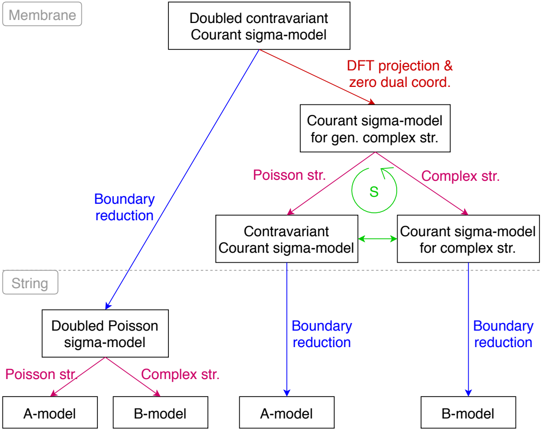
- And the B-model for $\lambda \ll 1$

$$\mathcal{S}^{\lambda\,(3)}_{\mathrm{A/B}} \xrightarrow{\lambda \ll 1} \frac{1}{\lambda} \oint_{T[1]\partial\Sigma_3} \mathrm{d}^2\hat{z} \left(\boldsymbol{\chi}_i \boldsymbol{D}\boldsymbol{X}^i + \widetilde{\boldsymbol{X}}_i \boldsymbol{D}\widetilde{\boldsymbol{\chi}}^i + \boldsymbol{J}^i{}_j \boldsymbol{\chi}_i \widetilde{\boldsymbol{\chi}}^j - \partial_j \boldsymbol{J}^i{}_k \widetilde{\boldsymbol{X}}_i \widetilde{\boldsymbol{\chi}}^j \widetilde{\boldsymbol{\chi}}^k \right)$$

$$\lambda = g_{\mathrm{B}} = \frac{1}{g_{\mathrm{A}}} \qquad \Rightarrow \qquad \text{Topological S-duality!}$$

Summary

Different reductions and connections between AKSZ string and membrane sigma-models related to A- and B-models



Outlook

- What physical quantities does our AKSZ membranes calculate? (In general, it is not easy to compute something in a membrane sigma-model).
- What is the relevance of geometric and non-geometric fluxes in topological string theory?
- Have our constructions any relevance in flux compactifications? (topological strings appear in compactifications without fluxes)
- Has it any relation to topological mirror symmetry?
- Further study of the new classes of Courant algebroid (also their twists).
- Does there exist a Courant algebroid for the general version of generalized complex structure?
- Our S-duality arises from the T-duality inspired generalized complex geometry. Whether there is a physical origin behind this relation or whether it is just a coincidence found in the topological field theories.
- Lift up our approach to topological M-theory and exceptional generalized geometry? (First step: [1802.04581] by ZK, Sinkovics and Szabo)

Thank you for your attention!