Higher spin particles: theory and phenomenology

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hep-ph: 2010.02224, 2102.13652, 2104.03231, 2106.09031

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Outline

Introduction:

- Motivation for higher spin
- History of higher spin
- Problems with higher spin

Our work:

- Our multispinor formalism for any spin

Phenomenological applications:

- Dark matter
- Collider signatures
- Δ-resonance of nuclear physics

Motivation for higher spin

Motivation for higher spin

(Higher spin is $s = \frac{3}{2}$, 2, $\frac{5}{2}$, 3...)

Motivation for higher spin

- Higher spin particles exist in nature as hadronic resonances
- Well behaved theories are known for spins 0, $1/2$ and 1, but not for higher.
- Higgs is spin 0, quarks and leptons spin $1/2$, gauge bosons spin 1. If gravitino exists it is spin 2. How about elementary spin $3/2$? Does it exist?

History of higher spin

History of higher spin

History of higher spin

- Dirac (1936): Relativistic wave equations for arbitrary spin. Problems with interactions for $s \geq 3/2$.
- Pauli & Fiertz (1939): Again relativistic wave equations for arbitrary spin. Problems with interactions that can be solved with additional fields.
- Rarita & Schwinger (1941): spin-3/2 and simpler half-integer spin. Predominant theory for spin-3/2 today.
- Freedman, Nieuwenhuizen, Ferrara, Deser, Zumino (1976): Supergravity. Supergravity predicts spin-3/2 field, gravitino and its spin-2 superpartner graviton. Gravitino is described by a Rarita-Schwinger field.

Problems with higher spin

Problems with higher spin

(And there are plenty...)

Notorious No-Go theorems

No-Go theorems have poisoned people's minds!!!

- Weinberg theorem
- Coleman-Mandula theorem
- Weinberg-Witten theorem

The message: there cannot be massless fields with spin higher than 2 that interact with "normal" particles.

One can avoid no-go theorems by having massive higher spin fields!

Ostrogradsky theorem

The Ostrograsky theorem (1850):

"Any theory that has more than one time-derivative acting on a field has an instability"

Instability = arbitrary low negative energies = ghosts!

Ghosts are particles with negative energies
$\Rightarrow$ One can produce them from vacuum with a normal particle
$\Rightarrow$ Vacuum is immediately filled with particles!
$\Rightarrow$ Bad!

Higher derivative theories (spin ≥ 2)

- The higher spin fields are typically described as high rank Lorentz tensors (multiple Lorentz indices)
- These indices need to be contracted with something and that something is derivatives
- Multiple time derivatives will then act on a field $\Rightarrow$ Ostrogradsky instability!
- High rank fields will contain more degrees of freedom than necessary. They need to be projected out with symmetries.
- Problem: interactions often do not respect the symmetries of the free Lagrangian $\Rightarrow$ everything breaks down!

Lorentz representations

Poincare symmetry

- Poincare symmetry is the symmetry of space-time
- It consists of rotations (3 generators), translations (time + space: 4 generators) and boosts (3 generators)
- Subgroup of Poincare symmetry is Lorentz symmetry that can be written as two direct products of $SU(2)$: $SU(2)_L \times SU(2)_R$
- This is the origin of spin: paticles transform under some representation of Lorentz group: (j_L, j_R)

Lorentz representation: spin-0 and spin-1/2

Both spin-0 and spin-1/2 Lorentz representations have the same number of degrees of freedom (DOF) as the physical particle:

Rep	DOF	Example	DOF
$(0,0)$	1	ϕ	1
$(1/2,0)$	2	e_L	2
$(0,1/2)$	2	e_R	2

Everything good!

Free spin-1

- Spin-1 particles are described by an vector field, A^{μ}, in Lorentz repsentation $(1/2, 1/2)$.
- Problem: A^{μ} has 4 DOFs. However, physical massive spin-1 particle has 3 DOFs.
  $\Rightarrow$ One must remove 1 DOF somehow.
- Free massive vector is described by Proca equation:
  $\Box A^{\mu} - \partial^{\mu}(\partial_{\nu} A^{\nu}) + m^2 A^{\mu} = 0$
- Operate from left by ∂_{μ}: $\Rightarrow$ Lorentz condition $\partial_{\mu} A^{\mu} = 0$. This eliminates one degree of freedom.
- All good!

Interacting spin-1

- Interacting vector has a current on the left-hand side of the Proca equation: $\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) + m^2 A^\mu = j^\mu$
- Interaction potentially ruins DOF counting!
- Again operate with ∂_μ from the left: $\Rightarrow \partial_\mu A^\mu = \frac{1}{m^2}\partial_\mu j^\mu$
- We want Lorentz condition $\partial_\mu A^\mu = 0$ for correct DOFs!
- Therefore current must be conserved: $\partial_\mu j^\mu = 0$.
- For gauge theories this happens automatically.
  $\Rightarrow$ Spin-1 seems to need gauge symmetry!

Spin-3/2

- Spin-3/2 is described by a Rarita-Schwinger field ψ^μ.
- Rarita-Schwinger field is a vector spinor in following Lorentz representation:

$$\overbrace{(1/2,1/2)}^{\text{vector}} \otimes \overbrace{\Big((1/2,0) \oplus (0,1/2)\Big)}^{\text{spinor}} =$$
$$\underbrace{(1,1/2)}_{\text{spin}-3/2} \oplus \underbrace{(1/2,1)}_{\text{spin}-3/2} \oplus \underbrace{(1/2,0) \oplus (0,1/2)}_{\text{spin-1/2 background}}$$

- $\psi^\mu = \begin{pmatrix} \psi_1^\mu \\ \psi_2^\mu \\ \psi_3^\mu \\ \psi_4^\mu \end{pmatrix}$, 16 DOFs!
- Physical spin-3/2 has only 4 DOFs!
  $\Rightarrow$ Constraints required!

Spin-3/2

- Rarita-Schwinger Lagrangian is:
$\mathcal{L} = \bar{\psi}_\mu \left[-\epsilon^{\mu\nu\alpha\beta}\gamma_\beta\gamma_5\partial_\alpha - \frac{1}{2}m[\gamma^\mu, \gamma^\nu] \right] \psi_\nu$
- Correponding equation of motion is:
$\left(-\epsilon^{\mu\nu\alpha\beta}\gamma_\beta\gamma_5\partial_\alpha - \frac{1}{2}m[\gamma^\mu, \gamma^\nu] \right)\psi_\nu = 0$
- This equation of motion contains Dirac equation: $(i\gamma^\nu\partial_\nu - m)\psi^\mu$ and subsidiary conditions $\gamma^\mu\psi_\mu = 0$ and $\partial^\mu\psi_\mu = 0$
- With these constraints correct DOFs are obtained. Good!
- Interactions hoewever spoil this as they do not respect these conditions! $\Rightarrow$ Interacting Rarita-Schwinger does not work! (Johnson-Sudarshan problem and Velo-Zwanziger problem)
- Disaster for nuclear physics! They only use Rarita-Schwinger field eventhough it is not consistent!

Rarita-Schwinger field in SUSY

- In supergravity the spin-3/2 gravitino is the superpartner of spin-2 graviton
- In supergravity the interaction terms have same symmetries as Rarita-Schwinger kinetic term (and that symmetry is supersymmetry) $\Rightarrow$ The interactions do not break the counting of DOFs!
- Only known way to have consistent interactions for Rarita-Schwinger field is local supersymmetry, i.e. supergravity.
- Rarita-Schwinger field does not work in non-supersymmetric flat space-time.
- This is a disaster as the nuclear physics is full of spin-3/2 nucleon resonances (Δ, ...)

How to proceed?

How to proceed?

(What representation to pick?)

Representations

Spin	Rep	DOF	Example	Rep	DOF
0	$(0,0)$	1	KG	$(0,0)$	1
$\frac{1}{2}$	$(\frac{1}{2},0)$	2	ψ_L	$(\frac{1}{2},0)$	2
1	$(\frac{1}{2},\frac{1}{2})$	4	gauge	$(1,0)$	3
$\frac{3}{2}$	$(\frac{1}{2},1)$	8	SUSY	$(\frac{3}{2},0)$	4
2	$(1,1)$	16	gravity	$(2,0)$	5
$j>2$	?	?	?	$(j,0)$	$2j+1$

$\Rightarrow$ $(j,0)$ and $(0,j)$ representations have automatically the same DOFs as the corresponding massive particle!

$\Rightarrow$ Let us use them!

Multispinor formalism

Multispinor formalism

Introduced in:
J.C.Criado, NK, M.Raidal,H.Veermäe: hep-ph: 2010.02224 (PRD),

and elaborated in
A.Djouadi, J.C.Criado, NK, M.Raidal,H.Veermäe: hep-ph: 2102.13652 (JHEP),
A.Djouadi, J.C.Criado, NK, K.Müürsepp, M.Raidal,H.Veermäe: hep-ph: 2104.03231 (Phys.Lett.B),
A.Djouadi, J.C.Criado, NK, K.Müürsepp, M.Raidal,H.Veermäe: hep-ph: 2106.09031

$(j,0)\oplus(0,j)$

Background:

- Originally Weinberg used $(j,0)\oplus(0,j)$ to describe any spin (1964)[1]
- The representation $(j,0)\oplus(0,j)$ has the correct number of degrees of freedom for massive fields
- This allows to avoid the usual problems of higher-spin fields (non-physical degrees of freedom tend to reappear as ghost in interacting theories)
- Practically useful reformulation of Weinberg's original was presented in *Criado, Koivunen, Raidal, Veermäe*, arXiv:2010.02224 [hep-ph] (PRD)

[1] Weinberg, Phys. Rev. **133**, B1318 (1964)

Weinberg's approach to any spin

What Weinberg did?

- Use Lorentz representation $(j, 0) \oplus (0, j)$ for all the spins j.
- Perturbation theory: Lorentz invariant S-matrix can be calculated from Dyson's formula:

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 ... d^4x_n T\{\mathcal{H}(x_1)...\mathcal{H}(x_n)\}$$

- Particle interpretation: demand correct Lorentz transformation properties of any spin field ψ and that field commutators/anticommutators vanish for space-like separation.
- No Lagrangian or free-Hamiltonian required!

Weinberg's approach to any spin

- Particle states are built by acting on vacuum with creation operators like usual. Demand that creation and annihilation operators, $\hat{a}^\dagger$ and $\hat{a}$, transform properly under Lorentz transformation.
- $\hat{a}^\dagger$ and $\hat{a}$ must satisfy correct commutation/anticommutation rules
- $\Rightarrow$ Any spin field can be constructed:

$$\psi_\sigma^{(j)}(x) = \int \frac{d^3p}{(2\pi^3)(2E_p)} \sum_{\sigma'} \left[D^j_{\sigma\sigma'} \hat{a}_\sigma(p) e^{ipx} + \bar{D}^j_{\sigma\sigma'} \hat{a}^\dagger_\sigma(p) e^{-ipx} \right]$$

- Calculate the propagator from $\langle 0 | T\{\psi_\sigma^{(j)} \psi_{\sigma'}^{(j)\dagger}\} |0\rangle$
  (Becomes super-complicated as spin increses!!!)
- Equation of motion:

$$\left(\gamma^{\mu_1\mu_2\ldots\mu_{2j}} \partial_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_{2j}} + m^{2j}\right)\psi = 0$$

Why Weinberg's approach needs improving?

- Weinberg's approach is notoriously complicated!
- No phenomenological computation has ever been done using his formalism!
- Weinberg indices the any spin field with spin-j rotation group index.
- We propose to use Lorentz group indices instead!
  (Dotted and undotted indices, familiar from SUSY)

Recall two component Weyl spinors

- Undotted index a: left-handed Lorentz group index $({}^{a}{}_{a})$
  Dotted index $\dot{a}$: right-handed Lorentz group index $({}_{\dot{a}}{}^{\dot{a}})$
- Example: Dirac field and gamma-matrix with 2-spinors:

$$\Psi_D = \begin{pmatrix} \psi_{L,a} \\ \psi_R^{\dot{a}} \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{a\dot{a}} \\ \bar{\sigma}^{\mu\dot{a}a} & 0 \end{pmatrix}$$

- Dirac Lagrangian with 2-spinors:

$$\mathcal{L}_{\mathrm{Dirac}} = i\bar{\Psi}_D\gamma^\mu\partial_\mu\Psi_D - m\bar{\Psi}_D\Psi_D$$
$$= i\psi^\dagger_{L,\dot{a}}\bar{\sigma}^{\mu\dot{a}a}\partial_\mu\psi_{L,a} + i\psi_R^{\dagger a}\sigma^\mu_{a\dot{a}}\partial_\mu\psi_R^{\dot{a}} - m\left[\psi_R^{\dagger a}\psi_{L,a} + \psi^\dagger_{L,\dot{a}}\psi_R^{\dot{a}}\right]$$

Our multispinor formalism

Our multispinor formalism

(Finally!)

Multispinor formalism $(j,0)\oplus(0,j)$

- Use fully symmetric "multispinor", $\psi_{a_1\ldots a_{2j}} = \psi_{(a)}$, to represent spin-j field (j = any spin!):

$$\psi_{(a)}(x) = \int \frac{d^3p}{(2\pi^3)(2E_p)} \sum_\sigma \left[\hat{a}_\sigma(p)u_{(a)}(p,\sigma)e^{ipx} + \hat{a}^\dagger_\sigma(p)v_{(a)}(p,\sigma)e^{-ipx}\right]$$

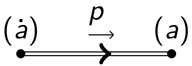
- $\psi_{(a)}(x)$ is taken to be neutral and Majorana
- Satisfies equations of motion (order-2j in derivatives(!)):

$$i\partial^{(\dot{a})(a)}\psi_{(a)} = m^{2j}\psi^{\dagger(\dot{a})} \quad \text{and} \quad (\Box + m^2)\psi_{(a)} = 0$$

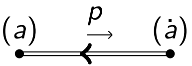
- Mass dimension of $\psi_{(a)}$ is $j+1 \Rightarrow$ EFT

Feynman rules (propagators)

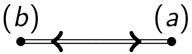
Propagators



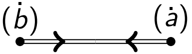
$= i\,\frac{p_{(a)(\dot{a})}}{p^2-m^2}\,,$



$= i\,\frac{p^{(\dot{a})(a)}}{p^2-m^2}\,,$



$= i\,\frac{m^{2j}\delta^{(b)}_{(a)}}{p^2-m^2}\,,$

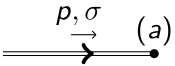


$= i\,\frac{m^{2j}\delta^{(\dot{a})}_{(\dot{b})}}{p^2-m^2}\,.$

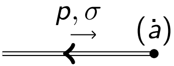
- Spin-1/2: $p_{(a)(\dot{a})} = p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}}$
- Spin-1: $p_{(a)(\dot{a})} = p_{a_1 a_2 \dot{a}_1 \dot{a}_2} = \frac{1}{2!}\left[p_{a_1\dot{a}_1}p_{a_2\dot{a}_2} + p_{a_1\dot{a}_2}p_{a_2\dot{a}_1}\right]$
- Spin-j: $p_{(a)(\dot{a})} = \frac{1}{(2j)!}\left[p_{a_1\dot{a}_1}\dots p_{a_{2j}\dot{a}_{2j}} + \text{all permutations}\right]$
- Propagator has only single pole.
- Only physical degrees of freedom propagate (Well of course, we have only physical degrees of freedom!)
  $\Rightarrow$ No ghosts!

Feynman rules (external lines)

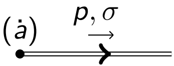
External lines



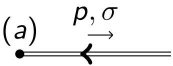
$= u_{(a)}(p,\sigma),$



$= v^*_{(\dot{a})}(p,\sigma),$



$= u^*_{(\dot{a})}(p,\sigma),$



$= v_{(a)}(p,\sigma).$

Completeness relations

$$
\begin{aligned}
\sum_\sigma u_{(a)}(p,\sigma) u^*_{(\dot{a})}(p,\sigma) &= p_{(a)(\dot{a})}, \\
\sum_\sigma v_{(a)}(p,\sigma) v^*_{(\dot{a})}(p,\sigma) &= p_{(a)(\dot{a})}, \\
\sum_\sigma u_{(a)}(p,\sigma) v^{(b)}(p,\sigma) &= m^{2j} \delta^{(b)}_{(a)}, \\
\sum_\sigma u^{*(\dot{a})}(p,\sigma) v^*{}_{(\dot{b})}(p,\sigma) &= m^{2j} \delta^{(\dot{a})}_{(\dot{b})}.
\end{aligned}
$$

No problems

- Correct number of degrees of freedom (the higher derivatives in equations of motion are not a problem)
- The propagator is well defined (no additional poles, no ghosts!)
- Simple to use: multispinor: $S(p) = i\frac{p_{(a)(\dot{a})}}{p^2 - m^2}$
  Weinberg:
  $$S(p)^{(2)} = -i\frac{(-p^2)^2 - 2p^2(\vec{p}\cdot\vec{J})(\vec{p}\cdot\vec{J} - p^0) + \frac{2}{3}(\vec{p}\cdot\vec{J})\left[(\vec{p}\cdot\vec{J})^2 - |\vec{p}|^2\right]\left[\vec{p}\cdot\vec{J} - 2p^0\right]}{p^2 - m^2}$$
- The multispinor framework is effective field theory that allows fully consistent computations bellow the cut-off scale Λ (just like in any EFT)
- This formulation is the first practical and consistent framework that allows the study of higher spin particles!

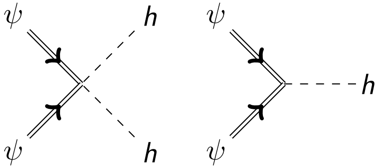
Phenomenology

Phenomenology

Higgs portal

- The free theory is well behaving
- One can add interactions at will! Interaction do not cause trouble, as only physical DOFs are present from the start.
- Simplest interaction to study: Higgs portal:

$$\mathcal{L} = \frac{\lambda}{\Lambda^{2j}}(H^\dagger H)\psi^{(a)}\psi_{(a)} + h.c.$$

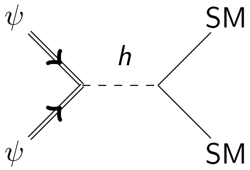


Any spin dark matter

- The relic abundance is set by Boltzmann equation:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v_{\mathrm{rel}} \rangle (n^2 - n_{\mathrm{eq}}^2)$$

- The is thermally averaged DM annihilation cross section. The annihilation channels are:



- We can produce any spin DM through freeze-out of freeze-in

Freeze-out

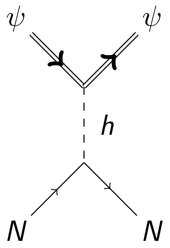
- DM initially in thermal equilibriom with SM
- When temparature drops bellow DM mass, DM equilibrium number density, $n_{\rm eq}$, becomes Boltzmann suppressed
  $\Rightarrow$ DM cannot annihilate fast enough to keep up with SM bath
  $\Rightarrow$ DM freezes out
- Neglect equilibrium number density:

$$\frac{dn}{dt} + 3Hn \simeq -\langle \sigma v_{\rm rel} \rangle n^2$$

$\Rightarrow$ DM abundance:

$$\Omega h^2 = \frac{8.7 \times 10^{-11}}{\mathrm{GeV}^2} \left(\int_{x_f}^{\infty} \mathrm{d}x \langle \sigma v_{\mathrm{rel}} \rangle \frac{g_{*s}(T)}{x^2} \right)^{-1}$$

Direct detection



- $$\sigma_N = \frac{2m_N^2\mu_N^2 f_N^2|\lambda|^2}{\pi m_h^4 m^2}\left[1 + c_{2\theta} + \frac{4}{3}j(j+1)\frac{\mu_N^2 v_{\mathrm{rel}}^2}{m^2}\right],$$
- Direct detection cross section is velocity suppressed for purely imaginary coupling! $(\lambda = |\lambda| e^{i\theta})$
- One can avoid crippling direct detection constraints!

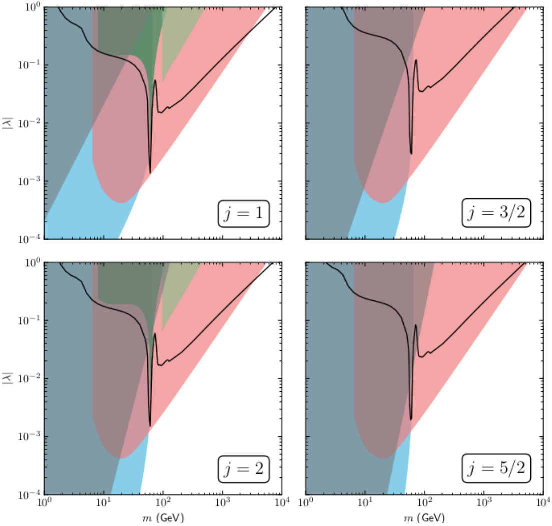
Indirect detection

- Indirect detection is based on detecting gamma-ray bursts in spheroidal dwarf galaxies
- DM would annihilate in them and produce photons
- Example $\psi\psi \to f\bar{f}$:

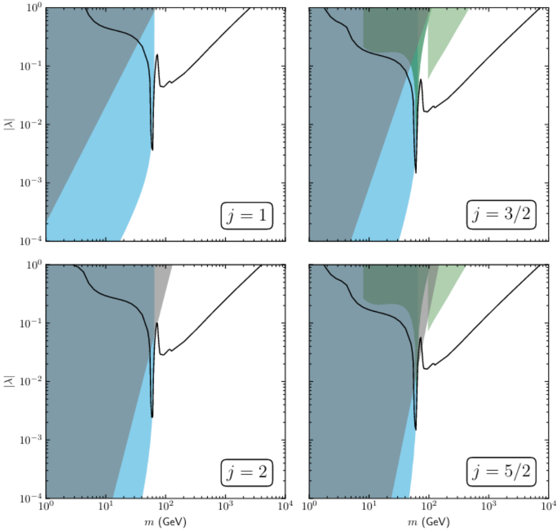
$$\sigma_{\psi\psi\to\bar{f}f} v_{\text{rel}} \sim \frac{|\lambda|^2 m_f^2}{8\pi(2j+1)m^4}\left[1 + (-1)^{2j} c_{2\theta} + \frac{2}{3} j(j+1) v_{\text{rel}}^2\right]$$

- Real coupling: s-wave ($\sim v_{\mathrm{rel}}^0$) annihilation for bosons, p-wave ($\sim v_{\mathrm{rel}}^2$) for fermions
- For imaginary couplings this is reversed: s-wave ($\sim v_{\mathrm{rel}}^0$) annihilation for fermions, p-wave ($\sim v_{\mathrm{rel}}^2$) for bosons

DM abundance, real portal coupling λ



DM abundance, imaginary portal coupling λ



Collider phenomenology, spin-3/2

- Assume spin-3/2 is SM gauge singlet
- Lowest order operator linear in any spin field is dim-7:

$$-\mathcal{H}_{\text{linear}} = \tfrac{1}{\Lambda^3}\psi_{3/2}^{abc}\Big[\, c_q\epsilon^{IJK}u^*_{Ia}d^*_{Jb}d^*_{Kc} + c_l(l_a^T\epsilon l_b)e^*_c + c_{lq}(q_{Ia}^T\epsilon l_b)d_c^{*I}$$
$$+c_B\tilde{\phi}^\dagger\sigma^{\mu\nu}_{ab}B_{\mu\nu}l_c + c_W\tilde{\phi}^\dagger\sigma^{\mu\nu}_{ab}\sigma_i W^i_{\mu\nu}l_c + c_\phi\sigma^{\mu\nu}_{ab}(D_\mu\tilde{\phi})^\dagger D_\nu l_c\,\Big] + \text{h.c.}.$$

Spin-3/2 decay modes

- 3-body:

$$\psi_{3/2} \to udd \ , \ \bar{u}\bar{d}\bar{d}$$

$$\psi_{3/2} \to e^+e^-\nu_e \ , \ e^+e^-\bar{\nu}_e, d\bar{d}\nu_e \ , \ d\bar{d}\bar{\nu}_e \ , \ u\bar{d}e^- \ , \ \bar{u}de^+$$

$$\Gamma(\psi_{3/2} \to f_1 f_2 f_3) = \frac{\kappa_{f_1 f_2 f_3}}{7680\pi^3} \frac{m_{3/2}^7}{\Lambda^6}$$

- 2-body:

$$\psi_{3/2} \to W^+e^- \ , \ W^-e^+ \ , \ Z\nu_e \ , \ Z\bar{\nu}_e \ , \ \gamma\nu_e \ , \ \gamma\bar{\nu}_e \ , \ H\nu_e \ , \ H\bar{\nu}_e$$

$$\Gamma(\psi_{3/2} \to \gamma\nu_e) = \frac{c_\gamma^2 v^2}{192\pi\Lambda^6} m_{3/2}^5$$

Spin-3/2 production in LHC

- $\Delta B = 1$ (resembles R-parity violating SUSY):

$$u_R d_R \to \psi_{3/2} \bar{d}_R\,, \quad \bar{u}_R \bar{d}_R \to \psi_{3/2} d_R\,,$$
$$d_R d_R \to \psi_{3/2} \bar{u}_R\,, \quad \bar{d}_R \bar{d}_R \to \psi_{3/2} u_R$$

$$qq \to \psi_{3/2} q \to 4q \quad \Rightarrow \quad pp \to 4j$$

- $\Delta L = 1$ (resembles SUSY):

$$d_L \bar{d}_R \to \psi_{3/2} \bar{\nu}_L\,, \quad \bar{d}_L d_R \to \psi_{3/2} \nu_L\,, \quad u_L \bar{d}_R \to \psi_{3/2} e_L^+\,, \quad \bar{u}_L d_R \to \psi_{3/2} e_L^-$$

$$\begin{cases} qq \to \psi_{3/2} \bar{\nu} \to ee\nu\bar{\nu}\,, \; qq\nu\bar{\nu}\,, \; qqe\nu \\ qq \to \psi_{3/2} e \to eee\nu\,, \; qqe\nu\,, \; qqee \end{cases}$$

$$\Rightarrow \quad pp \to ee E_{\mathrm{T}}^{\mathrm{mis}}, eee E_{\mathrm{T}}^{\mathrm{mis}}, qq E_{\mathrm{T}}^{\mathrm{mis}}, qqe E_{\mathrm{T}}^{\mathrm{mis}}, qqee$$

Collider phenomenology, spin-2

- Lowest order operator linear in spin-2 field is dim-7:

$$-\mathcal{H}_{\rm linear} = \frac{1}{\Lambda^3}\psi_2^{abcd}\Big[c_B\sigma_{ab}^{\mu\nu}\sigma_{cd}^{\rho\lambda}B_{\mu\nu}B_{\rho\lambda} + c_W\sigma_{ab}^{\mu\nu}\sigma_{cd}^{\rho\lambda}W_{i\mu\nu}W^i_{\rho\lambda}$$
$$+c_G\sigma_{ab}^{\mu\nu}\sigma_{cd}^{\rho\lambda}G_{A\mu\nu}G^A_{\rho\lambda}\Big] + \text{h.c.}$$

- Significant differences to Kaluza-Klein graviton
- Decay modes:

$$\psi_2 \to \gamma\gamma,\ ZZ,\ Z\gamma,\ WW,\ gg.$$

- Produced in gluon-gluon fusion:

$$\hat\sigma(gg\to\psi_2) = \frac{16\pi c_G^2}{3}\frac{m_2^8}{\hat s^{3/2}\,\Lambda^6}\delta(\sqrt{\hat s}-m_2).$$

Δ-resonance

- Spin-3/2 Δ-resonance ($m_\Delta = 1.232$ GeV) is the most important baryon resonance (excited nucleon)
- It is predominantly described as Rarita-Schwinger field, which has inconsistent interactions as stated earlier
- This needs fixing! $\Rightarrow$ Multispinor formalism!
- Δ-resonance has isospin-3/2:

$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix}$$

Δ-resonance

Δ-resonance mediates pion-nucleon scattering and pion photo-production from nucleon:

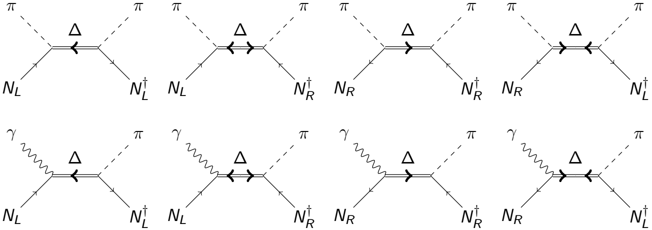


Figure: Feynman diagrams contributing to the resonant $\pi N \to \pi N$ (top) and $\gamma N \to \pi N$ (bottom) processes.

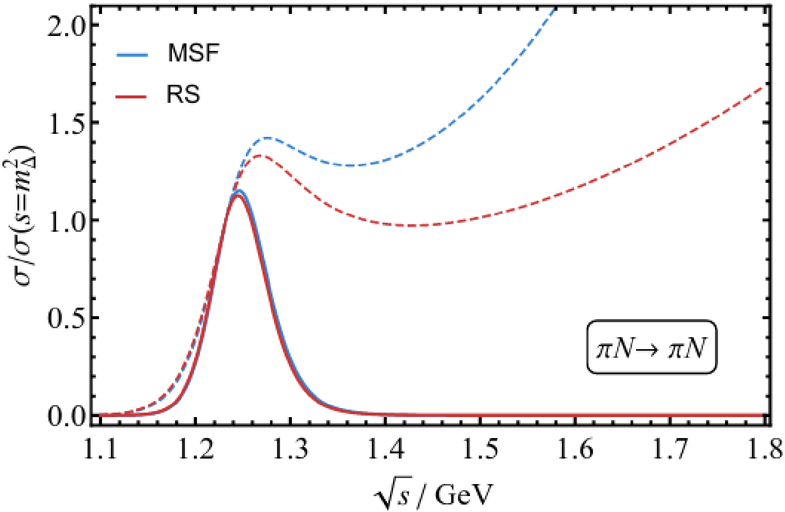
Δ-resonance interactions in multispinor formalism

Field	Lorentz irrep	Isospin	Hypercharge	Dimension
π	$(0,0)$	1	0	1
N_L	$(1/2,0)$	$1/2$	$1/2$	$3/2$
N_R	$(0,1/2)$	$1/2$	$1/2$	$3/2$
Δ_L	$(3/2,0)$	$3/2$	$1/2$	$5/2$
Δ_R	$(0,3/2)$	$3/2$	$1/2$	$5/2$
F_L	$(1,0)$	0	0	2
F_R	$(0,1)$	0	0	2

$$-\mathcal{H}_{\pi N\Delta} = \tfrac{c_L}{\Lambda^3}\left[\partial^a{}_{\dot b}(N_R)^{\dagger\, b}\partial^{\dot b c}\pi_A T_A(\Delta_L)_{abc} + \partial_{\dot a}{}^b(N_L)^\dagger_{\dot b}\partial_{b\dot c}\pi_A T_A(\Delta_R)^{\dot a\dot b\dot c}\right]$$

$$-\mathcal{H}_{\gamma N\Delta} = \tfrac{c_\gamma}{\Lambda^2}\left[F^{ab}(N_L)^c T_3(\Delta_L)_{abc} + F_{\dot a\dot b}(N_L)_{\dot c} T_3(\Delta_L)^{\dot a\dot b\dot c}\right] + \text{h.c.}$$

Multispinor formalism vs Rarita-Schwinger



Summary

- We have written down an effective field theory for generic massive particle of any spin particle.
- We call this the multispinor formalism
- Multispinor formalism: always correct number of degrees of freedom $\Rightarrow$ Consistent interactions for any spin!
- Interactions are always consistent, unlike in most higher spin theories (like Rarita-Schwinger)
- People are finally free to study higher spin phenomenology using the multispinor formalism!