Unexpected alliance: 331-model with Froggatt-Nielsen and linear seesaw mechanisms

Katri Huitu[1], Niko Koivunen[2] and Timo J. Kärkkäinen[3]

[1] University of Helsinki & Helsinki Institute of Physics
[2] National Institute Of Chemical Physics And Biophysics
[3] ELTE Eötvös Loránd university

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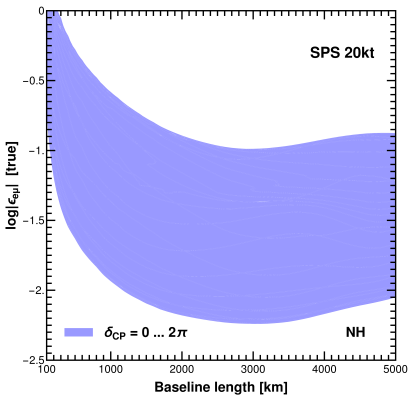
Timo J. Kärkkäinen

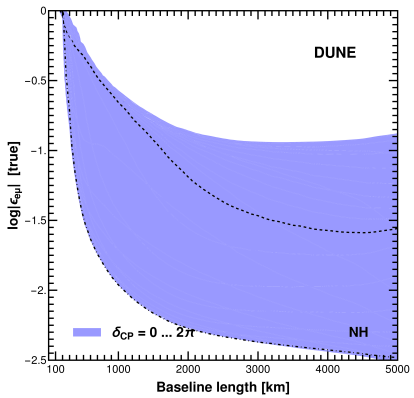
- University of Helsinki & Helsinki Institute of Physics
  - 03/2012: BSc in theoretical physics, *CP violation in K^0 decay*
  - 12/2013: MSc in theoretical physics, *Neutrino mass hierarchy and leptonic CP violation confidence limits in LAGUNA*
  - 11/2018: PhD in theoretical physics, *Neutrino masses and oscillations in theories beyond the Standard model*
- Eötvös Loránd university - 10/2019: Postdoctoral researcher

Nonstandard interactions on accelerator ν oscillations

Huitu, Kärkkäinen, Maalampi, Vihonen

(1601.07730, 1710.01080, 1711.02971)

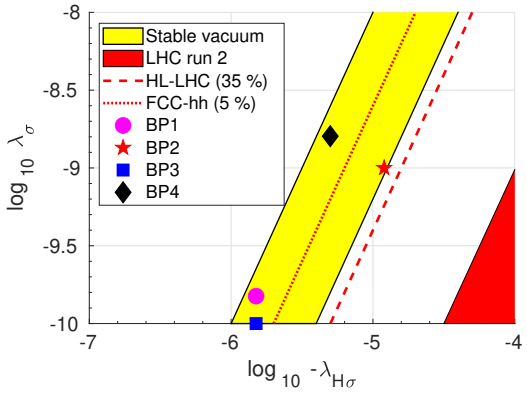




Scalar couplings and RGE analysis

Das, Huitu, Kärkkäinen

(1810.23232, 1911.XXXXX)



Three problems

> Why there are three generations of matter?

> Why Yukawa couplings are hierarchical?

> Why neutrinos are massive?

Three problems, three solutions

> Why there are three generations of matter?
$\Rightarrow$ $SU(3)_C \times SU(3)_L \times U(1)_X$ (331-model)

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We propose a combined model with smallest possible extension.

Solution to first problem

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We propose a combined model with smallest possible extension.

First ingredient: $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group

- Electric charge is defined by $Q = T_3 + \beta T_8 + Q_X$.
  - $\beta = \pm\sqrt{3}$: three scalar triplets and one scalar sextet
    Pisano and Pleitez (1992), Frampton (1992)
  - $\beta = \pm\frac{1}{\sqrt{3}}$: three scalar triplets
    Georgi and Pais (1979), Singer, Valle and Schechter (1980), Valle and Singer (1983)
  - $\beta = 0$: new gauge bosons, scalars and leptons with $\pm\frac{1}{2}e$ charge and new quarks with $\pm\frac{1}{6}e$ charge
    Hue and Ninh (2016)
- We choose $\beta = -\frac{1}{\sqrt{3}}$, preferring to construct a minimal model with integer charges for gauge bosons, scalars and leptons.
- Triangle anomaly induced by $SU(3)_L$ fermion triplets cancels only if the number of $SU(3)_L$ triplets equals the number of antitriplets. In 331-models this requires that the number of generations N_g is integer multiple of number of colours N_C.

Number of generations in 331-model

$$\beta(\alpha_s) = \overbrace{-(33-2n_f)\frac{\alpha_s^2}{12\pi^2}}^{\text{leading order}} - (153-19n_f)\frac{\alpha_s^3}{24\pi^2} - \left(2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2\right)\frac{\alpha_s^4}{128\pi^3}$$

$$\Rightarrow \alpha_s(\mu_R^2) = \frac{1}{(33-2n_f)\ln\frac{\mu_R^2}{\Lambda^2}}$$

≥ 6 generations of colour fermions change the running of QCD coupling in a way the strong interaction is not asymptotically free $\Rightarrow$ in 331-models only $N_g = 3$ is possible.

Fermion content: SM $+ \nu' + N_R + U + D_1 + D_2$

Field ($i = 1, 2, 3$)	$SU(3)_C$	$SU(3)_L$	$U(1)_X$
$L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ \nu'_i \end{pmatrix}_L$	**1**	**3**	$-\frac{1}{3}$
$e_R,\ \mu_R,\ \tau_R$	**1**		-1
$N_{R,1},\ N_{R,2},\ N_{R,3}$			0
$Q_{L,1} = \begin{pmatrix} u_1 \\ d_1 \\ U \end{pmatrix}_L$	**3**		$\frac{1}{3}$
$Q_{L,2} = \begin{pmatrix} d_2 \\ -u_2 \\ D_1 \end{pmatrix}_L$	**3**	$\mathbf{3}^*$	0
$Q_{L,3} = \begin{pmatrix} d_3 \\ -u_3 \\ D_2 \end{pmatrix}_L$			
$u_R,\ c_R,\ t_R,\ U_R$		**1**	$2/3$
$d_R,\ s_R,\ b_R,\ D_{R,1},\ D_{R,2}$			$-1/3$

$SU(3)_L$ is anomaly free!

Scalar content

The vacuum symmetry allows us to rotate away one of the vacuum expectation values (VEV).

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix}, \qquad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \qquad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix}$$

$$\langle\eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v' \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \\ v_2 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

- We assign VEVs with hierarchy $v', v_1 \sim v_{\mathrm{SM}} \ll u, v_2$.

$$\mathrm{SU}(3)_L \times \mathrm{U}(1)_X \xrightarrow{u, v_2} \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \xrightarrow{v', v_1} \mathrm{U}(1)_Q$$

- Minimal scalar content! 2 triplets $\Rightarrow m_u, m_d = 0$ (1905.05278)

Scalar fields

$$
\begin{aligned}
V = {} & \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 \\
& + \lambda_{12} (\eta^\dagger \eta)(\rho^\dagger \rho) + \lambda_{13} (\eta^\dagger \eta)(\chi^\dagger \chi) + \lambda_{23} (\rho^\dagger \rho)(\chi^\dagger \chi) \\
& + \widetilde{\lambda}_{12} (\eta^\dagger \rho)(\rho^\dagger \eta) + \widetilde{\lambda}_{13} (\eta^\dagger \chi)(\chi^\dagger \eta) + \widetilde{\lambda}_{23} (\rho^\dagger \chi)(\chi^\dagger \rho) \\
& + \sqrt{2} f (\varepsilon_{ijk} \eta^i \rho^j \chi^k + \text{h.c.}) + (b \rho^\dagger \chi + \text{h.c.})
\end{aligned}
$$

- There are 14 independent real scalar fields (the five complex neutral scalar fields can be written by their real and imaginary parts).
- Since all Higgs potential parameters are real, there is no mixing between real and imaginary parts of the neutral scalar field, allowing us to disentangle the mass matrix to CP-even and CP-odd part.
- From the full Higgs potential we obtained 5×5, 5×5 and 4×4 mass matrices for CP-even, CP-odd and charged scalars.

There are 4 CP-even, 2 CP-odd and 2 charged scalars

Six eigenvalues of the mass matrices are zero

$\Rightarrow$ massless Goldstone bosons

$\Rightarrow$ massive gauge bosons $W^{\pm}, Z, V^{\pm}, Z', X^0$, and X^{0*}.

$$m_{H_1}^2 = \frac{1}{2}\left(-\tilde{\lambda}_{23} - \frac{2b}{uv_2}\right)\left(\frac{v_1^2}{v'^2}u^2 + u^2 + v_2^2\right) + \mathcal{O}\left(v_{\text{light}}^2/v_{\text{heavy}}^2\right)$$

$$m_{H_2}^2 = A - \sqrt{A^2 - B} + \mathcal{O}\left(v_{\text{light}}^2/v_{\text{heavy}}^2\right)$$

$$m_{H_3}^2 = A + \sqrt{A^2 - B} + \mathcal{O}\left(v_{\text{light}}^2/v_{\text{heavy}}^2\right)$$

$$m_{A_1}^2 = \frac{1}{2}\left(-\tilde{\lambda}_{23} - \frac{2b}{uv_2}\right)\left(\frac{v_1^2}{v'^2}u^2 + u^2 + v_2^2 + v_1^2\right)$$

$$m_{A_2}^2 = -\frac{b(u^2 + v_2^2 + v_1^2)}{uv_2}$$

$$m_{H_1^{\pm}}^2 = \frac{1}{2}\left(-\tilde{\lambda}_{23} - \frac{2b}{uv_2}\right)\frac{v_1^2}{v'^2}u^2 + \frac{1}{2}\left(\tilde{\lambda}_{12}v_2^2 + \tilde{\lambda}_{13}u^2\right)$$

$$m_{H_2^{\pm}}^2 = \frac{1}{2}\left(-\tilde{\lambda}_{23} - \frac{2b}{uv_2}\right)\left(\frac{v_1^2}{v'^2}u^2 + u^2 + v_2^2\right)$$

Gauge sector

- Symmetry breaking pattern in 331-model:

$$\mathrm{SU}(3)_L \times \mathrm{U}(1)_X \to \mathrm{SU}(2)_L \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}(1)_\mathrm{Q}$$

- Covariant derivative:

$$D_\mu = \partial_\mu - ig_3 \sum_{a=1}^{8} T_a W_{a\mu} - ig_x X B_\mu,$$

where g_3 and g_x are the $SU(3)_L$ and $U(1)_X$ gauge couplings. $T_a = \lambda_a/2$ are the $SU(3)_L$ generators. The $SU(3)_L$ gauge bosons are

$$\sum_{a=1}^{8} T_a W_{a\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & W'^{+}_{\mu} & X'^{0}_{\mu} \\ W'^{-}_{\mu} & -\frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & V'^{-}_{\mu} \\ X'^{0*}_{\mu} & V'^{+}_{\mu} & -\frac{2}{\sqrt{6}} W_{8\mu} \end{pmatrix},$$

$$W'^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}), \quad V'^{\mp}_{\mu} = \frac{1}{\sqrt{2}}(W_{6\mu} \mp iW_{7\mu}),$$

$$X'^{0}_{\mu} = \frac{1}{\sqrt{2}}(W_{4\mu} - iW_{5\mu}).$$

Charged gauge boson masses

4 singly charged gauge bosons: $W^\pm$ and $V^\pm$.

$$
\begin{aligned}
m_{W^\pm}^2 &= \frac{g_3^2}{8}\left[(2v'^2 + u^2 + v_1^2 + v_2^2) - \sqrt{(v_1^2 + v_2^2 + u^2) - 4v_1^2u^2}\right] \\
&= \frac{g_3^2}{4}(v'^2 + \frac{v_1^2u^2}{v_2^2 + u^2}) + \mathcal{O}\left(\frac{v_{\text{light}}^2}{v_{\text{heavy}}^2}\right) \\
m_{V^\pm}^2 &= \frac{g_3^2}{8}\left[(2v'^2 + u^2 + v_1^2 + v_2^2) + \sqrt{(v_1^2 + v_2^2 + u^2) - 4v_1^2u^2}\right] \\
&= \frac{g_3^2}{4}(v_2^2 + u^2) + \mathcal{O}\left(\frac{v_{\text{light}}^2}{v_{\text{heavy}}^2}\right)
\end{aligned}
$$

Mixing between $W^\pm$ and $V^\pm$

$$\tan 2\theta = -\frac{2v_1v_2}{v_2^2 + u^2 - v_1^2} \approx 0.02 \text{ in our model}$$

Neutral gauge boson masses

5 neutral gauge bosons: γ, Z^0, Z'^0, X^0 and X^{0*}.

$$
\begin{aligned}
m_\gamma^2 &= 0 \\
m_X^2 &= \frac{g_3^2}{4}(u^2 + v_2^2 + v_1^2) \\
m_Z^2 &= \frac{g_3^2}{4}\left(\frac{3g_3^2 + 4g_x^2}{3g_3^2 + g_x^2}\right)\left(v'^2 + \frac{v_1^2 u^2}{v_2^2 + u^2}\right) + \mathcal{O}\left(\frac{v_{\text{light}}^2}{v_{\text{heavy}}^2}\right), \\
m_{Z'}^2 &= \frac{3g_3^2 + g_x^2}{9}(v_2^2 + u^2) + \mathcal{O}\left(\frac{v_{\text{light}}^2}{v_{\text{heavy}}^2}\right)
\end{aligned}
$$

In total, 9 $\mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ gauge bosons. Note that since X^0 and X^{0*} are antiparticles of each other, they are identified as physical neutral nonhermitian gauge bosons, having the same mass.

Nonstandard interactions in 331FN

Neutrino production and propagation properties are affected via effective operators

$$\mathcal{L}_{\rm NSI}^{\rm CC} = -2\sqrt{2}G_F\varepsilon_{\alpha\beta}^{\ell\ell'}(\overline{\nu}_{L\alpha}\gamma^\mu\nu_{L\beta})(\overline{\ell}_L\gamma_\mu\ell'_L)$$

which produce distortions to nuclear β decay energy spectrum, neutrino oscillation events and charged lepton flavour violating decays.

- Gauge boson $V^\pm$ driven operator has effective NSI couplings $|\varepsilon| \sim (m_{W^\pm}/m_{V^\pm})^2$.
- Charged Higgs $H_i^\pm$ driven operator has scalar-scalar form, which via Fierz transform gains the V-A operator form, with $|\varepsilon| \sim |Y_\ell|^2 \left(m_{W^\pm}/m_{H_i^\pm}\right)^2$.

Both sources for NSI are negligible with our chosen benchmarks for new gauge bosons and scalars ($m_{V^\pm}, m_{H_i^\pm} > 5$ TeV).

Solution to second problem

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Second ingredient: Froggatt-Nielsen (FN) mechanism

Froggatt and Nielsen (1979)

- FN mechanism requires the existence of a SM gauge singlet **flavon**, charged under new global[1] FN symmetry, $\mathrm{U}(1)_{\mathrm{FN}}$.
- Imposition of FN symmetry forbids direct Yukawa terms in the Lagrangian. Instead they are generated as effective couplings.
- 331-model already contains the needed fields for FN mechanism: flavon in **FN331** is an effective gauge singlet combination $\rho^\dagger\chi$ with VEV $v_2 u/2$ and FN charge -1.

	η	ρ	χ	L^c_α	e_R	μ_R	τ_R	$N_{R,i}$
Q_{FN}	-1	1	0	L	$10-L$	$7-L$	$5-L$	0

- There will be only one charge assignment we can freely plug in. We choose this to be the FN charge of lepton triplet, L.
- Quark sector studied in Huitu, Koivunen (1706.09463, 1905.05278).

[1] There are variants where the new symmetry is a local gauge symmetry or a discrete symmetry.

Yukawa couplings emerge

The effective operator that generates the Yukawa couplings of the charged fermions is

$$\Delta\mathcal{L}_\ell^Y = \underbrace{(c_s^f)_{ij}}_{\mathcal{O}(1)} \left(\frac{\rho^\dagger\chi}{\Lambda^2}\right)^{(n_f^s)_{ij}} \bar{\psi}_{L,i}^f s f_{R,j} + h.c.,$$

where Λ is the new physics scale and $s = \eta, \rho, \chi$. The power $(n_f^s)_{ij}$ is positive integer determined by the FN charge conservation:

$$(n_f^s)_{ij} = Q_{\text{FN}}(\bar{\psi}_{L,i}^f) + Q_{\text{FN}}(f_{R,j}) + Q_{\text{FN}}(s)$$

The corresponding Yukawa couplings are

$$(y_s^f)_{ij} = (c_s^f)_{ij} \left(\frac{v_2 u}{2\Lambda^2}\right)^{(n_f^s)_{ij}} \equiv (c_s^f)_{ij} \varepsilon^{(n_f^s)_{ij}},$$

and the suppression is provided choosing $v_2 u/2\Lambda^2 \equiv \varepsilon < 1$.

Lepton Yukawa couplings and neutrino mixing

Smaller ε provides stronger suppression. FN charge assignment replaces the arbitrary Yukawa fine-tuning in the SM. We have chosen $\varepsilon = \sin\theta_C \approx 0.23$ as the suppression factor:

$$Y_\ell = \begin{pmatrix} c_{11}\varepsilon^9 & c_{12}\varepsilon^6 & c_{13}\varepsilon^4 \\ c_{21}\varepsilon^9 & c_{22}\varepsilon^6 & c_{23}\varepsilon^4 \\ c_{31}\varepsilon^9 & c_{32}\varepsilon^6 & c_{33}\varepsilon^4 \end{pmatrix}, \quad Y_\nu = \begin{pmatrix} c'_{11}\varepsilon^{18} & c'_{12}\varepsilon^{18} & c'_{13}\varepsilon^{18} \\ c'_{21}\varepsilon^{18} & c'_{22}\varepsilon^{18} & c'_{23}\varepsilon^{18} \\ c'_{31}\varepsilon^{18} & c'_{32}\varepsilon^{18} & c'_{33}\varepsilon^{18} \end{pmatrix}$$

These will be diagonalized by biunitary transform:

$$U_L^\ell Y_\ell U_R^{\ell\dagger} = Y_\ell^{\text{diag}}, \quad U_\nu^\dagger Y_\nu U_\nu = Y_\nu^{\text{diag}}$$

PMNS matrix elements are $\mathcal{O}(1)$, since $U_L^\ell, U_\nu \sim \mathcal{O}(1)$ due to same FN charge of left-handed lepton triplets.

$$U_{\text{PMNS}} \approx \overbrace{\cos\theta}^{\sim 0.9-1.0} \; U_L^\ell U_\nu \sim \mathcal{O}(1)$$

Second way to generate Yukawa couplings

$$\Delta\mathcal{L}_\nu = (c^N_{\eta^*})_{ij}\left(\frac{\rho^\dagger\chi}{\Lambda^2}\right)^{(n^{\eta^*}_N)_{ij}}\varepsilon_{\alpha\beta\gamma}\bar{L}^\alpha_{L,i}(L^c_{L,j})^\beta(\eta^*)^\gamma + h.c.$$

Together with the previous effective operators, these produce the following Yukawa Lagrangian for neutrinos:

$$\mathcal{L}^Y_\nu = e_{ij}\varepsilon_{\alpha\beta\gamma}\bar{L}^\alpha_{L,i}(L^c_{L,j})^\beta(\eta^*)^\gamma + y^N_{ij}\bar{L}_{L,i}\rho N_{R,j} + y'^N_{ij}\bar{L}_{L,i}\chi N_{R,j} + \cdots$$

The resulting Yukawa couplings are $(Q(\bar{L}_{L,i}) \equiv L)$

$$\begin{aligned}
y^N_{ij} &= c^N_{ij}\varepsilon^{Q(\bar{L}_{L,i})+Q(N_{R,j})+Q(\rho)} = c^N_{ij}\varepsilon^{L+1} \\
y'^N_{ij} &= c'^N_{ij}\varepsilon^{Q(\bar{L}_{L,i})+Q(N_{R,j})+Q(\chi)} = c'^N_{ij}\varepsilon^{L} \\
e_{ij} &= (c^N_{\eta^*})_{ij}\varepsilon^{Q(L^c_{L,i})+Q(L^c_{L,j})+Q(\eta^*)} = (c^N_{\eta^*})_{ij}\varepsilon^{2L+1}
\end{aligned}$$

Mass generation of heavy neutrinos

The right-handed neutrino singlet $N_{R,i}$ will also have a Majorana mass term which is generated by the following operator,

$$\Delta\mathcal{L} \supset M_0 c_{ij}^M \left(\frac{\rho^\dagger \chi}{\Lambda^2}\right)^{Q_{\rm FN}(N_{R,i})+Q_{\rm FN}(N_{R,j})} \overline{(N_{R,j})^c} N_{R,j} + \text{h.c.},$$

M_0 is a free parameter. We choose it to be same as the FN-messenger scale, Λ, in order not to introduce new mass scales into the model. The Majorana mass term becomes,

$$\mathcal{L}_{\rm Majorana} = \frac{1}{2} M_{ij} \overline{(N_{R,j})^c} N_{R,j} + h.c.,$$

where the Majorana mass matrix is

$$M_{ij} = \Lambda c_{ij}^M \varepsilon^{q(N_{R,i})+q(N_{R,j})}.$$

The messenger scale Λ is

$$\Lambda = \sqrt{\frac{u v_2}{2\varepsilon}}.$$

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Third ingredient: Linear seesaw mechanism

After spontaneous symmetry breaking, the neutrino mass terms can be collected, and they form a familiar neutrino mass matrix without any new input: FN331 already contains the ν mass generation mechanism.

Akhmedov, Lindner, Schnapka, Valle (1996), Albright, Barr (2004)...

$$\mathcal{L}_\nu^m = \frac{1}{2}\begin{pmatrix} \overline{(\nu_L)^c} \\ \overline{(\nu'_L)^c} \\ \overline{N_R} \end{pmatrix}^T \begin{pmatrix} 0 & 2(m^D)^\dagger & (m^N)^* \\ 2(m^D)^* & 0 & (m'^N)^* \\ (m^N)^\dagger & (m'^N)^\dagger & M^* \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu'_L \\ (N_R)^c \end{pmatrix} + \text{h.c.}$$

$$m_{ij}^N = \frac{v_1}{\sqrt{2}} y_{ij}^N, \qquad m'^N_{ij} = \frac{v_2}{\sqrt{2}} y_{ij}^N + \frac{u}{\sqrt{2}} y'^N_{ij} \qquad m_{ij}^D = \frac{v'}{\sqrt{2}} e_{ij}$$

- m_1, m_2, m_3: Three light sub-eV-scale neutrinos
- m_4, m_5, m_6: Three medium eV or keV-scale neutrinos
- m_7, m_8, m_9: Three heavy TeV-scale neutrinos

Neutrino masses after diagonalization

The light, medium and heavy blocks can be written at lowest order as:

$$\begin{aligned} m_{\text{light}} &= 2m^{D\dagger}(m'^{N\dagger})^{-1}M^{*}(m'^{N*})^{-1}2m^{D*} \\ &\quad - \left[m^{N*}(m'^{N*})^{-1}2m^{D*} + 2m^{D\dagger}(m'^{N\dagger})^{-1}m^{N\dagger}\right] \\ m_{\text{medium}} &= m'^{N*}(M^{*})^{-1}m'^{N\dagger} \\ m_{\text{heavy}} &= M^{*}. \end{aligned}$$

The order of magnitude of light-, medium- and heavy-neutrino masses can now be estimated

$$m_{ij}^{\text{light}} \sim \frac{v_{\text{light}}^{2}}{v_{\text{heavy}}}\varepsilon^{2L+2}, \quad m_{ij}^{\text{medium}} \sim v_{\text{heavy}}\varepsilon^{2L}, \quad \text{and} \quad m_{ij}^{\text{heavy}} \sim v_{\text{heavy}}.$$

Example benchmark point

$$y_{ij}^N = c_{ij}^N \varepsilon^{L+1} \qquad y'^N_{ij} = c'^N_{ij} \varepsilon^L \qquad e_{ij} = (c_{\eta^*}^N)_{ij} \varepsilon^{2L+1}$$

- We generated all the c matrices with elements $0.5 < |c_{ij}| < 5$.
- We fit the observed lepton masses, Δm_{ij}^2 and $|U_{\ell i}^{\rm PMNS}|$ from neutrino oscillation data and $m_1 + m_2 + m_3 < 0.12$ eV.

v' (GeV)	204.117
v_1 (GeV)	204.117
u (TeV)	5
v_2 (TeV)	5.5
L	9
$Q_{\rm FN}(e_R)$	1
$Q_{\rm FN}(\mu_R)$	-2
$Q_{\rm FN}(\tau_R)$	-4

- Active-sterile mixing bounds from β decay kink searches, $0\nu\beta\beta$ experiments and neutrino oscillation experiments.

Neutrino masses in FN331

$$m_1 = 1.49 \text{ meV}, \qquad m_2 = 8.60 \text{ meV} \qquad m_3 = 50.4 \text{ meV}$$

$$\Delta m_{21}^2 = 7.18 \cdot 10^{-5} \text{ eV}^2 \quad \Delta m_{32}^2 = 2.54 \cdot 10^{-3} \text{ eV}^2 \quad \sum_{i=1}^{3} m_i = 0.06 \text{ eV}$$

$$m_4 = 11.8 \text{ eV}, \qquad m_5 = 18.7 \text{ eV} \qquad m_6 = 35.1 \text{ eV}$$

$$m_7 = 21.1 \text{ TeV}, \qquad m_8 = 37.0 \text{ TeV} \qquad m_9 = 54.2 \text{ TeV}$$

- In other benchmarks, $m_4, m_5, m_6 \sim$ keV, making the medium-energy neutrinos a dark matter candidate.
- TeV scale neutrinos might be responsible for leptogenesis scenario, producing observed baryon-to-photon ratio (matter-antimatter asymmetry) of the universe.

Can we see the medium-energy neutrinos?

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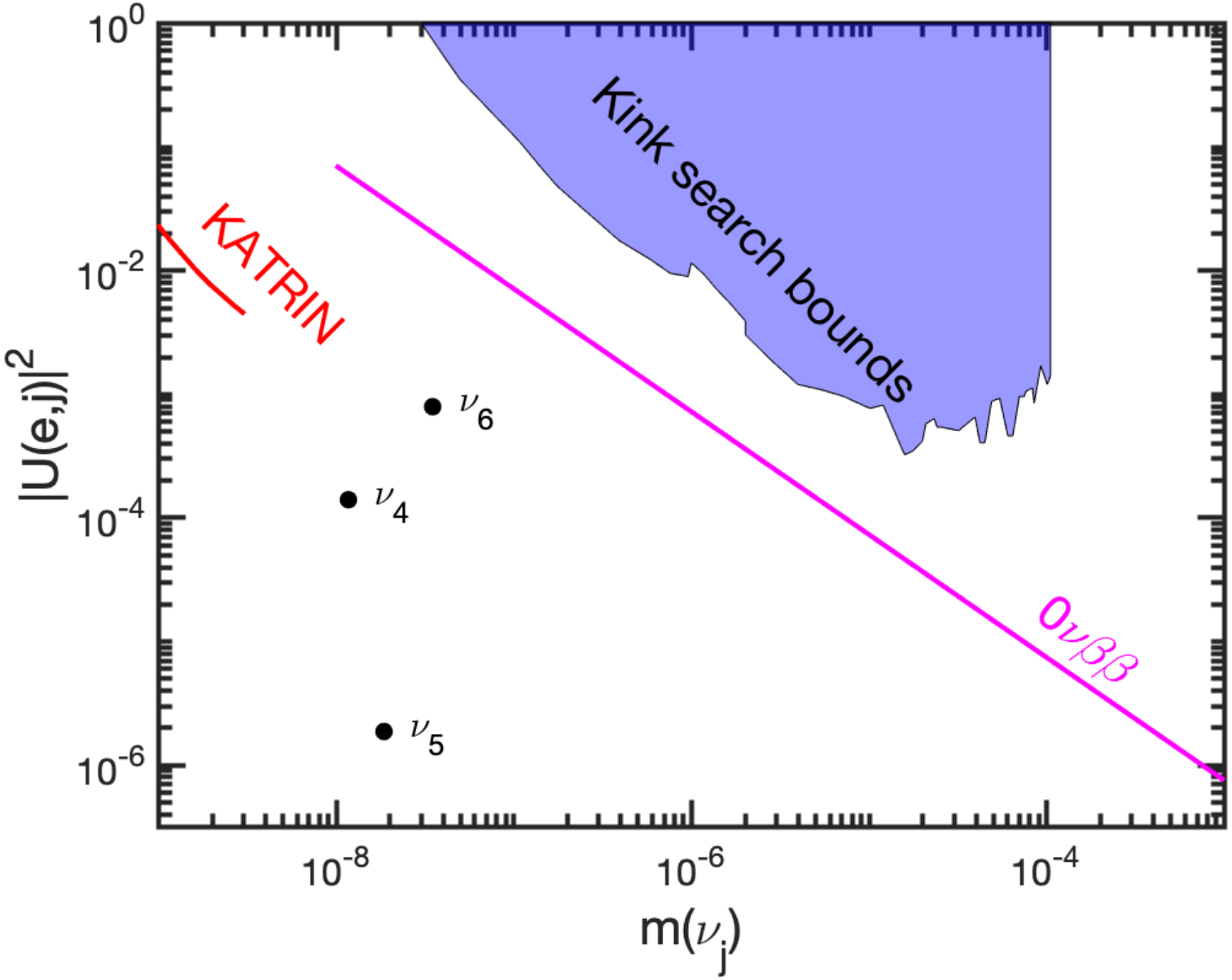
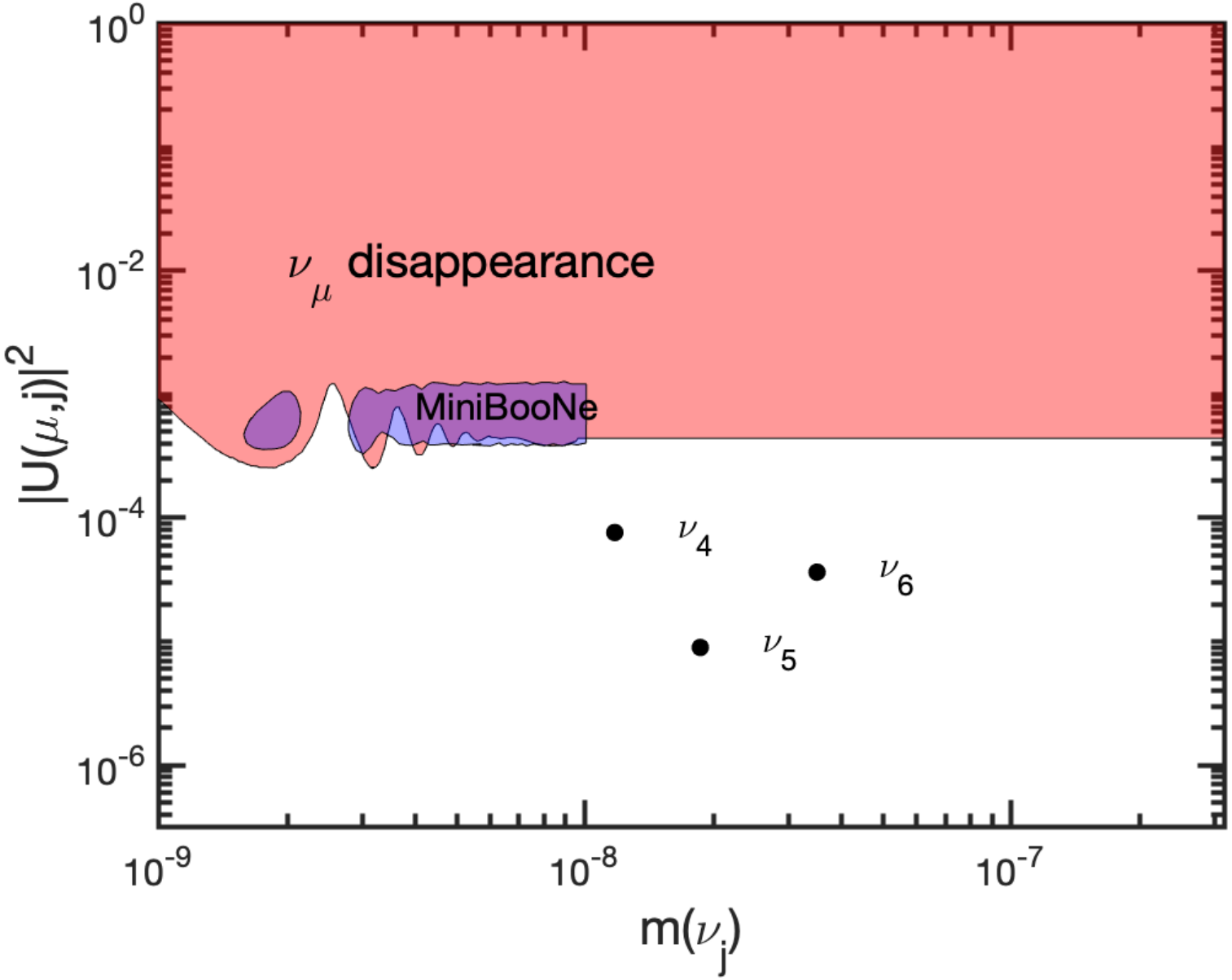
- TeV scale neutrinos might be responsible for leptogenesis scenario, producing observed baryon-to-photon ratio (matter-antimatter asymmetry) of the universe.

Constraints from ν_e—ν_s mixing



Constraints from ν_μ—ν_s mixing



Conclusions

Three problems of the Standard Model can be solved within the TeV scale, combining three known beyond-the-standard-model frameworks.

> Why there are three generations of matter?
> $\Rightarrow$ $SU(3)_L \times SU(3)_C \times U(1)_X$ (331-model)

> Why Yukawa couplings are hierarchical?
> $\Rightarrow$ Froggatt-Nielsen mechanism

> Why neutrinos are massive?
> $\Rightarrow$ (linear) seesaw mechanism

Experimental sensitivities should be constrained by a factor of 10 to probe the medium-massive neutrino sector. Unfortunately not a candidate for the detected neutrino oscillation anomalies.

Thank you!