Scalar mass stability bound in a simple Yukawa-theory from renormalisation group equations

(arXiv:1508.06774)

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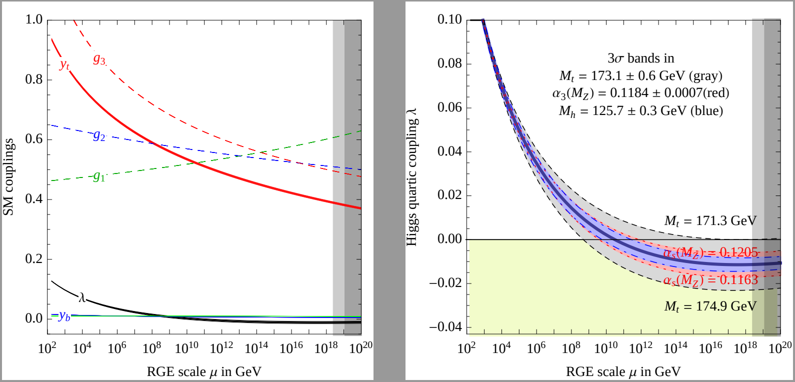
Motivation

Stability Higgs bounds with perturbative RGE in SM.

$$\frac{d\lambda(Q^2)}{d\log Q^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda h_t^2 - 3h_t^4 + \frac{3}{2}\lambda(3g_2^2+g_1^2) + \frac{3}{16}(2g_2^4 + (g_2^2+g_1^2)^2)\right)$$



arXiv:1512.01222 [hep-ph]

Motivation

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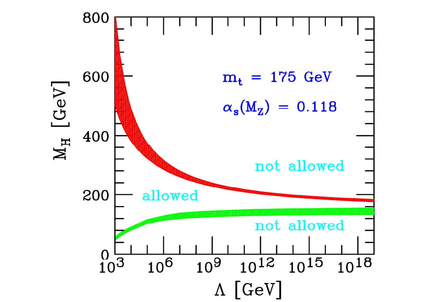
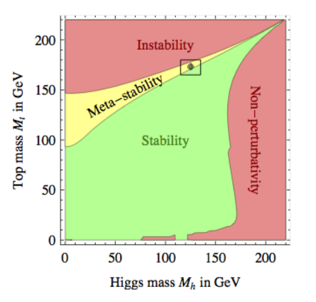




Non-perturbative studies in simplified models (see below):

Lattice: Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, 2007
D.Y.-J. Chu, K. Jansen, B. Knippschild, C.-J. D. Lin and A. Nagy, 2015

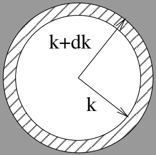
FRG: H. Gies, C. Gneiting and R. Sondenheimer, 2014

Outline

- Functional Renormalization Group Equations
- Higgs-top model with discrete chiral symmetry
- Functional Renormalization Group Equations of the system
- The Local Potential Approximation (LPA)
- Consistent solutions
- Results

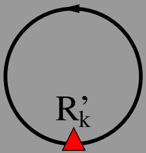
The Wetterich equation

Calculating the contributions to the effective action from momentum shell to momentum shell:



$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr}\, \partial_t R_k \left(\Gamma_k^{(2)} + R_k \right)^{-1}$$

The regulator term suppresses $p < k$ terms by giving them large mass.
Γ_k contains the quantum fluctuations with momenta higher than k.



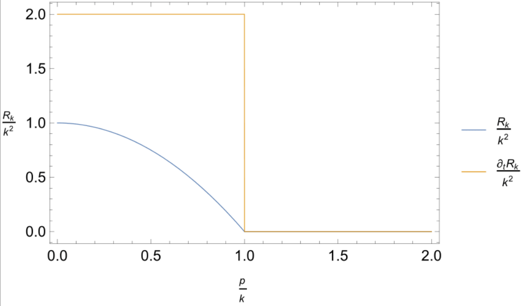
The Optimized Regulator

used in this paper for bosons

$$R_B(p) = (k^2 - p^2)\,\Theta\left(\frac{k^2}{p^2} - 1\right)$$

for fermions

$$R_F(p) = \not{p}\left(\frac{k}{\sqrt{p^2}} - 1\right)\Theta\left(\frac{k^2}{p^2} - 1\right)$$



Effective action of the Higgs-top toy model

Classical action of a scalar-fermion Yukawa bound system:

$$S = \int \mathrm{d}^4x \left[\frac{1}{2}(\partial_\mu\sigma)^2 + U(\rho) + \bar{\psi}\partial\!\!\!/\psi + h\sigma\bar{\psi}\psi\right], \quad \rho = \frac{1}{2}\sigma^2, \quad I = \sigma\bar{\psi}\psi$$

ρ and I are invariant under the discrete chiral symmetry:

$$\sigma(x) \to -\sigma(x), \qquad \psi(x) \to \gamma_5\psi(x), \qquad \bar{\psi}(x) \to -\bar{\psi}(x)\gamma_5$$

Scale-dependent effective-action for $k < \Lambda$:

$$\Gamma_k = \int \mathrm{d}^4x \left[\frac{Z_{\sigma,k}}{2}(\partial_\mu\sigma)^2 + U_k(\rho) + Z_{\psi,k}\bar{\psi}\partial\!\!\!/\psi + h_k\sigma\bar{\psi}\psi\right], \quad \rho = \frac{Z_{\sigma,k}}{2}\sigma^2$$

Infrared observables (obtained by fine-tuning the initial conditions):

$$v = Z_{\sigma 0}^{1/2}\sigma_0 = 246 GeV, \qquad m_\psi = h_0\sigma_0 = 173 GeV$$

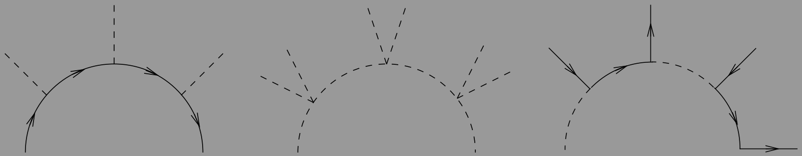
$$m_\sigma^2 = U_0'(\rho_0) + 2\rho_0 U_0''(\rho_0)$$

The Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\,\partial_t R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} = \frac{1}{2}\hat{\partial}_t \mathrm{STr}\log(\Gamma_k^{(2)} + R_k)$$

The right hand side of the Wetterich equation:

$$\frac{1}{2}\mathrm{STr}\log(\Gamma_k^{(2)} + R_k) = -\frac{1}{2}\mathrm{Tr}\log(\Gamma_{\Psi^T\Psi}^{(2)} + R_k^F) + \frac{1}{2}\mathrm{Tr}\log(\Gamma_{\sigma\sigma}^{(2)} + R_k^B)$$
$$+\frac{1}{2}\mathrm{Tr}\log\left[1 - \left(\Gamma_{\sigma\sigma}^{(2)} + R_k^B\right)^{-1}\Gamma_{\sigma\Psi}^{(2)}\left(\Gamma_{\Psi^T\Psi}^{(2)} + R_k^F\right)^{-1}\Gamma_{\Psi^T\sigma}^{(2)}\right]$$



ψ-loop in σ-bgd $\quad$ σ-loop in σ-bgd $\quad$ mixed loop in ψ-bgd

The Local Potential Approximation (LPA)

Constant background values, no field renormalization:

$$\sigma(x) \to v_k, \quad \psi(x) \to \psi_k, \quad \bar{\psi}(x) \to \bar{\psi}_k, \quad Z_{\sigma,k} \to 1, \quad Z_{\psi,k} \to 1$$

$$\partial_k [U_k(\rho_k) + h_k I_k] = \frac{1}{2} \hat{\partial}_k \int_q \Bigg[-4 \log(q_R^2 + m_\psi^2) + \log(q_R^2 + m_\sigma^2) + \\ + \log \left\{ 1 - h_k^2 \frac{1}{q_R^2 + m_\sigma^2} \frac{2 h_k I_k}{q_R^2 + m_\psi^2} \right\} \Bigg].$$

$$m_\sigma^2 = U_k'(\rho_k) + 2\rho_k U_k''(\rho_k), \qquad m_\psi^2 = 2 h_k^2 \rho_k$$

At a given scale ρ_k and I_k are connected by the equation of motion of the scalar field:

$$\sigma \frac{\delta \Gamma_k}{\delta \sigma} \Big|_{\sigma = v_k, \psi = \psi_k} = h_k I_k + 2\rho_k U_k'(\rho_k) = 0.$$

Consistent solutions

The consistency of the two sides of the LPA equation can be ensured in different ways.

Version *A*: Complete elimination of I_k on RHS

$$\partial_k h_t = 0,$$

$$\partial_t U_k(\rho_k) = \frac{1}{2}\hat{\partial}_t \int_q \Big\{ -5\log(q_R^2 + m_\psi^2) + \\ + \log\left[(q_R^2 + m_\sigma^2)(q_R^2 + m_\psi^2) + 4h_k^2 \rho_k U'(\rho_k)\right] \Big\}$$

After rewriting the equation using dimensonless variables and perform the $\hat{\partial}_t$ operation on RHS:

$$\partial_t u_r = (d-2)\rho_r u_r' - d u_r + \\ + v_d \left(-\frac{5}{1+\mu_\psi^2} + \frac{2+\mu_\psi^2+\mu_\sigma^2}{(1+\mu_\psi^2)(1+\mu_\sigma^2) + 4h_r^2\rho_r u_r'} \right)$$

Consistent solutions

Version B: Linearization of RHS in I_k

$$\partial_t(h_k I_k) = -\frac{1}{2}\hat{\partial}_t \int_q \frac{2h_k^3 I_k}{(q_R^2 + m_\psi^2)(q_R^2 + m_\sigma^2)},$$

$$\partial_t U_k(\rho_k) = \frac{1}{2}\hat{\partial}_t \int_q \Bigg[-4\log(q_R^2 + m_\psi^2) + \log(q_R^2 + m_\sigma^2) + + \log\left\{1 + h_k^2 \frac{1}{q_R^2 + m_\sigma^2} \frac{4\rho_k U_k'(\rho_k)}{q_R^2 + m_\psi^2}\right\}\Bigg] - \frac{1}{2}\hat{\partial}_t \int_q \frac{4h_k^2 \rho_k U_k'(\rho_k)}{(q_R^2 + m_\psi^2)(q_R^2 + m_\sigma^2)}.$$

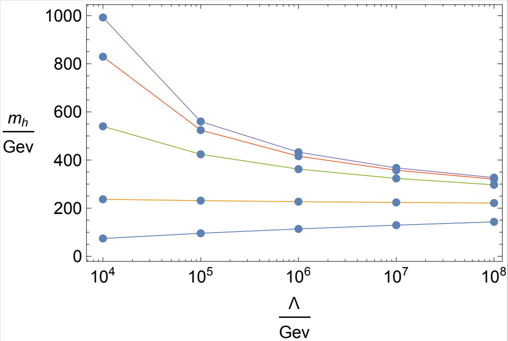
Gies *et al.* (2014): no ψ background, last two terms missing.

Results

The potential used in the study for the symmetric (SYM) and symmetry broken (SB) regime respectively:

$$U_k^{(\mathrm{SYM})}(\rho_k) = \sum_{n=1}^{N_p} \frac{\lambda_n \rho_k^n}{n!}, \qquad U_k^{(\mathrm{SB})}(\rho_k) = \sum_{n=2}^{N_p} \frac{\lambda_n (\rho_k - \kappa_k)^n}{n!}$$

Results for $\lambda_2 = 0.001, 1, 10, 50, 100$ from bottom to top respectively:



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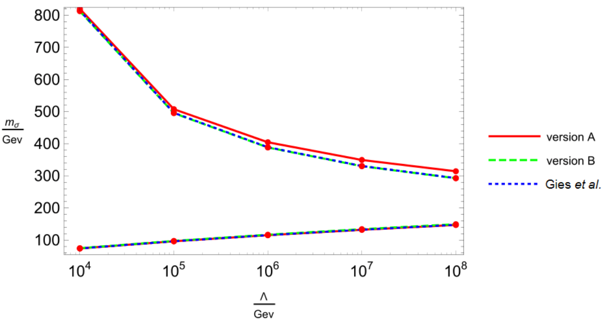
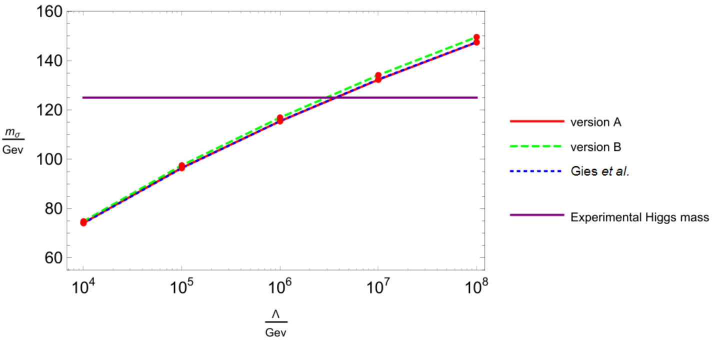
Higgs mass stability and triviality bounds for $N_p = 2$:



Results

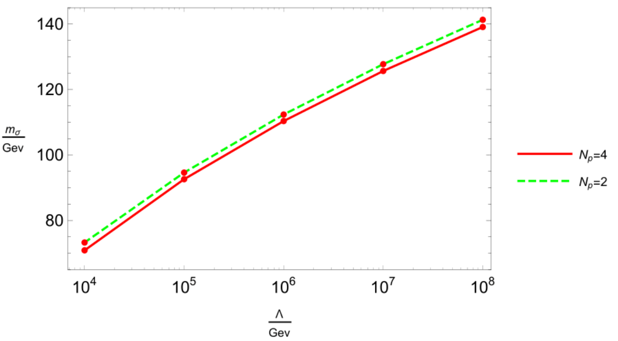


Including Z_ψ, $Z_\sigma \neq 1$ leads to a percent level changes.
The maximum allowed value of the cutoff

$N_p = 2$: $$2.9 \times 10^6 \text{GeV} < \Lambda^{(2)}_{max} < 3.7 \times 10^6 \text{GeV}$$

Results

Lower bound with quadratic and quartic truncations of $U_k(\rho_k)$ using Version A:



The maximum allowed value of the cutoff

$$N_p = 2: \qquad 2.9 \times 10^6 \text{GeV} < \Lambda_{max}^{(2)} < 3.7 \times 10^6 \text{GeV}$$

$$N_p = 4: \qquad 3.7 \times 10^6 \text{GeV} < \Lambda_{max}^{(4)} < 5.3 \times 10^6 \text{GeV}$$

Summary

- The allowed range of the Higgs mass has been determined with FRG in presence of a pointlike composite fermion background.
- Close agreement of all approximation signals a robust determination of Λ_{max}

Outlook

- Investigation of a more general ansatz for Γ_k with the effects of heavy neutrinos of the seesaw mechanism
- Inclusion of the multiplet structure and the gauge interactions of the SM