Integrable boundary states in AdS/CFT

Tamás Gombor

Contents

1) Introduction to AdS/CFT

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

2) Application of integrability for 2pt functions

Δ : spectrum of an integrable model

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3) Defect in AdS/CFT

$$\langle \mathcal{O}(x)\rangle = \frac{C_\Delta}{x_\perp^\Delta}$$

4) 1pt functions and integrable boundary states

$$C_\Delta = \frac{\langle B|\Delta\rangle}{\sqrt{\langle \Delta|\Delta\rangle}}$$

Motivation

Two important question in theoritical physics:

Understanding of QFTs beyond perturbation theory

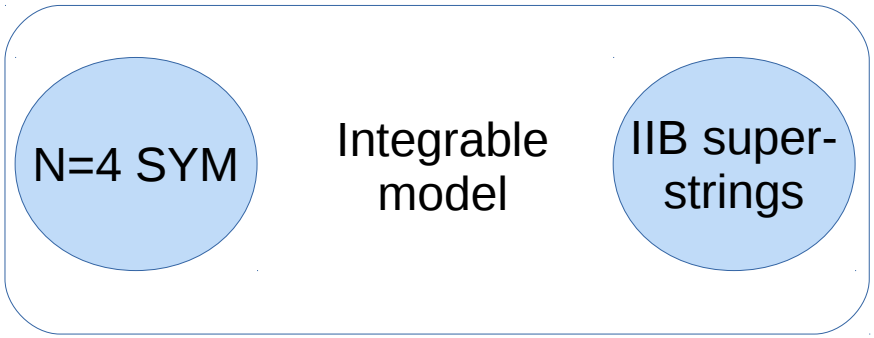
Quantization of gravity

Gauge/gravity duality
or
AdS/CFT

Possibilities for check the gauge/gravity conjecture

- Compare measurements to gauge/gravity predictions
- Compare to lattice gauge theory results
- Integrability in AdS/CFT

Integrability is a tool to validate AdS/CFT

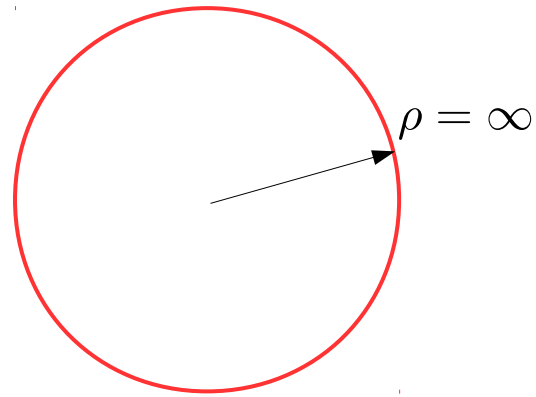


$$AdS_{d+1}: \quad -X_{-1}^2 - X_0^2 + X_1^2 + \cdots + X_d^2 = -L^2$$

Global coordinates

$$X_{-1} = L\cosh\rho\cos\tau$$
$$X_0 = L\cosh\rho\sin\tau$$
$$X_i = L\sinh\rho\hat{x}_i$$

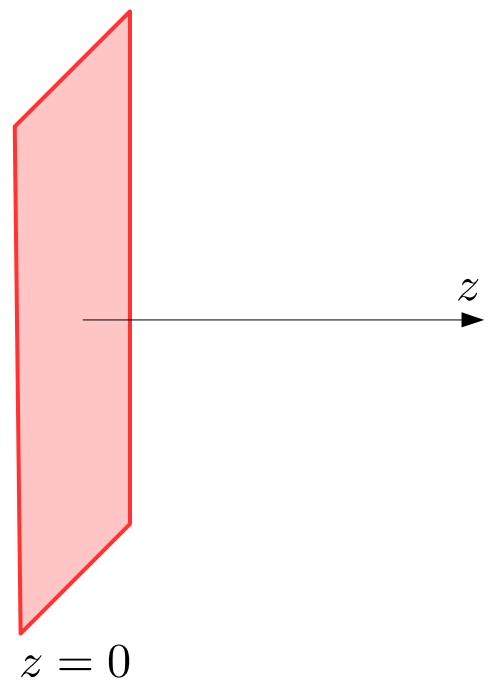
$$ds^2 = L^2(d\rho^2 - \cosh^2\rho d\tau^2 + \sinh^2\rho d\Omega_{d-1}^2)$$



Poincaré coordinates

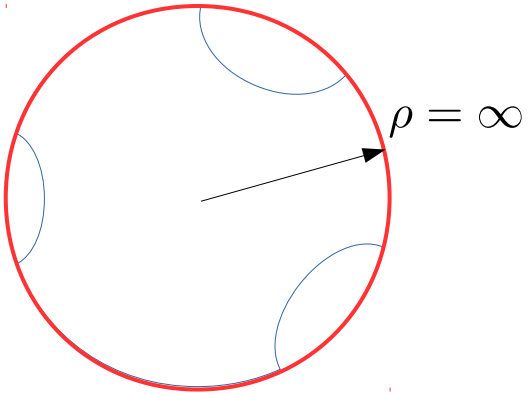
$$ds^2 = \frac{L^2}{z^2}(dz^2 - dt^2 + dx_i dx_i)$$

$z = 0$ boundary = Minkowsi spacetime

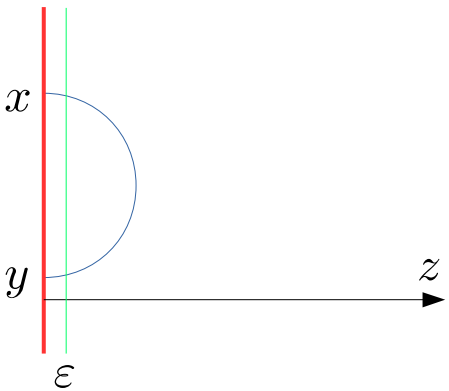


Symmetry: $SO(2,d)$ ⟷ Symmetry of CFT_d

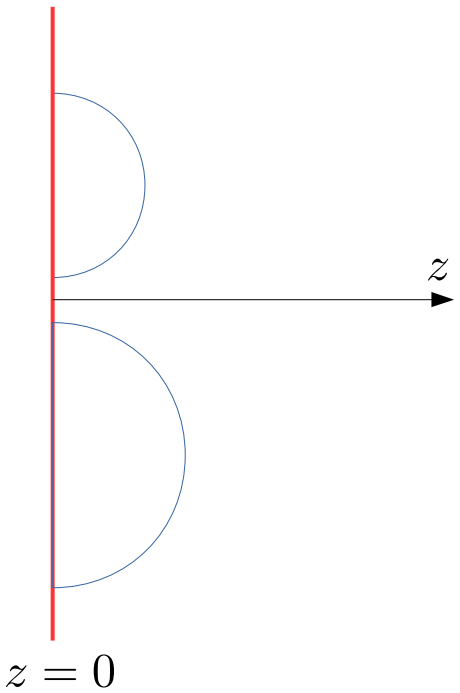
Euclidean AdS_{d+1} $\longrightarrow$ Geodesics:



Particle propagation from (0,x) to (0,y)



$$S = m\int_{0,y}^{0,x} ds = mL\int_{\pi}^{0} \frac{d\alpha}{sin\alpha} \to \infty$$

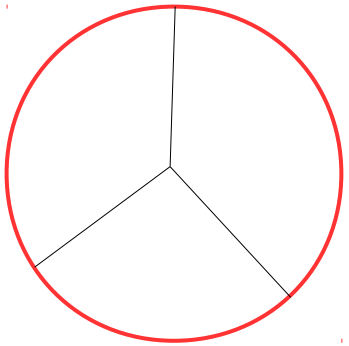


Regularization: $S = mL\int_{\pi-2\varepsilon/(x-y)}^{2\varepsilon/(x-y)} \frac{d\alpha}{\sin\alpha} = 2m\log(\cot((x-y)/\varepsilon))$

$$e^{-S} = \frac{\varepsilon^{2mL}}{(x-y)^{2mL}} = \langle \mathcal{O}^{bare}(x)\mathcal{O}^{bare}(y)\rangle$$

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}} \longrightarrow \Delta = mL$$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle \longrightarrow$$

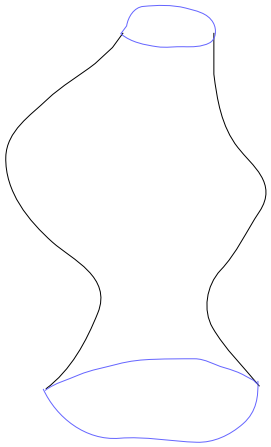


String theory and low energy effective actions

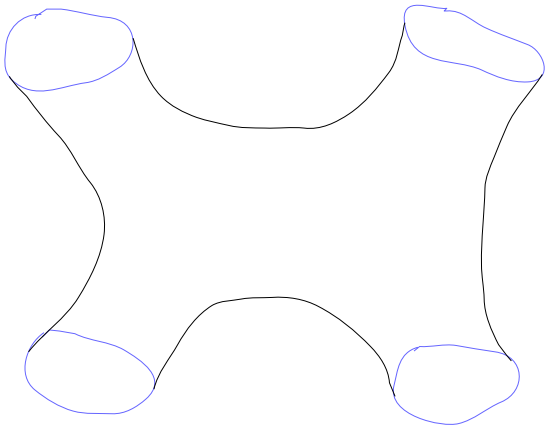
String action: $$S = -\frac{1}{4\pi l_s^2}\int_\Sigma d^2\sigma\sqrt{-g}g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + fermions$$

Σ worldsheet: $\sigma^\alpha, g_{\alpha\beta}$ spacetime: $X^\mu(\sigma), \eta_{\mu\nu} \to (-1, 1, 1 \dots)$

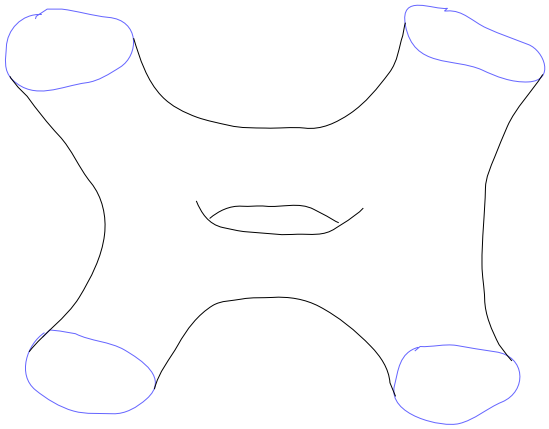
Σ :



free string

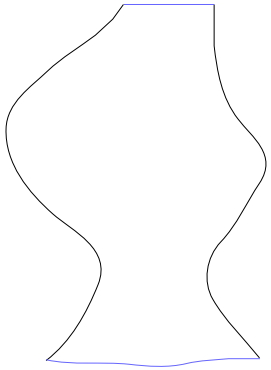


tree level

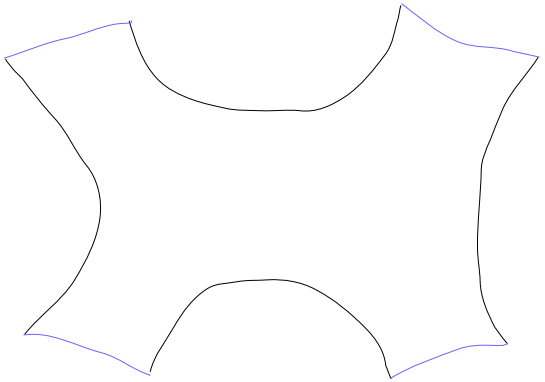


one loop

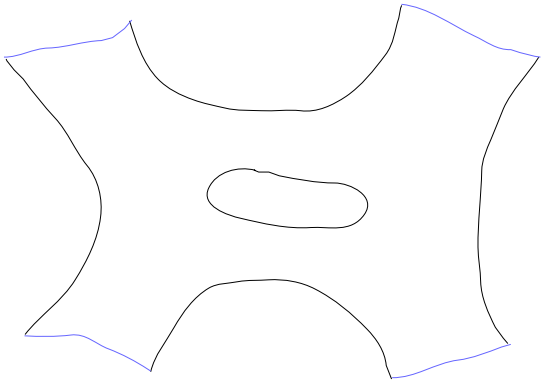
Σ :



free string



tree level



one loop

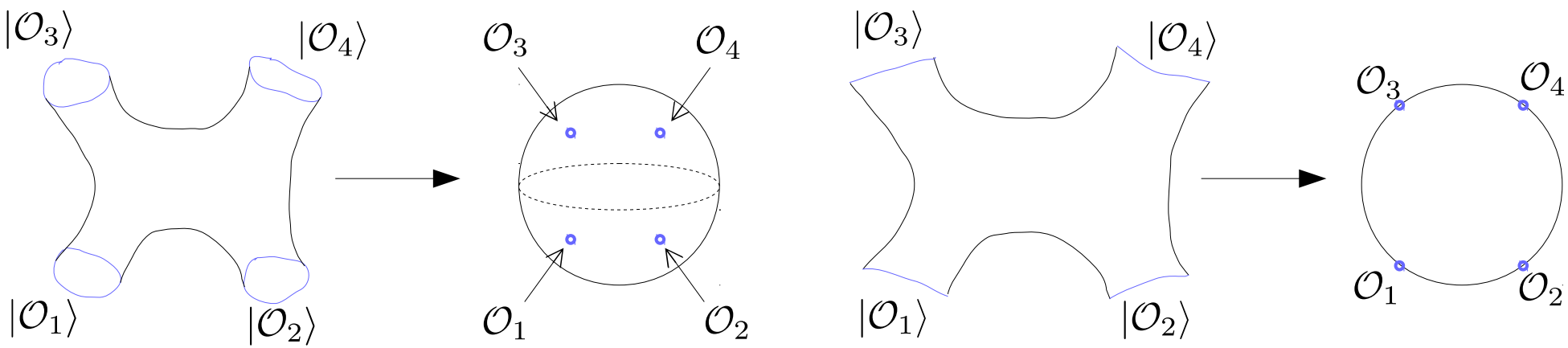
Σ is a cylinder $\longrightarrow$ closed string spectrum $\longrightarrow$ massless spacetime spin 2 particle = graviton

Σ is a strip $\longrightarrow$ open string spectrum $\longrightarrow$ massless spacetime spin 1 particle = photon

Worldsheet is not physical $\longrightarrow$ Diffeo. invariance $\longrightarrow$ Flat world sheet metric

Gauge fixing

Remaining diffeo. $\longrightarrow$ conformal inv. $\longrightarrow$ worldsheet CFT $\longrightarrow$ State/operator map



graviton with spacetime momentum p: $\mathcal{O} \sim \int d^2\sigma \partial_\alpha X^\mu \partial^\alpha X^\nu e^{ip\cdot X} \zeta_{\mu\nu}$

photon with spacetime momentum p: $\mathcal{O} \sim \int_{\partial\Sigma} ds \partial_\alpha X^\mu e^{ip\dot{X}} \zeta_\mu$

String theory in curved spacetime: (sigma model) $S = -\frac{1}{4\pi l_s^2}\int_\Sigma d^2\sigma\sqrt{-g}g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu G_{\mu\nu}(X) + fermions$

Spacetime metric: $G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \longrightarrow S = S_{Poly} + V$

Path integral: $Z = \int \mathcal{D}X\mathcal{D}g e^{-S_{Poly}-V} = \int \mathcal{D}X\mathcal{D}g e^{-S_{Poly}}(1 - V + \frac{1}{2}V^2 + \dots)$

$V = \frac{1}{4\pi l_s^2}\int_\Sigma d^2\sigma\sqrt{-g}g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu h_{\mu\nu}(X) \longrightarrow$ graviton vertex operator

$$\mathcal{O} \sim \int d^2\sigma\partial_\alpha X^\mu\partial^\alpha X^\nu e^{ip\cdot X}\zeta_{\mu\nu}$$

$$h_{\mu\nu}(X) = \int d^D p e^{ip\cdot X}\zeta_{\mu\nu}(p)$$

Inserting V = single graviton state $\longrightarrow$ Inserting exp(-V) = coherent state of gravitons

Consistent string theory $\longleftrightarrow$ worldsheet conf. sym. $\longleftrightarrow$ Vanishing beta function

$\beta_{\mu\nu}(G) = \mathcal{R}_{\mu\nu} = 0$ Ricci flat spacetime

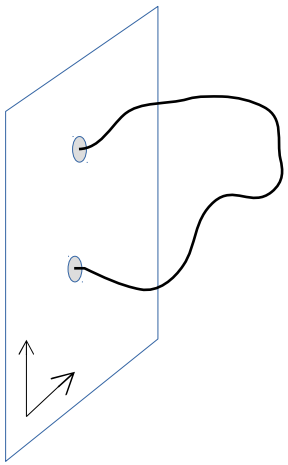
Type IIB: zero mass bosons $h_{\mu\nu},\ B_{\mu\nu},\ \Phi,\ C,\ C_{\mu\nu},\ C_{\mu\nu\rho\sigma}$ $\longrightarrow$ four-form

$B_{\mu\nu}$, $C_{\mu\nu}$ $\longrightarrow$ two-forms

Low energy effective action $\longrightarrow$ Type IIB SUGRA

$$S \sim \int d^{10}X\sqrt{-G}e^{-2\Phi}\left(\mathcal{R} - \frac{1}{2}|H_3|^2 + 4\partial_\mu\Phi\partial^\mu\Phi + \dots\right)$$

Dp-branes: p+1 dimensional surfaces:



$$\partial_\sigma X^a = 0 \quad a = 0, 1, \ldots p$$
$$X^I = \phi^I \quad I = p+1, \ldots 9$$

SO(1,9)
$\downarrow$
SO(1,p)×SO(9-p)

Inserting coherent state of photons $\longrightarrow$ $S = S_{Poly} + S_b$

$$S_b = \frac{1}{4\pi l_s^2} \int_{\partial\Sigma} d\tau \dot{X}^a A_a(X) \qquad A_a(X) = \int d^{p+1}k e^{ik\cdot X} \zeta_a(k)$$

Vanishing beta function $\longrightarrow$ DBI action $\xrightarrow[\text{limit}]{\text{low energy}}$ $S \sim \int d^{p+1}\xi \left(\frac{1}{4} F_{ab}F^{ab} + \frac{1}{2}\partial_a\phi^I \partial^a \phi^I + \ldots \right)$

$\longrightarrow$ p+1 dimensional U(1) gauge theory with 9-p scalars

N coincident D-branes: $\longrightarrow$ massless fields = N×N Hermitian matrcies

$$S \sim \int d^{p+1}\xi \ \mathrm{Tr} \left(\frac{1}{4} F_{ab}F^{ab} + \frac{1}{2}\mathcal{D}_a\phi^I \mathcal{D}^a \phi^I - \frac{1}{4}\sum [\phi^I, \phi^J]^2 + fermions \right)$$

$\longrightarrow$ p+1 dimensional U(N) YM with 9-p scalars, every field in adjoint rep

Another aspect of p-branes: they can be charges under p+1-form

point particle: $\int d\tau \dot{X}^\mu A_\mu = \int_{V_0} A_1 \longrightarrow \int_{V_p} C_{p+1}$

Two aspects of N coincident D3-brane in type IIB, when $N >> 1$

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						

$g_s N << 1$

Closed and open strings in flat 9+1 dim Minkowski space

open-closed << open-open

Open and closed strings are decoupled

Low energy excitations:

I. Open strings

Low energy effective action:

3+1 dimensional U(N) YM with 6 scalars + fermions $\quad g_{YM}^2 = 4\pi g_s$

3+1 dim. $\mathcal{N} = 4$ Super Yang-Mills

II. Closed strings

Free super-gravitons in flat 9+1 dim. spacetime

$g_s N >> 1$

D3-branes are charged under the Ramond-Ramond four form C_4

Type IIB SUGRA: $\quad G_{\mu\nu}, F_5 = dC_4, e^{\Phi} = g_s$

$$ds^2 = H(r)^{-\frac{1}{2}} \eta_{ab} dX^a dX^b + H(r)^{\frac{1}{2}} dX^I dX^I$$

r is the distance from the branes: $X^I X^I = r^2$

$$H(r) = 1 + \frac{L^4}{r^4}, \qquad \frac{L^4}{l_s^4} = 4\pi g_s N$$

$$r << L \to ds^2 = \boxed{\frac{r^2}{L^2}\eta_{ab} dX^a dX^b + \frac{L^2}{r^2} dr^2} + \boxed{L^2 d\Omega_5^2}$$

Low energy excitations: $\quad AdS_5 \qquad S^5$

I. Closed strings at $r \to 0$

IIB superstrings on $AdS_5 \times S^5$

II. Closed strings at $r \to \infty$

Free super-gravitons in flat 9+1 dim. spacetime

Maldacena conjecture:

$\mathcal{N}=4$ super Yang-Mills $\equiv$ IIB superstrings on $AdS_5 \times S^5$

$$g_{YM}^2 = 4\pi g_s \qquad \frac{L^4}{l_s^4} = 4\pi g_s N$$

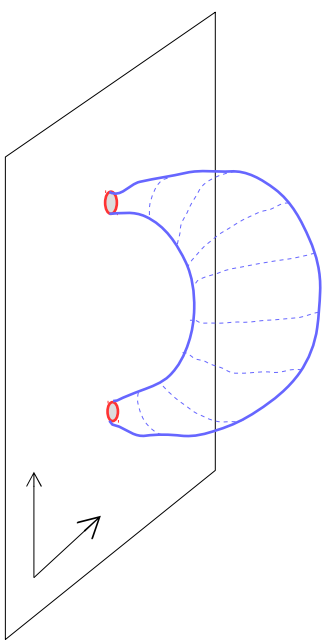
$\mathcal{N}=4$ SYM generating function: $Z[\phi_0] = \left\langle e^{i\int d^4x \phi_0^i(x)\mathcal{O}_i(x)} \right\rangle$

Duality: $Z[\phi_0] = Z_{\rm string}[\phi^i \to r^{\Delta_i}\phi_0^i] \qquad \approx e^{iS_{\rm sugra}[\phi^i \to r^{\Delta_i}\phi_0^i]}$

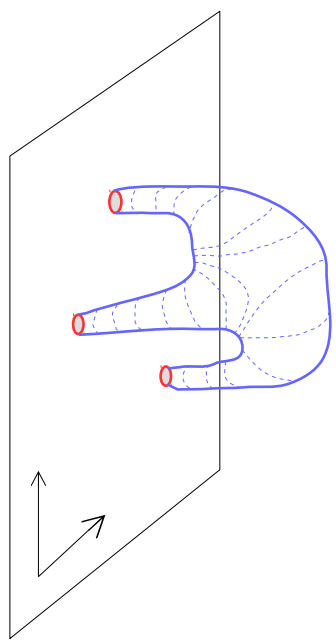
$$\lambda >> 1, N >> 1$$

Examples:

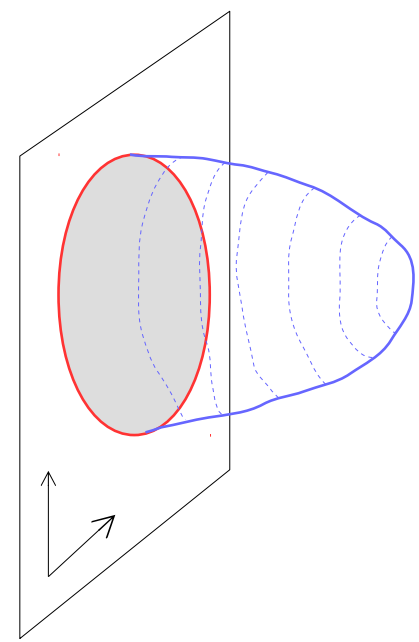
Two-point functions:



three-point functions:



Wilson loops:

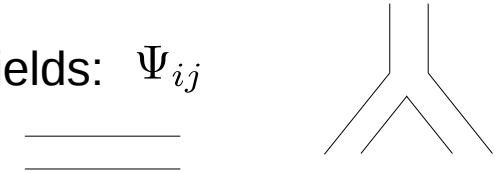


Perturbation theory

$$g_{YM}^2 = 4\pi g_s \qquad \frac{L^4}{l_s^4} = 4\pi g_s N \longrightarrow \lambda = g_{YM}^2 N = \frac{L^4}{l_s^4}$$

Gauge theory

Fields: Ψ_{ij}



Feynman graphs:

$$\lambda^{M+\varepsilon/2-1} N^{\chi-\varepsilon/2} \cdot \text{Feynman integral}$$

M: #loops

χ: Euler character

ε: external lines

$$\chi = 2 - 2g - h$$

Double expansion: genus(1/N) and loops(λ)

$$N >> 1, \lambda << 1$$

Planar limit: $N \to \infty, \quad \lambda = fix << 1$

String theory

$$S = -\frac{L^2}{4\pi l_s^2} \int_\Sigma d^2\sigma \sqrt{-g}\,(\dots)$$
$$= -\frac{\sqrt{\lambda}}{4\pi} \int_\Sigma d^2\sigma \sqrt{-g}\,(\dots)$$

String sigma model with coupling: $1/\sqrt{\lambda}$

Woldsheet perturbation theory + genus expansion (string perturbation theory)

$$g_s << 1, \lambda >> 1$$

Free strings: $g_s \to 0, \quad \lambda = fix >> 1$

Two-point function in the planar limit at weak coupling

$$\langle \mathcal{O}_I(x)\mathcal{O}_{I'}(y)\rangle = \frac{\delta_{I,I'}}{|x-y|^{2\Delta}} = \frac{\delta_{I,I'}}{|x-y|^{2\Delta_0}}(1 - 2\gamma \log|x-y| + \dots) \qquad \Delta(\lambda) = \Delta_0 + \gamma(\lambda)$$

$$\mathcal{O}_I = \mathcal{F}_I^{i_1\dots i_J}\,\mathrm{Tr}\,[\phi_{i_1}\dots\phi_{i_J}] \longrightarrow \Delta_0 = J$$

$\mathrm{span}\,\{\phi_i\} = V$ one site

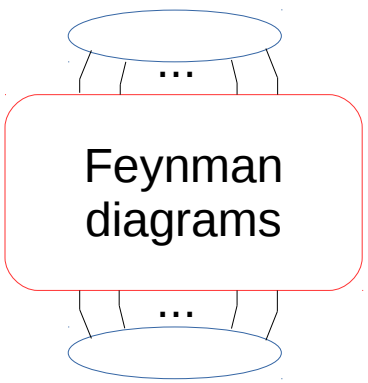
$\mathcal{F}_I \in V^{\otimes J}$ spin chain with length J

$$\langle \mathcal{O}_I(x)\mathcal{O}_{I'}(y)\rangle = \frac{1}{|x-y|^{2\Delta_0}}(\delta_{I,I'} - 2\mathcal{F}_I\cdot\Gamma(\lambda)\mathcal{F}_{I'}\log|x-y| + \dots)$$

$\Gamma(\lambda) \in \mathrm{End}V^{\otimes J}$ Hamilton operator

$\mathcal{O}$:

$\langle \mathcal{O}_I(x)\mathcal{O}_{I'}(y)\rangle$:



$\longrightarrow \Gamma(\lambda)$

$$\Gamma(\lambda) = \lambda\Gamma^{(1)} + \lambda^2\Gamma^{(2)} + \dots$$

one-loop: nearest neighbor interaction

two-loop: next-to-nearest neighbor interaction

Scalar one-loop Hamiltonian:

$$\Gamma^{(1)} \sim \sum_{k=1}^{J} 2P_{k,k+1} - K_{k,k+1}$$

Integrable SO(6) spin chain

Integrability also holds for higher loops and the full spectrum

$\downarrow$

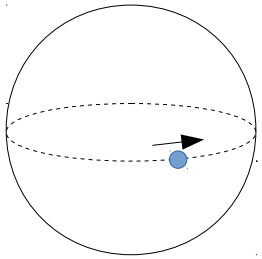
PSU(2,2|4) spin chain

Anomalous dimensions at strong coupling

Single-trace operators $\longrightarrow$ Closed strings

Global bosonic symmetry: $SO(2,4) \times SO(6)$ $\longrightarrow$ $(\Delta, S_1, S_2 | J_{12}, J_{34}, J_{56})$

Charges in involution



Gauge theory interpretaion:

$\mathrm{Tr} Z^J$

Perturbative expansion around the point-like string: $(J, 0, 0 | J, 0, 0)$

Anomalous dimension: $\gamma = \Delta - J = \mathcal{H}$ $\longrightarrow$ Light-cone Hamiltonian

Anomalous dimensions = energy levels of 1+1 dim QFT with size J

Decompactification limit $J \to \infty$ $\longrightarrow$ worldsheet S-matrix

String-sigma model $\longrightarrow$ Classical integrability

Assuming quantum integrability $\longrightarrow$ Factorized S-matrix which is fixed by the symmetry

It agrees with the perturbation theory of string-sigma model.

All loop S-matrix $\mathbb{S}(p_1, p_2)$

Gauge theory interpretaion:

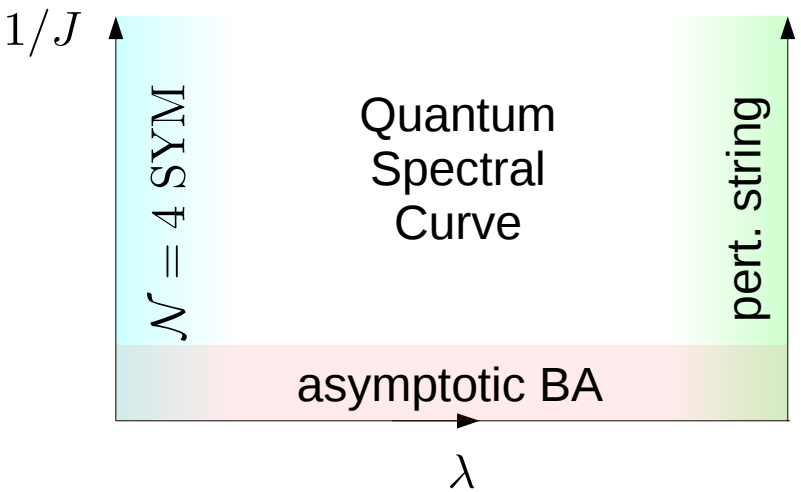
p_1 p_2

ZZ...ZZΦZZ...ZZΨZZ...ZZ $\longrightarrow$ ZZ...ZZΨ'ZZ...ZZΦ'ZZ...ZZ

$\mathbb{S}_{\Phi\Psi}^{\Phi'\Psi'}(p_1, p_2)$

p_2 p_1

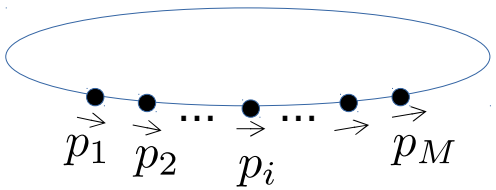
Anomalous dimensions at finite coupling



In asymptotically large volume $J \to \infty$

$\longrightarrow$ point-like interaction

Bethe-Yang equations



$$e^{ip_k J} \prod_{l \neq k} S(p_k, p_l) = 1$$

$$E = \sum_{k=1}^{M} E(p_k)$$

Integrable QFT at finite volume $\longrightarrow$ Termodynamical Bethe Ansatz

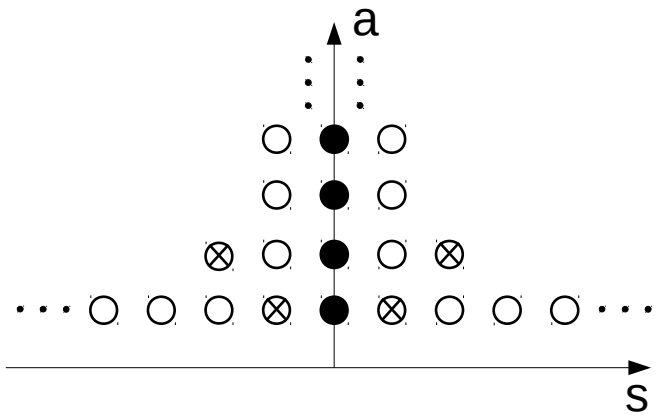
$\longrightarrow$ Y-system $\longrightarrow$ T-system $\longrightarrow$ Q-system

$$Y^+_{a,s} Y^-_{a,s} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+1/Y_{a+1,s})(1+1/Y_{a-1,s})}$$

$$f^{\pm}(u) = f(u \pm i/2)$$

$$T^+_{a,s} T^-_{a,s} = T_{a,s+1} T_{a,s-1} + T_{a+1,s} T_{a-1,s}$$

$$Y_{a,s} = \frac{T_{a,s+1} T_{a,s-1}}{T_{a+1,s} T_{a-1,s}}$$

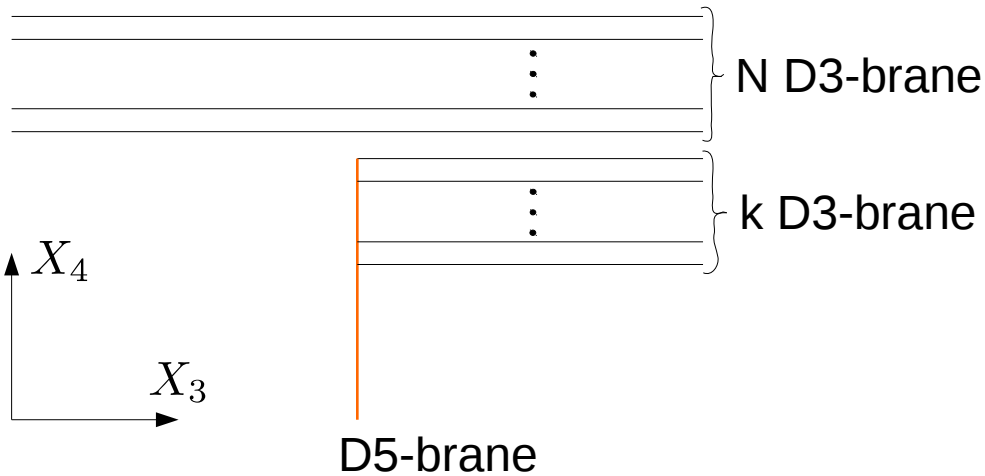


$$Q_I Q_{Iab} \propto \begin{vmatrix} Q^+_{Ia} & Q^+_{Ib} \\ Q^-_{Ia} & Q^-_{Ib} \end{vmatrix}$$

Finite number of Q functions

Surface defects in AdS/CFT

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D5	X	X	X		X	X	X			
D7	X	X	X		X	X	X	X	X	



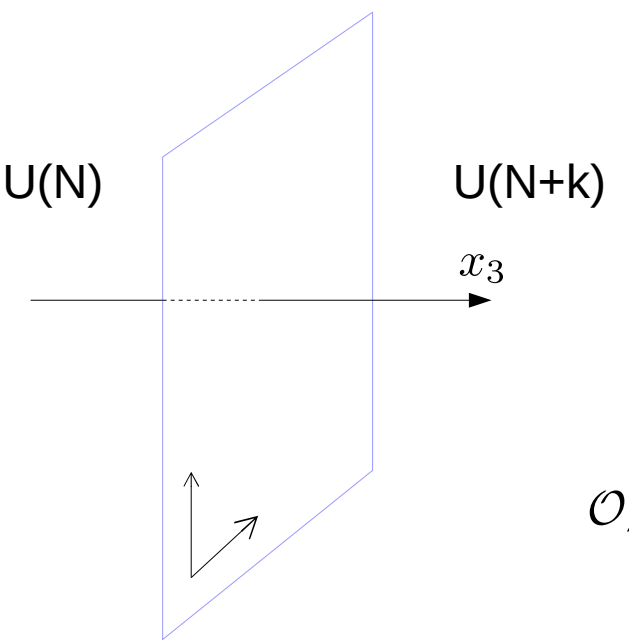
$$PSU(2,2|4) \to OSp(4|4)$$

Bosonic subalgebras:

$$SO(2,4) \to SO(2,3)$$

$$SO(6) \to SO(4)$$

weak coupling



$$\phi_i(x) = \frac{1}{x_3}\begin{pmatrix} (t_i)_{(k\times k)} & \mathbf{0}_{(k\times N)} \\ \mathbf{0}_{(N\times k)} & \mathbf{0}_{(N\times N)} \end{pmatrix} \quad ; \quad i = 1,2,3$$

$$\langle \mathcal{O}_\Delta(x)\rangle = \frac{C_\Delta}{x_3^\Delta} \xrightarrow{\text{tree level}} C_\Delta = \frac{\langle \mathrm{MPS}_k|\mathcal{F}\rangle}{\sqrt{\langle \mathcal{F}|\mathcal{F}\rangle}}$$

$$\mathcal{O}_\Delta \propto \mathcal{F}^{i_1\ldots i_J}\,\mathrm{Tr}\left[\phi_{i_1}\ldots\phi_{i_J}\right] \longrightarrow \langle \mathcal{O}_\Delta(x)\rangle \propto \frac{\mathcal{F}^{i_1\ldots i_J}\,\mathrm{Tr}\left[t_{i_1}\ldots t_{i_J}\right]}{x_3^J}$$

$$\langle \mathrm{MPS}_k| = \sum_{i_1,\ldots i_L} \langle i_1| \otimes \cdots \otimes \langle i_L|\,\mathrm{Tr}\left[t_{i_1}\ldots t_{i_L}\right]$$

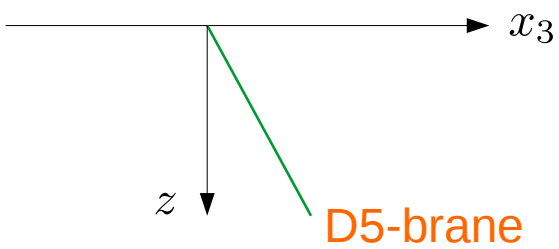
Strong coupling $k << N$

The background is same as before $\longrightarrow$ $AdS_5 \times S^5$ near horizon limit

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{ab}dx^a dx^b + z^2\right) + L^2 d\Omega_5^2$$

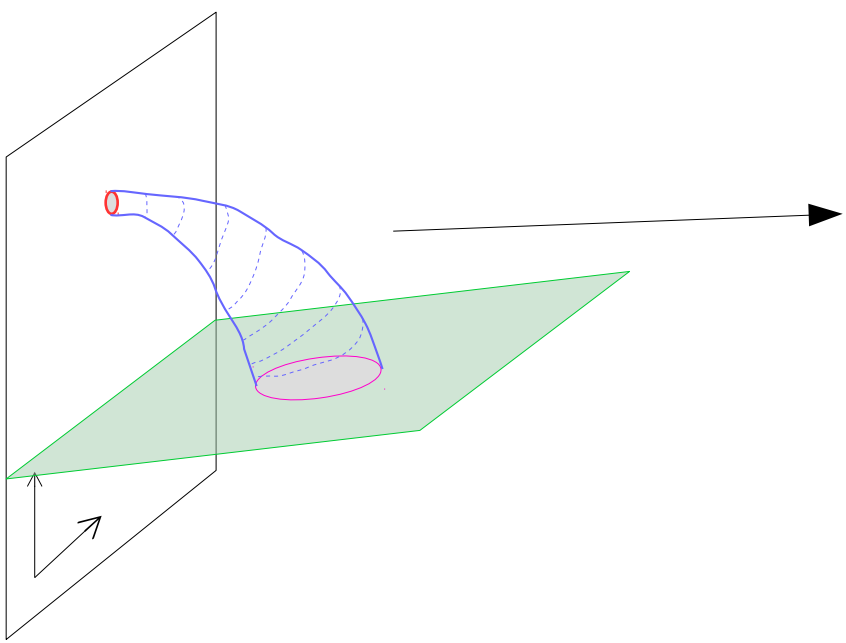
D5-brane effective action: DBI+WZ action

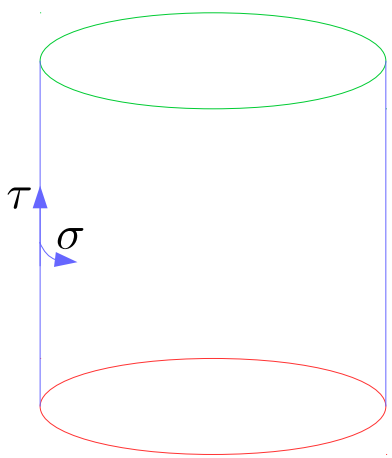
$$x_3 = \kappa z \qquad \kappa = \frac{\pi}{\sqrt{\lambda}}k$$



One-point function:

1+1 dim field theory picture





$\langle B|$ boundary state

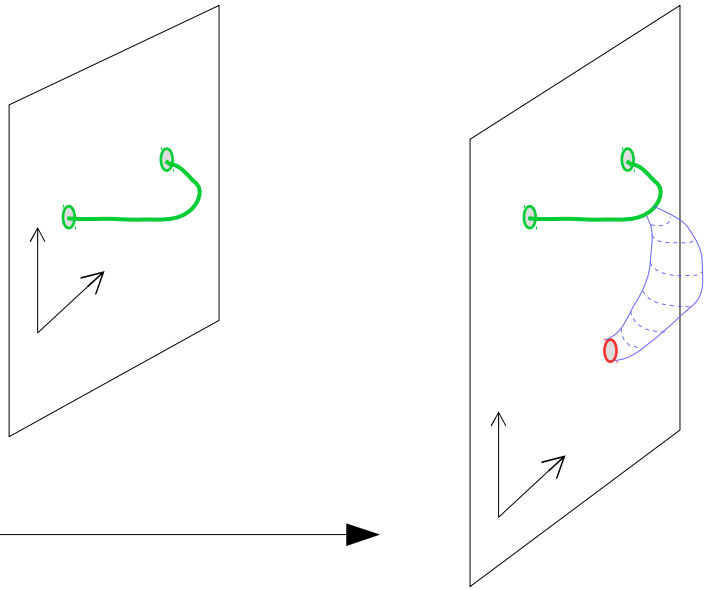
$|\mathcal{F}\rangle$ finite volume state

$$C_\Delta = \frac{\langle B|\mathcal{F}\rangle}{\sqrt{\langle \mathcal{F}|\mathcal{F}\rangle}}$$

Other applications of overlaps in AdS/CFT

Determinant operators = giant graviton

$$\left\langle \det \bar{Z}(x) \det Z(y) \right\rangle$$

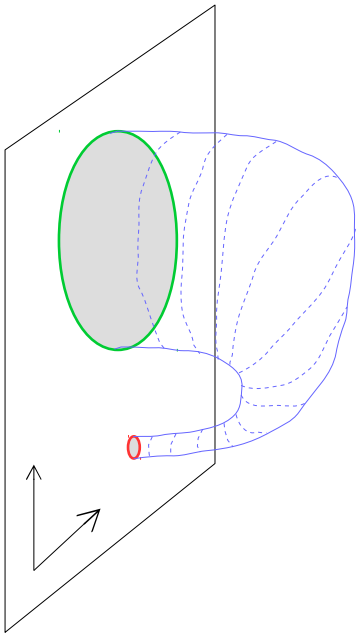


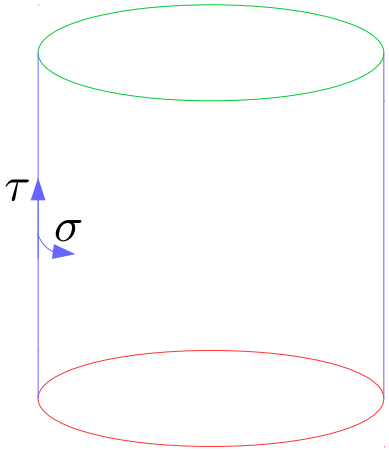
Three-point function

$$\left\langle \det \bar{Z}(x_1) \det Z(x_2) \mathcal{O}(x_3) \right\rangle$$

Wilson loops and single trace operators

$$\langle W_{\mathcal{C}} \mathcal{O}(x) \rangle$$





$\langle B|$ boundary state

$|\mathcal{F}\rangle$ finite volume state

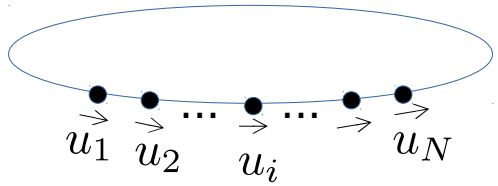
$$\frac{\langle B|\mathcal{F}\rangle}{\sqrt{\langle \mathcal{F}|\mathcal{F}\rangle}}$$

Factorized overlaps $\dfrac{\langle\Psi|\mathbf{u}\rangle}{\sqrt{\langle\mathbf{u}|\mathbf{u}\rangle}}$

$\langle\Psi|$: boundary state $\quad |\mathbf{u}\rangle$: on-shell Bethe state

BA equations:

$$e^{ip(u_k)L}\prod_{l\neq k} S(u_k-u_l)=1$$



$$\mathbf{u}=\{u_1,u_2,\ldots,u_{N-1},u_N\}$$

Neél state in XXZ model

$$\langle\Psi| = \underbrace{\langle\uparrow|\otimes\langle\downarrow|\otimes\langle\uparrow|\otimes\langle\downarrow|\otimes\cdots\otimes\langle\uparrow|\otimes\langle\downarrow|\otimes\langle\uparrow|\otimes\langle\downarrow|}_{\text{L (even)}}$$

$$S_z=0$$

$$|\mathbf{u}\rangle \longrightarrow S_z=L/2-N \longrightarrow N=L/2$$

Non-vanishing overlap $\longrightarrow$ Bethe roots in pair: $\{u_j\}_{j=1}^N=\{-u_j\}_{j=1}^N$

We assume that N is even $\longrightarrow$ $\{u_j\}_{j=1}^N=\{u_j\}_{j=1}^{N/2}\bigcup\{-u_j\}_{j=1}^{N/2}$

$$\boxed{\frac{\langle\Psi|\mathbf{u}\rangle}{\sqrt{\langle\mathbf{u}|\mathbf{u}\rangle}}=\prod_{j=1}^{N/2}\frac{\sqrt{(\tan(u_j+i\eta/2)(\tan(u_j-i\eta/2)}}{2\sin(2u_j)}\sqrt{\frac{\det G_+}{\det G_-}}}$$

The derivation based on the off-shell overlap formula

Gaudin determinants

$$\boxed{\frac{\langle\Psi|\mathbf{u}\rangle}{\sqrt{\langle\mathbf{u}|\mathbf{u}\rangle}} = \prod_{j=1}^{N/2} \frac{\sqrt{(\tan(u_j+i\eta/2)(\tan(u_j-i\eta/2)}}{2\sin(2u_j)} \sqrt{\frac{\det G_+}{\det G_-}}}$$

$$\exp(i\phi_{u_j}) = -\left(\frac{\sin(u_j-i\eta/2)}{\sin(u_j+i\eta/2)}\right)^L \prod_{k=1}^{N} \frac{\sin(u_j-u_k+i\eta)}{\sin(u_j-u_k-i\eta)} = 1$$

$$G_{jk} = \partial_{u_j}\phi_{u_k}$$

$$\langle\mathbf{u}|\mathbf{u}\rangle \sim \det G$$

$$G_{jk} = \delta_{jk}\left(LK_{\eta/2}(u_j) - \sum_{l=1}^{N} K_\eta(u_j-u_l)\right) + K_\eta(u_j-u_l) \qquad K_\eta(u) = \frac{\sinh(2\eta)}{\sin(u+i\eta)\sin(u-i\eta)}$$

$$\mathbf{u} = \mathbf{u}^+ \bigcup \mathbf{u}^- \qquad \partial_{u_j^+}\phi_{u_k^+} = \partial_{u_j^-}\phi_{u_k^-} \qquad \partial_{u_j^+}\phi_{u_k^-} = \partial_{u_j^-}\phi_{u_k^+}$$

$$\det G = \begin{vmatrix} \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^+} & \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^-} \\ \partial_{\mathbf{u}^-}\phi_{\mathbf{u}^+} & \partial_{\mathbf{u}^-}\phi_{\mathbf{u}^-} \end{vmatrix} = \begin{vmatrix} \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^+} + \partial_{\mathbf{u}^-}\phi_{\mathbf{u}^+} & \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^-} + \partial_{\mathbf{u}^-}\phi_{\mathbf{u}^-} \\ \partial_{\mathbf{u}^-}\phi_{\mathbf{u}^+} & \partial_{\mathbf{u}^-}\phi_{\mathbf{u}^-} \end{vmatrix} =$$

$$= \begin{vmatrix} \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^+} + \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^-} & 0 \\ \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^-} & \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^+} - \partial_{\mathbf{u}^+}\phi_{\mathbf{u}^-} \end{vmatrix} = \det G_+ \det G_-$$

$$(G_\pm)_{jk} = \partial_{u_j^+}\phi_{u_k^+} \pm \partial_{u_j^+}\phi_{u_k^-}$$

Other known overlaps

$$\frac{\langle \Psi_d | \mathbf{U} \rangle}{\sqrt{\langle \mathbf{U} | \mathbf{U} \rangle}} = \sum_{k=1}^{d} \prod_{u_j^{(a)} \in \mathbf{U}^+} h_k^{(a)}(u_j^{(a)}) \sqrt{\frac{\det G_+}{\det G_-}}$$

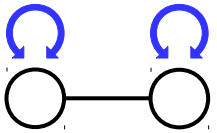
Integrability = pair structure

$$\mathbf{U} = \bigcup_a \mathbf{u}^{(a)} \quad \mathbf{U} = \mathbf{U}^+ \bigcup \mathbf{U}^- \quad G_\pm = \partial_{\mathbf{U}^+} \phi_{\mathbf{U}^+} \pm \partial_{\mathbf{U}^+} \phi_{\mathbf{U}^-}$$

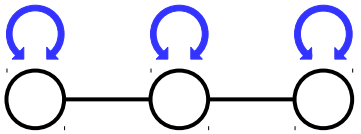
1pt func.

- General two-site states for XXZ spin chain — B. Pozsgay '18

- Integrable two-site states for SU(3) spin chain — L. Piroli, E. Vernier, P. Calabrese, B. Pozsgay '19



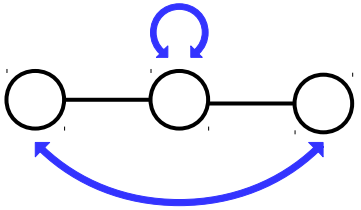
- Integrable MPS for SU(3) spin chain — M. de Leeuw, C. Kristjansen, S. Mori '16
- Integrable SO(3)×SO(3) symmetric MPS for SO(6) spin chain — M. De Leeuw, C. Kristjansen, G. Linardopoulos '18



- Integrable SO(5) symmetric MPS for SO(6) spin chain — M. De Leeuw, TG, C. Kristjansen, G. Linardopoulos, B. Pozsgay '19

3pt

- Integrable SO(2)×SO(4) symmetric two-site state for SO(6) spin chain — Y. Jiang, S. Komatsu, E. Vescovi '18



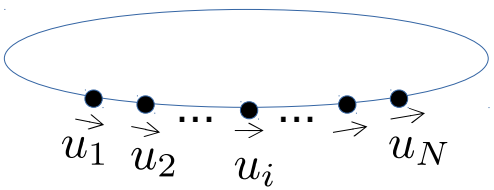
Physical interpretation of factorized overlaps

$$\frac{\langle\Psi|\mathbf{u}\rangle}{\sqrt{\langle\mathbf{u}|\mathbf{u}\rangle}} = \boxed{\prod_{u_j\in\mathbf{u}^+} h(u_j)} \times \boxed{\sqrt{\frac{\det G_+}{\det G_-}}}$$

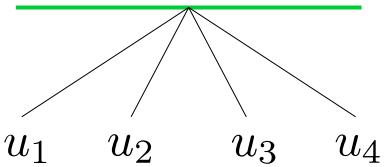
Depends on boundary state

Depends only on Bethe state

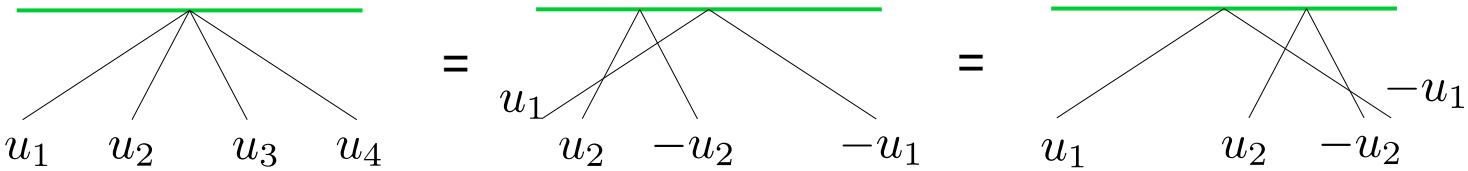
Bethe states: magnons on a circle



$\langle\Psi|$ annihilates magnons (boundary in time)



Since the annihilation amplitude is factorized $\prod_{u_j\in\mathbf{u}^+} h(u_j)$ → Annihilation only in pairs



Integrability = higher spin conserved charges = trajectories can be shifted

Piroli, Pozsgay, Vernier '17

Space reflection:

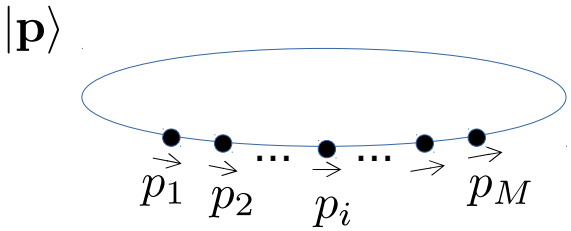
$$\Pi Q_n \Pi = (-1)^n Q_n$$

Integrability condition:

$$\langle\Psi|Q_{2n+1} = 0 \longrightarrow \langle\Psi|t(u) = \langle\Psi|\Pi t(u)\Pi$$

One-point function in the asymptotic limit

Closed string spectrum $\longrightarrow$ Asymptotic Bethe Ansatz



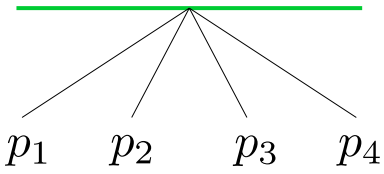
Gauge theory interpretation of $|\mathbf{p}\rangle$

$$\overset{p_1}{\longrightarrow} \qquad \overset{p_2}{\longrightarrow}$$

ZZ...ZZΦZZ...ZZΨZZ...ZZ

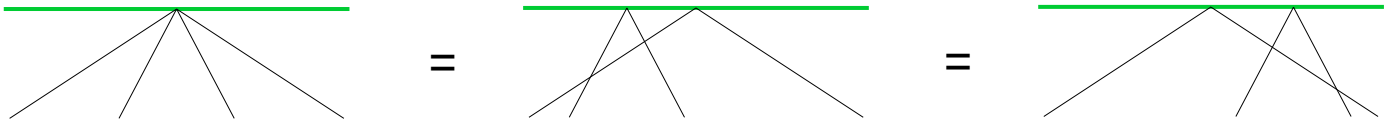
Boundary state and worldsheet K-matrix

$$\langle B| \sim \sum_{M=0}^{\infty} \mathbb{K}(p_1, \ldots, p_M) \qquad \mathbb{K}(p_1, p_2, p_3, p_4)$$



Assuming quantum integrability $\longrightarrow$ Factorized K-matrix which is fixed by the symmetry

$\longrightarrow$ All loop K-matrix $\mathbb{K}(p)$



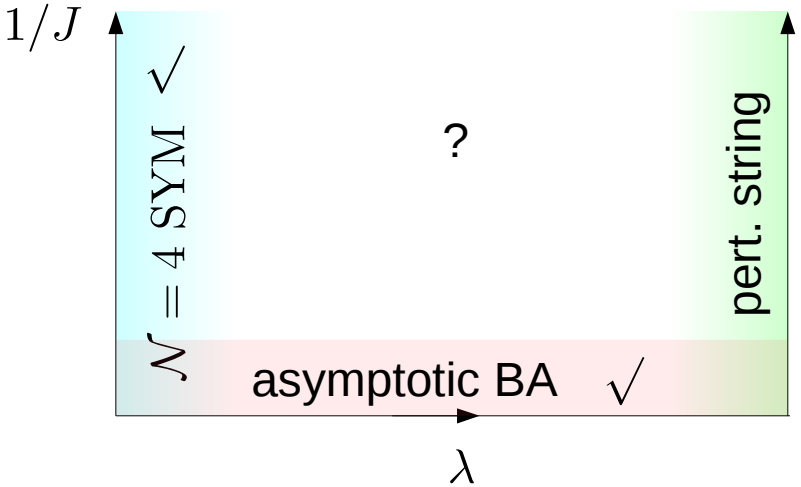
Gauge theory interpretaion:

$$\overset{p}{\longrightarrow} \qquad \overset{-p}{\longleftarrow}$$

<B|ZZ...ZZΦZZ...ZZΨZZ...ZZ> $\longrightarrow$ $\mathbb{K}_{\Phi\Psi}(p)$

The K-matrix and the asymptotic overlap was calculated in TG, Bajnok '20

Current status of one-point functions



Strong coupling

Ground state one point function when $\lambda \to \infty$ $\quad \mathrm{Tr} Z^J$

Asymptotic region

One-point function for the full spectrum

TG, Bajnok '20

Weak coupling

For D5 brane tree level overlaps were calculated for various sectors

For D7 brane tree level overlaps in SO(6) sector

M. De Leeuw, TG, C. Kristjansen, G. Linardopoulos, B. Pozsgay '19

One-loop results for ground state

$\mathrm{Tr} Z^J$

Finite size corrections?

Open questions

- Integrability at strong coupling? D7 case?
- Strong copling 1pt functions beyong the ground state
- Finite size effects

- Derivation of overlap formulas for nested systems
- Understanding the integrability of boundary states for twisted systems
- Connection between boundary states and separation of variables

Thank you for your attention!