New boundary transfer matrices for classical sigma models

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Contents

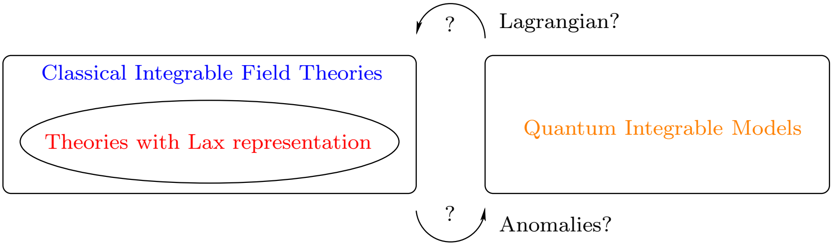
Outline

Definitions for bulk models

- Classical model with infinite set of local conserved charges $\Longrightarrow$ Classical Integrable Field Theory
- If $\exists \mathrm{L}(z)$ Lax operator where $\mathrm{dL}(z) + \mathrm{L}(z) \wedge \mathrm{L}(z) = 0 \Longleftrightarrow$ E.O.M is satisfied $\Longrightarrow$ infinitely many conserved charges.
- QFT with infinitely many local conserved charges $\Longrightarrow$
  - (i) no particle creation/annihilation
  - (ii) The n-particle scattering factorizes into 2-particle scattering uniquely (Yang-Baxter Equation)
- We can define IQFT with the quantum S-matrix bootstrap

Examples:

	Classic.Int.	Lax rep.	Q.Int.
PCM	✓	✓	✓
S^n sigma model	✓	✓	✓
$\mathbb{C}\mathrm{P}^n$ sigma model	✓	✓	✗



Sigma models

- The field of the sigma model $X : \Sigma \to \mathcal{M}$ where Σ is a 2 dimensional manifold e.g. $\mathbb{R}^2, \mathbb{R} \times S^1, \mathbb{R} \times (-\infty, 0], \mathbb{R} \times [0, \pi]$
- The Lagrangian: $\mathcal{L} = \frac{1}{2} G_{MN}(X) \partial_\alpha X^M \partial^\alpha X^N$

Principal Chiral Fields: $\mathcal{M} = G$ and $X(x) = g(x) \in G$ where G is a Lie group

- We can define two currents: $\mathrm{J}^R = g^{-1}\mathrm{d}g$ and $\mathrm{J}^L = g\mathrm{d}g^{-1} \implies \mathrm{J}^L = -g\mathrm{J}^R g^{-1}$.
- These currents are Lie-algebra valued one-forms which satisfy the flatness condition: $\mathrm{dJ}^{L/R} + \mathrm{J}^{L/R} \wedge \mathrm{J}^{L/R} = 0$.
- The Lagrangian is $\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\left[\mathrm{J}^L \wedge *\mathrm{J}^L\right] = -\frac{1}{4}\mathrm{Tr}\left[\mathrm{J}^R \wedge *\mathrm{J}^R\right]$.
- The E.O.M is $\mathrm{d} * \mathrm{J}^{L/R} = 0$.
- The PCMs have $G_L \times G_R$ global symmetry, the left/right group multiplication, $g(x) \to g_L g(x)$ and $g(x) \to g(x) g_R$.
- The transformation of the currents:

$$
\begin{aligned}
g_L &: \quad \mathrm{J}^L \to g_L \mathrm{J}^L g_L^{-1}, \quad && \mathrm{J}^R \to \mathrm{J}^R, \\
g_R &: \quad \mathrm{J}^L \to \mathrm{J}^L, \quad && \mathrm{J}^R \to g_R^{-1} \mathrm{J}^R g_R.
\end{aligned}
$$

- The $\mathrm{J}^{L/R}$ are the Noether currents of the left/right group multiplication symmetry.

Integrability of the PCMs

- Light-cone coordinates: $x^{\pm} = x^0 \pm x^1 \implies \partial_{\pm} = \frac{1}{2}(\partial_0 \pm \partial_1)$.
- The conservation laws in light-cone coordinates:

$$\mathrm{d} * \mathrm{J} = 0 \quad \implies \quad \partial_+ J_- + \partial_- J_+ = 0.$$

- The conservation of the energy-momentum tensor:

$$\partial_+ T_{--} + \partial_- T_{+-} = 0, \quad \partial_- T_{++} + \partial_+ T_{-+} = 0.$$

- The PCMs are (classically) conformal $\implies T_{+-} = T_{+-} = 0$ and $T_{\pm\pm} = \mathrm{Tr} J_{\pm}^2 \implies \partial_{\mp} T_{\pm\pm} = 0$.
- We have infinitely many conservation laws: $\partial_{\mp} T_{\pm\pm}^k = 0$. $(J_{\pm}^{(\pm k)} = T_{\pm\pm}^k, J_{\mp}^{(\pm k)} = 0)$
- In fact there are more conservation laws because of $\partial_{\mp} \mathrm{Tr} J_{\pm}^m = 0$.
- Goldschmidt and Witten showed that there is at least one higher spin conservation law at the quantum level[1]:

$$\partial_+ T_{--}^2 = \sum c_i B_i,$$

where B_is are total divergences $(B_i = \partial_{\pm} C_i) \implies \exists \tilde{J}^{(-2)} \to \partial_+ \tilde{J}_-^{(-2)} + \partial_- \tilde{J}_+^{(-2)} = 0$.
- At any case it is known that one higher conservation law is enough to ensure factorization of the S-matrix.

[1] Y. Y. Goldschmidt and Edward Witten. "Conservation Laws in Some Two-dimensional Models". In: *Phys. Lett.* 91B (1980), pp. 392–396.

Lax representation of PCM

- The E.O.M and the flatness condition is equivalent to the flatness condition of the Lax connection: $\mathrm{dL}(z) + \mathrm{L}(z) \wedge \mathrm{L}(z) = 0$ where

$$\mathrm{L}(z) = \frac{1}{1-z^2}\mathrm{J}^R + \frac{z}{1-z^2} * \mathrm{J}^R.$$

- We can define the monodromy matrix

$$T(z) = \mathcal{P}\overleftarrow{\exp}\left(-\int_{-\infty}^{\infty} L_1(z)\mathrm{d}x^1\right).$$

- It can be shown that the monodromy matrix is conserved: $\partial_0 T(z) = 0$ for all $z \Longrightarrow$ infinitely many conserved charges.
- The conserved charges are generated by expanding the monodromy matrix in z, for instance at infinity:

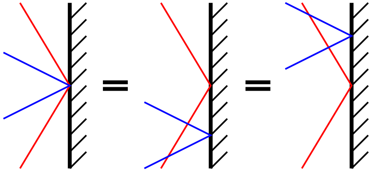
$$T(z) = \exp\left(\sum_{r=0}^{\infty}\left(\frac{-1}{z}\right)^{r+1} Q_r^R\right).$$

- These charges are non-local for $r > 0$ and the Q_0^R is the Noether charge of the right group multiplication. This infinite set of charges with the Poisson bracket as Lie bracket form a $\mathcal{Y}(\mathfrak{g})$ Yangian algebra.
- If we expand around $z = 0$ we get an other infinite set of conserved non-local charges ($\{Q_r^L\}$) which is an other $\mathcal{Y}(\mathfrak{g})$ Yangian and the Q_0^L is the Noether charge of the left group multiplication therefore the PCMs have $\mathcal{Y}(\mathfrak{g})_L \oplus \mathcal{Y}(\mathfrak{g})_R$ symmetry.
- Local conserved quantities are generated by Taylor expansion at $z = \pm 1$.

Outline

Definitions for the integrable theories at the half line $(-\infty, 0]$

- At the boundary case we have a surface term: $\partial_\alpha J^\alpha = 0$ and $Q = \int_{-\infty}^{0} J_0 dx^1$.
  $\dot{Q} = J_1(0) \Longrightarrow$ if $J_1(0) = \dot{f}$ then $Q - f$ is conserved.
- For example conformal BC: $\partial_\mp T_{\pm\pm} = 0$ and
  $T_{++}(0) = T_{--}(0) \Longrightarrow J_1 = T^m_{++} - T^m_{--} (= 0$ at the boundary$)$, $J_0 = T^m_{++} + T^m_{--}$
  $\Longrightarrow Q_m = \int_{-\infty}^{0} T^m_{++} + T^m_{--} dx^1$ are conserved.
- There is Lax representation also at boundary case.
- The boundary monodromy matrix (double row monodromy matrix) contains the bulk monodromy matrix and the reflection matrix.
- Infinitely many conserved charge at the quantum level:
  (i) no particle creation/annihilation at the boundary scattering
  (ii) The n-particle scattering at the boundary factorizes into 1-particle boundary scattering and 2-particle bulk scattering uniquely (boundary Yang-Baxter Equation)
- We can define boundary IQFT with the quantum R-matrix bootstrap.



$$R_{12} = S_{12} R_1 S_{21} R_2 = R_2 S_{12} R_1 S_{21}$$

Symmetric spaces

- G and $H < G$ are Lie groups.
- If $\exists\alpha \in \mathrm{Aut}(G)$ where $\alpha^2 = \mathrm{id}$ and $\forall h \in H$ $\alpha(h) = h$ then $\mathcal{M} = G/H$ is a symmetric space.
- The α is also an automorphism of $\mathfrak{g}$ and $\alpha(\mathfrak{h}) = \mathfrak{h}$.
- There is a $\mathbb{Z}_2$ grading: $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$ where $\alpha(\mathfrak{f}) = -\mathfrak{f}$ therefore

$$[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}, \qquad [\mathfrak{h},\mathfrak{f}] \subset \mathfrak{f}, \qquad [\mathfrak{f},\mathfrak{f}] \subset \mathfrak{h}.$$

G	$SO(n+m)$	$SO(2n)$	$SU(n+m)$	$SU(n)$	$SU(2n)$
H	$S(O(n)\times O(m))$	$U(n)$	$S(U(n)\times U(m))$	$SO(n)$	$Sp(n)$

G	$Sp(n+m)$	$Sp(n)$
H	$Sp(n)\times Sp(m))$	$U(n)$

Examples:

- $S^n \equiv SO(n+1)/SO(n)$
- $\mathbb{C}P^n \equiv SU(n+1)/U(n)$

- For most of these cases α is an inner automorphism $\alpha(g) = UgU^{-1}$ where

$$U_s = \begin{pmatrix} \mathbb{I}_n & 0 \\ 0 & -\mathbb{I}_m \end{pmatrix}, \quad \text{or} \quad U_a = \begin{pmatrix} 0 & \mathbb{I}_n \\ -\mathbb{I}_n & 0 \end{pmatrix}.$$

- For $SU(n)/SO(n)$ and $SU(2n)/Sp(n)$ αs are outer automorphisms: $\alpha(g) = \bar{g}$ and $\alpha(g) = U_a \bar{g} U_a^{-1}$.

Boundary condition for PCMs

Restricted field at the boundary

- $g \in H < G \implies J_0^R \in \mathfrak{h}$ and $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$.
- After varying the Lagrangian ($g \to g(1+\epsilon)$, $\epsilon(0) \in \mathfrak{h}$) we get a surface term: $\mathrm{Tr}[\epsilon J_1^R] \implies J_1^R \in \mathfrak{f}$.
- This BC is conformal because: $T_{++} = T_{--} \iff \mathrm{Tr}[J_0^R J_1^R] = 0$.
- Classically, there are infinity many conserved charges: $\partial_- T_{++}^k + \partial_+ T_{--}^k = 0$ with vanishing surface terms.
- At quantum level there are anomalies and we have seen at the bulk case:

$$\partial_- T_{++}^2 + \partial_+ T_{--}^2 = \sum c_i (B_i^+ + B_i^-). \tag{1}$$

- The surface terms vanish $\iff \mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$ is $\mathbb{Z}_2$ graded decomposition $\implies$ each symmetric space can be matched to an integrable BC.
- The residual symmetry is $G \times G \to H \times H$.

Boundary Lagrangian

- $L_b = \frac{1}{4}\mathrm{Tr}[M J_0^R]$ where $M \in \mathfrak{g}$.
- The boundary equation of motion: $J_1^R = \frac{1}{2}[M, J_0^R]$.
- We have an Lie-algebra endomorphism: ad_M and we can denote $\mathfrak{h} = \mathrm{Ker}(\mathrm{ad}_M)$ and $\mathfrak{f} = \mathrm{Im}(\mathrm{ad}_M)$. Therefore $J_1 \in \mathfrak{f}$ and the residual symmetry is $G \times G \to G \times H$.
- This BC is also conformal but the (1) has non vanishing surface terms therefore the we can not decide the quantum integrability.

Double row monodromy matrices

- We use the one and the double row monodromy matrix:

$$T(z) = \mathcal{P}\overleftarrow{\exp}\left(-\int_{-\infty}^{0} L_1(z)\mathrm{d}x^1\right) \qquad \Omega(z) = T(-z)^{-1}\kappa(z)T(z)$$

- The time derivative of the double row monodromy matrix:

$$\dot{\Omega}(z) = T(-z)^{-1}\Big(L_0(-z,0)\kappa(z) + \dot{\kappa}(z) - \kappa(z)L_0(z,0)\Big)T(z)$$

- Therefore the monodromy matrix is time independent for all z if

$$\kappa(z)L_0(z) - L_0(-z)\kappa(z) = \dot{\kappa}(z)$$

- This equation (reflection equation) is equivalent to with the boundary EOM just like the flatness condition $\mathrm{dL} + \mathrm{L}\wedge\mathrm{L} = 0$ is equivalent with the bulk EOM.

Reflection matrices for PCM

- Ansatz: $\kappa(z) = U$ where U is an constant group element.
- The reflection equation is equivalent with: $J_0^R = UJ_0^RU^{-1}$ and $J_1^R = -UJ_1^RU^{-1}$.
- We have a Lie-algebra automorphism $\alpha(X) = UXU^{-1}$ and let $\mathfrak{h}\subset\mathfrak{g}$ and $\mathfrak{f}\subset\mathfrak{g}$ where $\mathfrak{h} = \alpha(\mathfrak{h})$ and $\mathfrak{f} = -\alpha(\mathfrak{f}) \implies J_0^R \in \mathfrak{h}$ and $J_1^R \in \mathfrak{f}$.
- We must have $\dim(\mathfrak{g})$ independent of the BC $\implies \dim(\mathfrak{g}) = \dim(\mathfrak{h}) + \dim(\mathfrak{f}) \implies \mathfrak{g} = \mathfrak{h}\oplus\mathfrak{f}$

Outline

Quantum PCM

- All particles are bound states of "elementary" particles. They are the defining representation of the $G_L \times G_R$.
- The "elementary" S-matrix is a $(V_L \otimes V_R) \otimes (V_L \otimes V_R) \to (V_L \otimes V_R) \otimes (V_L \otimes V_R)$ matrix:

$$\tilde{S}(\theta) = \tilde{S}_0(\theta) P_{23} \left[S^L(\theta) \otimes S^R(\theta) \right] P_{23}.$$

- The $S^{L/R}(\theta)$ is a $V_{L/R} \otimes V_{L/R} \to V_{L/R} \otimes V_{L/R}$ matrix and P is the permutation matrix.
- The $S(\theta) = S^L(\theta) = S^R(\theta)$ has symmetry G and it is a rational solution of the Yang-Baxter equation $(\theta_{ij} = \theta_i - \theta_j)$:

$$S_{12}(\theta_{12}) S_{13}(\theta_{13}) S_{23}(\theta_{23}) = S_{23}(\theta_{23}) S_{13}(\theta_{13}) S_{12}(\theta_{12}).$$

- The integrable quantum reflection matrices $\tilde{R} : (V_L \otimes V_R) \to (V_L \otimes V_R)$ are the solutions of the boundary Yang-Baxter equation.
- If the $\tilde{R}$ matrix is factorized at the $(V_L \otimes V_R)$:

$$\tilde{R}(\theta) = \tilde{R}_0(\theta) \left[R^L(\theta) \otimes R^R(\theta) \right]$$

then the bYBE is also factorized $(\vartheta_{ij} = \theta_i + \theta_j)$:

$$S_{12}(\theta_{12}) R_1^{L/R}(\theta_1) S_{21}(\theta)(\vartheta_{12}) R_2^{L/R}(\theta_2) = R_2^{L/R}(\theta_2) S_{12}(\vartheta_{12}) R_1^{L/R}(\theta_1) S_{21}(\theta)(\theta_{12})$$

- The rational solutions are classified and they depend on what is the residual symmetry algebra $\mathfrak{h}$.

$\mathfrak{g} = \mathfrak{su}(n)$

$\mathfrak{h} = \mathfrak{su}(k) \oplus \mathfrak{su}(n-k) \oplus \mathfrak{u}(1)$

$$R = \begin{pmatrix} \frac{\alpha+\theta}{\alpha-\theta}\mathbb{I}_k & 0 \\ 0 & \mathbb{I}_{n-k} \end{pmatrix}$$

$\mathfrak{h} = \mathfrak{so}(n)$

This case belongs to a representation changing reflection where a particle goes to its anti-particle.

$\mathfrak{h} = \mathfrak{sp}(n)$

This case also belongs to a representation changing reflection.

$$R = \begin{pmatrix} 0 & \mathbb{I}_{n/2} \\ -\mathbb{I}_{n/2} & 0 \end{pmatrix}.$$

$\mathfrak{g} = \mathfrak{so}(n)$

$\mathfrak{h} = \mathfrak{so}(k) \oplus \mathfrak{so}(n-k)$

$$R = \begin{pmatrix} \frac{c+\theta}{c-\theta}\mathbb{I}_k & 0 \\ 0 & \mathbb{I}_{n-k} \end{pmatrix}$$

$\mathfrak{h} = \mathfrak{so}(2n-1) \oplus \mathfrak{so}(2)$

$$R = \begin{pmatrix} A_\alpha(\theta) & B_\alpha(\theta) & 0 \\ -B_\alpha(\theta) & A_\alpha(\theta) & 0 \\ 0 & 0 & \mathbb{I}_{n-2} \end{pmatrix}$$

$\mathfrak{h} = \mathfrak{su}(n/2) \oplus \mathfrak{u}(1)$

$$R = \begin{pmatrix} \mathbb{I}_{n/2} & i\alpha\theta\mathbb{I}_{n/2} \\ -i\alpha\theta\mathbb{I}_{n/2} & \mathbb{I}_{n/2} \end{pmatrix}$$

$\mathfrak{g} = \mathfrak{sp}(n)$

$\mathfrak{h} = \mathfrak{so}(k) \oplus \mathfrak{so}(n-k)$

$$R = \begin{pmatrix} \frac{c+\theta}{c-\theta}\mathbb{I}_k & 0 \\ 0 & \mathbb{I}_{n-k} \end{pmatrix}$$

$\mathfrak{h} = \mathfrak{su}(n/2) \oplus \mathfrak{u}(1)$

$$R = \begin{pmatrix} \mathbb{I}_{n/2} & i\alpha\theta\mathbb{I}_{n/2} \\ -i\alpha\theta\mathbb{I}_{n/2} & \mathbb{I}_{n/2} \end{pmatrix}$$

Some comments on the classification

- For all possible solutions G/H is a symmetric space.
- We can choose $R^L \neq R^R$ reflection matrices which have $H_L \times H_R$ residual symmetry where $H_L \neq H_R$.
- The so far known integrable boundary conditions with field theoretical description have $H_L \times H_R$ symmetry where $H_L = H_R$.
- We can see from the classificationi that some Rs contain parameters. The residual symmetries of these cases are not semi-simples.
- The known field theoretical integrable boundary conditions do not have any parameters.

Questions

- Is there any field theoretical description of the solutions with $H_L \times H_R$ symmetry where $H_L \neq H_R$.
- Is there any field theoretical description of the solutions which have free parameters.

Outline

The new $\kappa(z)$ for PCMs

- The ansatz is the following:

$$\kappa(z) = k(z)\left(\mathbb{I} + zM + z^2 N\right), \qquad \text{where } M \in \mathfrak{g} \text{ and } k(z) \in \mathbb{R}.$$

- The reflection equation is the following:

$$\kappa(z)L_0(z) - L_0(-z)\kappa(z) = \dot{\kappa}(z) \quad \Longrightarrow$$
$$\left(\mathbb{I} + zM + z^2 N\right)\left(J_0^R - zJ_1^R\right) - \left(J_0^R + zJ_1^R\right)\left(\mathbb{I} + zM + z^2 N\right) = 0.$$

- Which leads to the following equation system:

$$\begin{aligned} z^1 : \quad & [M, J_0^R] - 2J_1^R = 0 \\ z^2 : \quad & [N, J_0^R] - \{M, J_1^R\} = 0 \\ z^3 : \quad & \{N, J_1^R\} = 0 \end{aligned}$$

- These three equations have to be equivalent with $\dim(\mathfrak{g})$ independent boundary conditions $\Longrightarrow$

$$2N - M^2 \sim \mathbb{I} \qquad \text{and} \qquad N^2 \sim \mathbb{I}.$$

- And the boundary condition is

$$J_1^R = \frac{1}{2}[M, J_0^R] \qquad \Longrightarrow \qquad \{N, J_1^R\} = 0.$$

Some comments for these new $\kappa(z)$s

- If $N \neq 0$ then it defines an automorphism which leads to a $\mathbb{Z}_2$ graded decomposition: $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$.
- If $N = 0$ then $M^2 \sim \mathbb{I}$ so M defines the decomposition : $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$.
- The BCs on $\mathfrak{h}$ and $\mathfrak{f}$: $\Pi_{\mathfrak{h}}(J_1^R) = 0$ and $\Pi_{\mathfrak{f}}(J_1^R) = \beta(\Pi_{\mathfrak{f}}(J_0^R))$ where $\beta : \mathfrak{f} \to \mathfrak{f}$ is an invertible linear map.
- The $M \in \mathfrak{h}$ and $[M, \mathfrak{h}] = 0$ therefore $\mathfrak{h}$ has non-trivial center so it is not semi-simple: $\mathfrak{h} = \mathfrak{u}(1) \oplus \mathfrak{h}_s$ where $\mathfrak{u}(1)$ is generated by M.
- We have seen that this boundary condition belongs this boundary Lagrangian:

$$L_b = \frac{1}{4}\mathrm{Tr}\left[M J_0^R\right].$$

- The model has $G \times H$ symmetry and one free parameter.
- The right conserved charges are:

$$Q_R = \int_{-\infty}^{0} J_0^R \mathrm{d}x^1 \quad \Longrightarrow \quad \dot{Q}_R = J_1^R\Big|_{x=0} = 0 \Longleftrightarrow J_1^R \in \mathfrak{h}.$$

- The left conserved charges are ($\mathrm{J}^L = -g\mathrm{J}^R g^{-1}$):

$$Q_L = \int_{-\infty}^{0} J_0^L - \frac{1}{2}gMg^{-1}\delta(x)\mathrm{d}x \quad \Longrightarrow$$

$$\dot{Q}_L = \left(J_1^L - \frac{1}{2}g[J_0^R, M]g^{-1}\right)\Big|_{x=0} = \left(-gJ_1^R g^{-1} + \frac{1}{2}g[M, J_0^R]g^{-1}\right)\Big|_{x=0} = 0.$$

The solutions

- $\mathfrak{g} = \mathfrak{su}(n)$ and $\mathfrak{h} = \mathfrak{u}(1) \oplus \mathfrak{su}(m) \oplus \mathfrak{su}(n-m)$

$$M = i\frac{\lambda}{\frac{n}{2}-m}\begin{pmatrix}(n-m)\mathbb{I}_m & \\ & m\mathbb{I}_{n-m}\end{pmatrix}, \qquad N = \lambda^2\frac{n}{n-2m}\begin{pmatrix}-\mathbb{I}_m & \\ & \mathbb{I}_{n-m}\end{pmatrix}.$$

$$\kappa(z) = \begin{pmatrix}\frac{1+iz\lambda}{1-iz\lambda}\mathbb{I}_m & \\ & \mathbb{I}_{n-m}\end{pmatrix}.$$

- $\mathfrak{g} = \mathfrak{so}(n)$ and $\mathfrak{h} = \mathfrak{so}(2) \oplus \mathfrak{so}(n-2)$

$$M = 2\lambda\begin{pmatrix}U_2 & \\ & \mathbb{O}_{(n-2)\times(n-2)}\end{pmatrix}, \qquad N = \lambda^2\begin{pmatrix}-\mathbb{I}_2 & \\ & \mathbb{I}_{n-2}\end{pmatrix},$$

$$U_{2n} = \begin{pmatrix}\mathbb{O}_{n\times n} & -\mathbb{I}_n \\ \mathbb{I}_n & \mathbb{O}_{n\times n}\end{pmatrix}, \qquad \kappa(z) = \begin{pmatrix}\frac{1-\lambda^2z^2}{1+\lambda^2z^2} & \frac{-2\lambda z}{1+\lambda^2z^2} & \\ \frac{2\lambda z}{1+\lambda^2z^2} & \frac{1-\lambda^2z^2}{1+\lambda^2z^2} & \\ & & \mathbb{I}_{n-2}\end{pmatrix}.$$

- $\mathfrak{g} = \mathfrak{so}(2n)$ or $\mathfrak{g} = \mathfrak{sp}(n)$ and $\mathfrak{h} = \mathfrak{u}(1) \oplus \mathfrak{su}(n)$
  For this case $M = \lambda U_{2n}$ and $N = 0$ and

$$\kappa(z) = \frac{1}{\sqrt{1+\lambda^2z^2}}\begin{pmatrix}\mathbb{I}_n & -\lambda\mathbb{I}_n \\ \lambda\mathbb{I}_n & \mathbb{I}_n\end{pmatrix}.$$

Conclusions

- We have seen that there are quantum integrable boundary conditions which do not have so far known field theoretical description e.g. when the residual symmetry is $H_L \times H_R$ and reflections with free parameter.
- We have determined new κ matrices for the principal models whose residual symmetry is $G \times H$ or $H \times G$.
- We have seen that if the center of the residual symmetry is one dimensional then the boundary condition and the κ matrix contain one free parameter.

Other Results

- We found a higher spin conserved charge for $G = \mathrm{SU}(n)$ when $n > 3$.
- We also showed these new transfer matrices have $\mathcal{Y}(\mathfrak{g}, \mathfrak{g}) \oplus \mathcal{Y}(\mathfrak{g}, \mathfrak{h})$ symmetry.
- The $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ case can be used to determine the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ symmetric κ matrices for SO(4) sigma models.
- This can be generalized for $\mathrm{SO}(2N)$ sigma models with $\mathrm{U}(N)$ symmetric boundary condition which are also new solutions.

Open questions

- Are there field theoretical descriptions for the cases when the residual symmetry is $G \times H$ but the H is semi-simple?
- Are there field theoretical descriptions for general $H_L \times H_R$ with two free parameters?
- Are the new boundary conditions which are integrable at the quantum level for for SU(2), $\mathrm{SO}(n)$ or for $\mathrm{Sp}(n)$ principal models or the $\mathrm{O}(2n)$ sigma models?