

# Localised Dirac eigenmodes at the Roberge-Weiss transition

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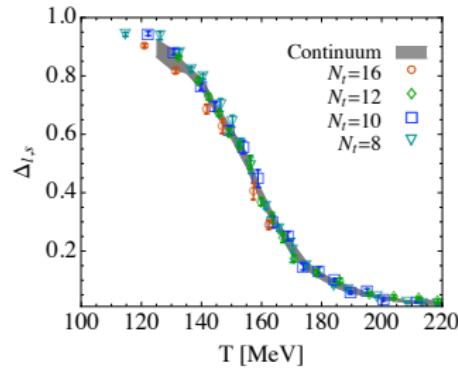
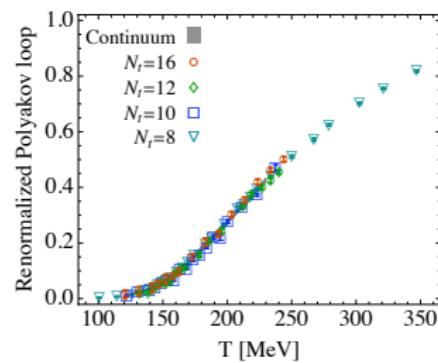
Budapest, 07 December 2021

Based on M. Cardinali, M. D'Elia, F. Garosi, MG, [arXiv:2110.10029](https://arxiv.org/abs/2110.10029)

# QCD at Finite Temperature

Analytic crossover in the range  $T \simeq 145 - 165$  MeV

Deconfinement, chiral symmetry restoration in the same temperature range



Renormalised Polyakov loop and chiral condensate  
Figures from [BW collaboration (2010)]

In other QCD-like gauge theories with genuine phase transitions

- deconfinement improves chiral symmetry properties if single transition (e.g., pure gauge theory,  $N_f = 3$  staggered fermions on coarse lattices)
- $T_{\text{dec}} < T_\chi$  if two transitions are present (e.g., adjoint fermions)

Relation between the two phenomena still not fully clear

# Finite-Temperature Gauge Theory

Partition function at finite  $T$  (imaginary time formulation)

$$Z = \int [DA] \int [D\psi D\bar{\psi}] e^{-S_G[A] - S_F[\psi, \bar{\psi}, A]} = \int [DA] \det(\not{D}[A] + m) e^{-S_G[A]}$$

- Euclidean gauge fields  $A_\mu^a$  and fermion field  $\psi, \bar{\psi}$
- compact temporal direction of size  $1/T$
- periodic/antiperiodic temporal b.c. for gauge/fermion fields

Yang-Mills action:  $S_G = \frac{1}{4} \int_0^{\frac{1}{T}} dt \int d^3x F_{\mu\nu}^a(t, \vec{x}) F_{\mu\nu}^a(t, \vec{x})$

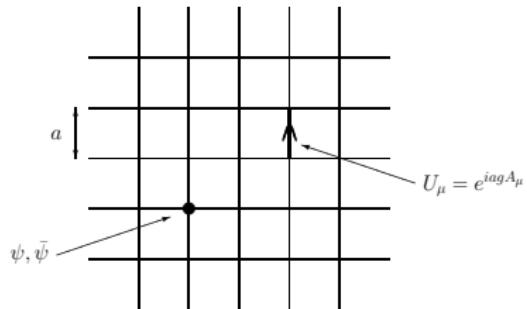
fermionic action:  $S_F = \int_0^{\frac{1}{T}} dt \int d^3x \bar{\psi}(t, \vec{x})(\not{D}[A] + m)\psi(t, \vec{x})$

Dirac operator:  $\not{D}[A] = (\partial_\mu + igA_\mu^a t^a)\gamma_\mu$

# Gauge Theories on the Lattice

Lattice approach: Euclidean continuum replaced with a discrete, finite lattice

$$Z_{\text{lat}} = \int [DU] \det(\not{D}_{\text{lat}}[U] + m) e^{-S_G^{\text{lat}}[U]}$$



- Dynamical fields associated with lattice elements:  
fermions  $\rightarrow$  sites, gauge fields  $\rightarrow$  edges
- Hypercubic  $N_t \times N_s^3$  lattice with periodic spatial boundary conditions on fields, temporal b.c. as above
- Thermodynamic ( $V \rightarrow \infty$ ) and continuum ( $a \rightarrow 0$ ) limits taken eventually at fixed  $T = (aN_t)^{-1}$

# Pure gauge theory, confinement, and centre symmetry

Quark mass  $m \rightarrow \infty$ : pure gauge theory, exact  $\mathbb{Z}_3$  centre symmetry

- Quark free energy from Polyakov loop  $\langle P \rangle \propto e^{-F_q/T}$

$$P(\vec{x}) \equiv \text{Pexp} \left\{ ig \int_0^{\frac{1}{T}} dt A_4(t, \vec{x}) \right\}$$

- Under singular gauge transformations  $U(\beta, \vec{x}) = zU(0, \vec{x})$

$$S_G \rightarrow S_G \quad P(\vec{x}) \rightarrow zP(\vec{x}) \quad z \in \mathbb{Z}_3$$

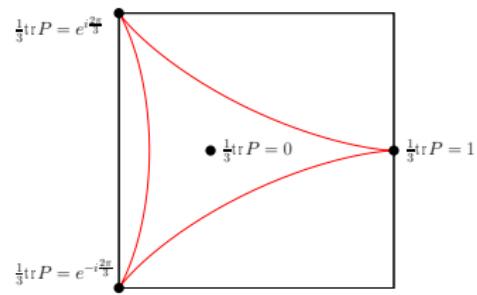
- Order parameter of centre symmetry/measure of confinement

$T < T_c$ :  $\langle P \rangle = 0 \Rightarrow F_q = \infty$ :

symmetry unbroken, quarks confined

$T > T_c$ :  $\langle P \rangle \neq 0 \Rightarrow F_q < \infty$ :

symmetry broken, quarks deconfined



- First-order deconfining transition at  $T_c \approx 290 \text{ MeV}$  [G. Boyd et al. (1996)]

# Massless quarks and chiral symmetry

Quark mass  $m \rightarrow 0$ : massless fermions, exact chiral symmetry

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \sim \mathrm{SU}(N_f)_V \times \mathrm{SU}(N_f)_A$$

Order par.: chiral condensate  $\propto$  *spectral density* of near-zero Dirac modes

$$|\langle \bar{\psi} \psi \rangle| = \int_0^\infty d\lambda \frac{2m\rho(\lambda)}{\lambda^2 + m^2} \underset{m \rightarrow 0}{\rightarrow} \pi\rho(0^+) \quad \rho(\lambda) = \lim_{V \rightarrow \infty} \frac{T}{V} \left\langle \sum_n \delta(\lambda - \lambda_n) \right\rangle$$

- Real world:  $u, d$  light and nearly degenerate  $\Rightarrow N_f = 2$  case as  $m \rightarrow 0$

$T < T_c$ :  $\langle \bar{\psi} \psi \rangle \neq 0 \Rightarrow$  3 massless pseudoscalar Goldstone bosons (pions)

$T > T_c$ :  $\langle \bar{\psi} \psi \rangle = 0 \Rightarrow$  Goldstone excitations no more massless

- Second-order restoring phase transition at  $T_c \approx 132 \text{ MeV}$

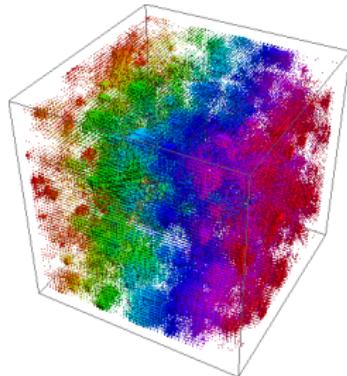
[Ding et al. (2019)]

Deconfinement and chiral transition governed by very different symmetries, both only approximate at the physical point: why do they stick together?

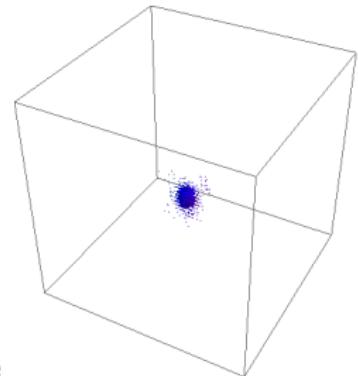
# Localisation of Dirac eigenmodes

One more thing happens at the QCD transition:  
low Dirac modes  $\not{D}\psi_n = i\lambda_n\psi_n$  become *localised*

- delocalised mode: extends throughout the whole system, spreading out like  $\|\psi(x)\|^2 \sim 1/V^\alpha$  with  $0 < \alpha < 1$
- localised mode: confined in finite region,  $\|\psi(x)\|^2 \sim 1/V^0$  inside, negligible outside



delocalised mode



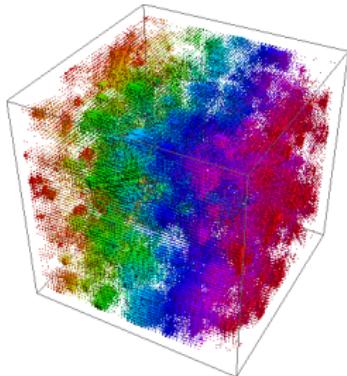
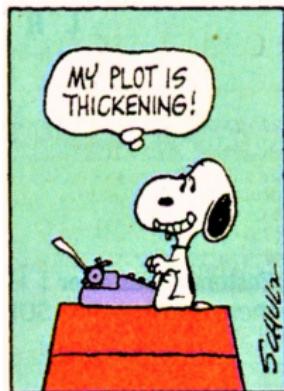
localised mode

Figure from [Ujfalusi et al. (2015)]

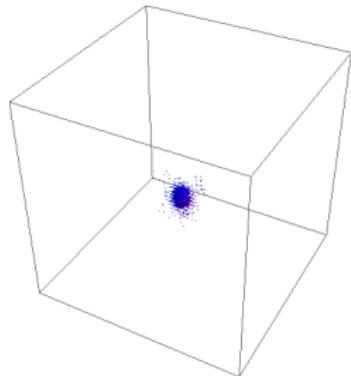
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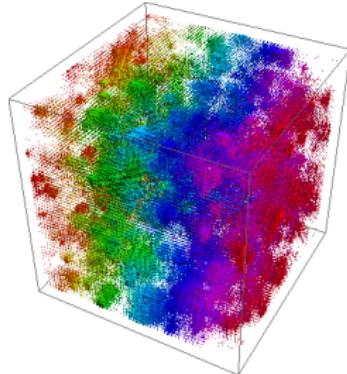
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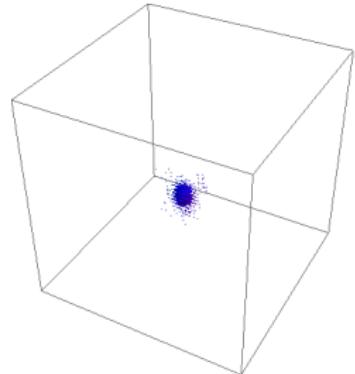
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Normalised modes

$$\int_0^\beta dt \int d^3x \|\psi_n(t, \vec{x})\|^2 = 1$$
$$\|\psi_n(t, \vec{x})\|^2 \equiv \sum_{c,\alpha} |\psi_{n c,\alpha}(t, \vec{x})|^2$$



delocalised mode

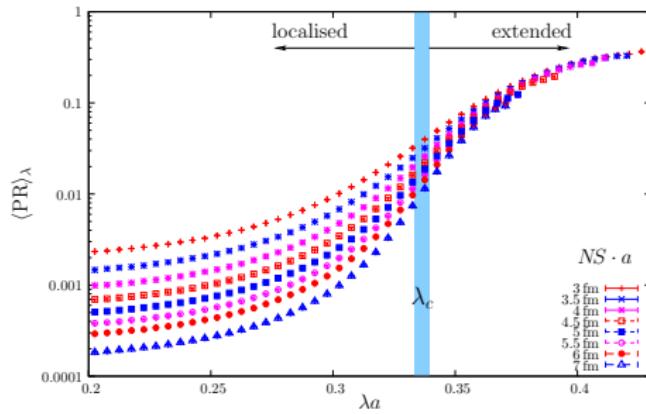


localised mode

Figure from [Ujfaluji et al. (2015)]

# Localisation of Dirac eigenmodes

Participation ratio  $\approx$  fraction of system occupied by a mode



Data for  $T \simeq 2.6 T_c$  from [MG et al. (2014)]

$$\text{IPR}_n = \int_0^{\frac{1}{T}} dt \int d^3x \|\psi_n(t, \vec{x})\|^4 \quad \text{PR}_n = \frac{T}{V} \text{IPR}_n^{-1}$$

Mobility edge  $\lambda_c$  separates localised and delocalised modes ( $T > T_c$ )

# Localisation of Dirac eigenmodes

Mobility edge extrapolates to zero in the crossover region, no localised modes in the confined/chirally broken phase at low  $T$

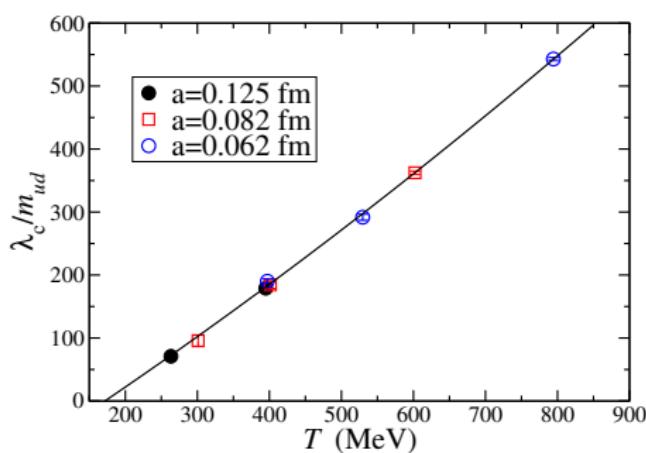


Figure from [Kovács, Pittler (2012)]

Ample numerical evidence from the lattice [Giordano, Kovács (2021)]

- Localised modes seen with various fermion discretisations
  - ▶ staggered [García-García, Osborn (2007)  
[Kovács, Pittler (2012)]]
  - ▶ overlap [Kovács (2010)]
  - ▶ domain wall [Cossu, Hashimoto (2016)]
  - ▶ twisted mass [Holicki *et al.* (2018)]
- Survive continuum limit  
⇒ not a lattice artefact

Ratio  $\lambda_c/m_{ud}$  expected to be RG-invariant [Kovács, Pittler (2012)]  
Proof in preparation [Giordano (202ish)]

# Localisation of Dirac eigenmodes

Formally identical to disordered systems of condensed matter

$$H(B, V)_{xy} = \delta_{xy} V_x + \sum_k B_{x,k} \delta_{x+\hat{k},y} + B_{x-\hat{k},k}^* \delta_{x-\hat{k},y}$$

disordered Hamiltonian $H(B, V)$	$\rightarrow$	Dirac operator $-i\cancel{D}[A]$
random hopping $B$ , potential $V$	$\rightarrow$	gauge fields $A$
ensemble average $\langle\langle \dots \rangle\rangle_{B,V}$	$\rightarrow$	path integral $\langle \dots \rangle$

- Same critical features as 3D unitary Anderson model
  - ▶ Anderson transition at the mobility edge [MG *et al.* (2014)]
  - ▶ Nontrivial multifractal exponents [Ujfalusi *et al.* (2015)]
  - ▶ Critical statistics [Nishigaki *et al.* (2014)]
- Localisation not surprising – appearing at the band centre at  $T_c$  is

# Localisation and deconfinement

Qualitative explanation of localisation: “sea/islands” picture

[Bruckmann *et al.* (2011); MG *et al.* (2015, 2016)]

- Polyakov loop = effective,  $\vec{x}$ -dependent temporal b.c.

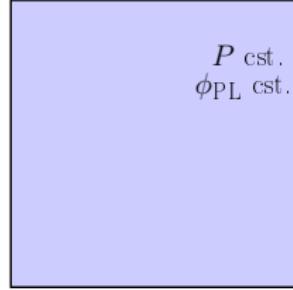
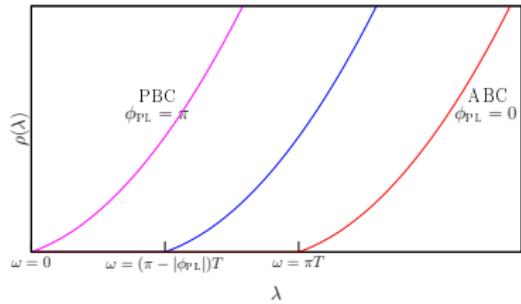
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$$\Leftrightarrow \not D_t \psi \text{ with } \psi\left(\frac{1}{T}, \vec{x}\right) = -P(\vec{x})\psi(0, \vec{x})$$

- Ordered phase  $\approx$  constant configs.

$$gA_\mu(x) = \delta_{\mu 4} \text{diag}(\phi_{PL})$$

- “Sea” of  $\phi_{PL} = 0$  selected by fermions, largest spectral gap  $\omega = (\pi - |\phi_{PL}|)T$
- “Islands” of fluctuations with local “energy”  $E(\vec{x}) = (\pi - |\phi_{PL}(\vec{x})|)T$  can support localised modes below the gap



Very general picture, applies to any gauge system with a deconfined phase

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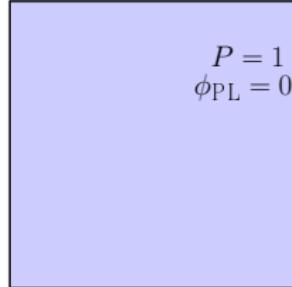
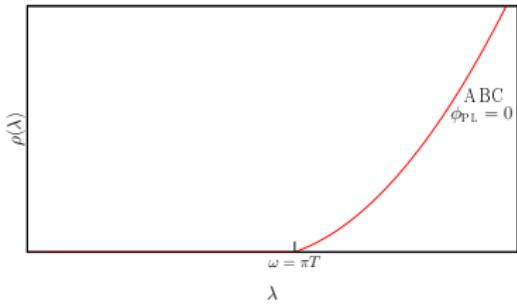
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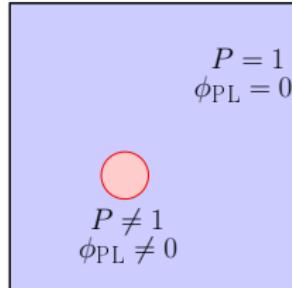
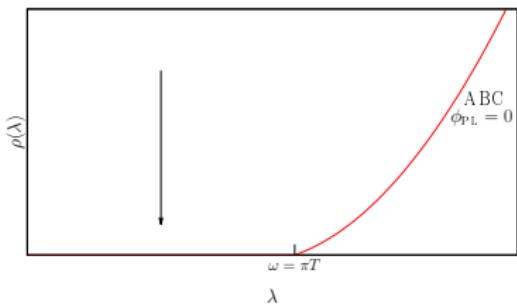
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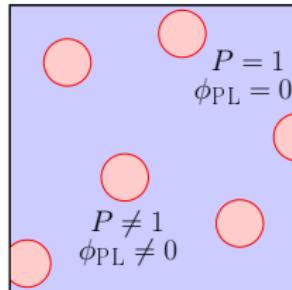
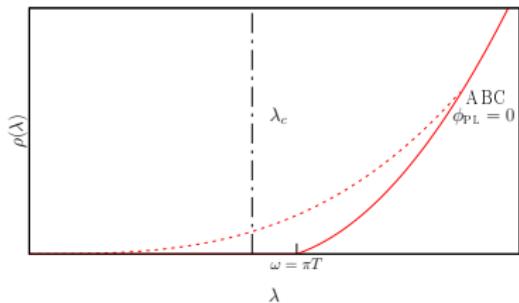
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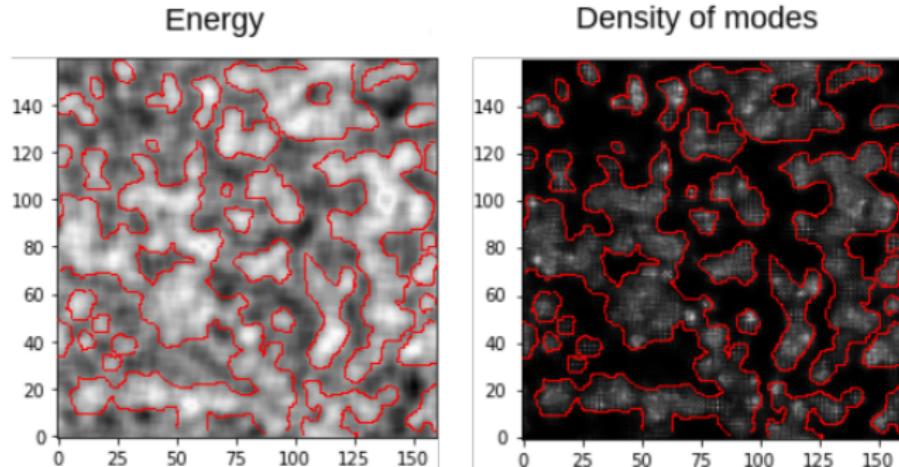
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# Sea/islands picture



From [D. Bugár, M.Sc. Thesis (2021)]

One  $160^2 \times 4$  lattice configuration of 2+1D U(1) gauge theory,  $\arg \sum_{\vec{x}} P(\vec{x}) = 0$

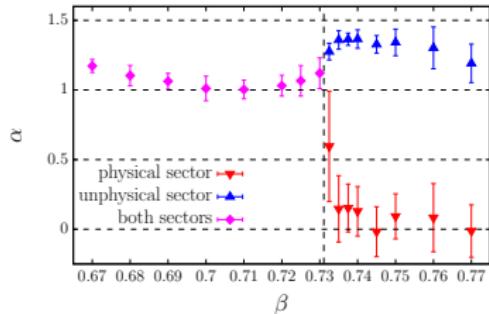
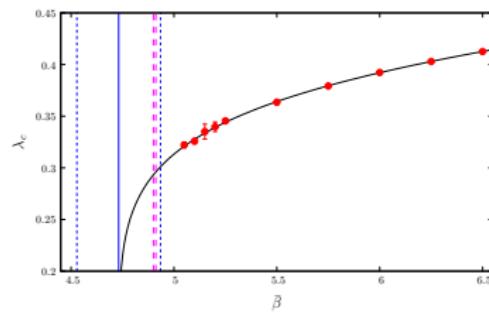
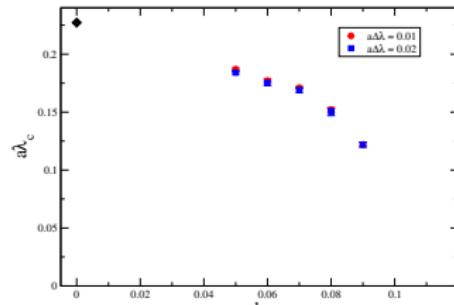
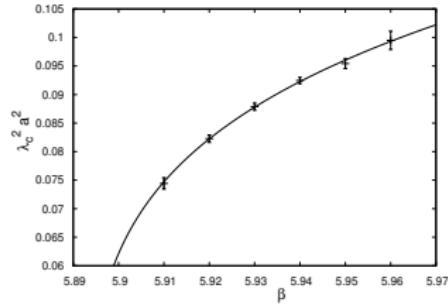
Energy:  $E_{\text{lat}}(\vec{x}) = \sin \frac{\pi - |\phi_{\text{PL}}(\vec{x})|}{N_t}$

Density of modes:  $D(\vec{x}) = \frac{\sum_{n, \lambda_n < \lambda_{\text{cut}}} \sum_t \|\psi_n(t, \vec{x})\|^2}{\sum_{n, \lambda_n < \lambda_{\text{cut}}} 1}$

# Localisation and deconfinement

Localisation observed in the deconfined phase in many pure gauge systems

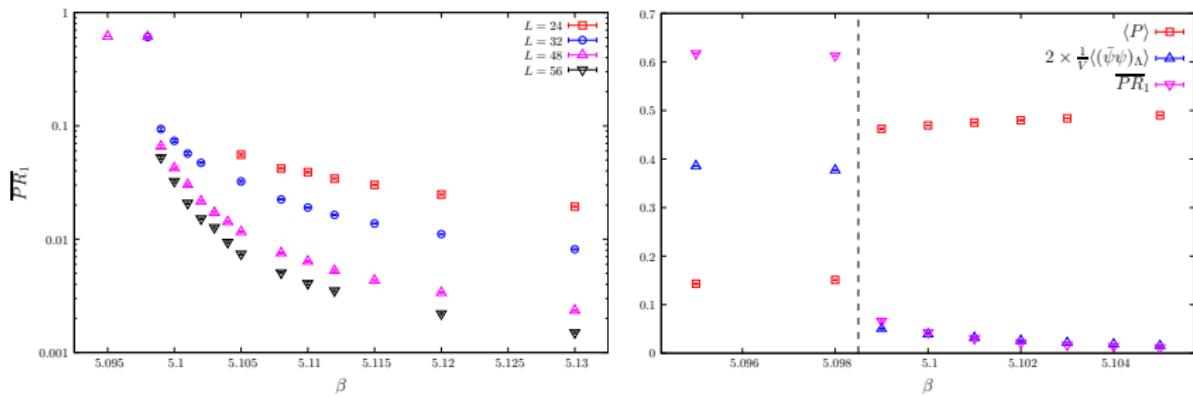
- SU(3) in 3+1D [Kovács, Vig (2018, 2020)] and 2+1D [MG (2019)]
- SU(3) + trace deformation [Bonati *et al.* (2021)]
- $\mathbb{Z}_2$  in 2+1D [Baranka, MG (2021)]



# Localisation and fermions

Localisation in a system with fermions displaying genuine phase transition:  
 $SU(3) + N_f = 3$  unimproved rooted staggered fermions on  $N_t = 4$  lattices

[De Forcrand, Philipsen (2003)]



Figures from [MG et al. (2017)]

- First-order, (partially) deconfining and (partially) chirally restoring p.t.
- Localised modes appear at the transition
- Transition is a lattice artefact, does not survive continuum limit

System with physical transition? Roberge-Weiss transition at imaginary  $\mu$

# Imaginary chemical potential and Roberge-Weiss symmetry

Add imaginary quark chemical potential  $\mu_I$  (to excite imaginary hadrons, perhaps?)

$$\not{D}[A] \longrightarrow \not{D}[A] + i\mu_I \gamma_4 = \not{D}[A] + i\hat{\mu}_I T \gamma_4$$

- $\mu_I$  enters like  $gA_4$ , modifies effective temporal boundary condition
- $\hat{\mu}_I \rightarrow \hat{\mu}_I \pm \frac{2\pi}{3}$  reabsorbed by centre transformation  $= gA_4 \rightarrow gA_4 \mp \frac{2\pi}{3} T$

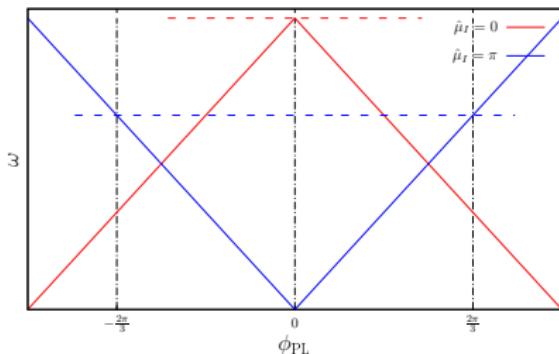
Partition function periodic  $Z(\hat{\mu}_I)$  under  $\hat{\mu}_I \rightarrow \hat{\mu}_I \pm \frac{2\pi}{3}$

[Roberge, Weiss (1986)]

Periodicity realised differently at low/high temperature:

- at low  $T$ ,  $Z(\hat{\mu}_I)$  smooth periodic function
- at high  $T$ , lines of first-order phase transitions at  $\hat{\mu}_I = \frac{\pi}{3} + \frac{2\pi}{3}n$ ,  $n \in \mathbb{Z}$

# Roberge-Weiss transition



- Effective boundary condition  $e^{i(\pi+\phi_{PL})} \rightarrow e^{i(\pi+\phi_{PL}+\hat{\mu}_I)}$
- Gap in uniform configuration modified:

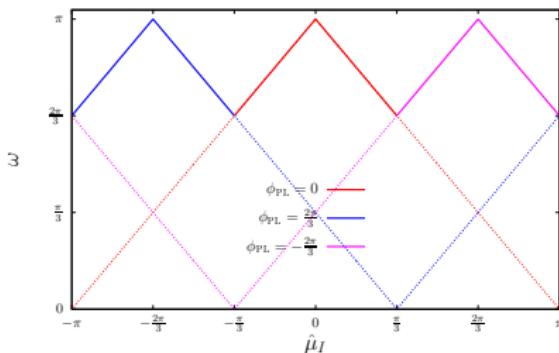
$$\hat{\mu}_I = 0: \omega = (\pi - |\phi_{PL}|)T \quad \hat{\mu}_I = \pi: \omega = |\phi_{PL}|T$$

- First-order transitions at  $\hat{\mu}_I = \frac{\pi}{3} + \frac{2\pi}{3}n$ 
  - ▶ centre sector favoured by fermions changes
  - ▶  $\mathbb{Z}_2$  centre symmetry since two centre sectors are equally favoured

First-order lines end at  $T_{RW} = 208(5)$  MeV, second-order Ising point

[C. Bonati et al. (2016)]

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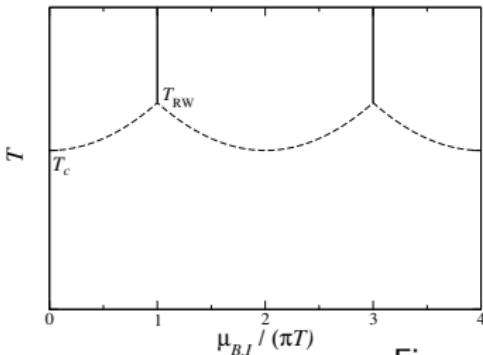


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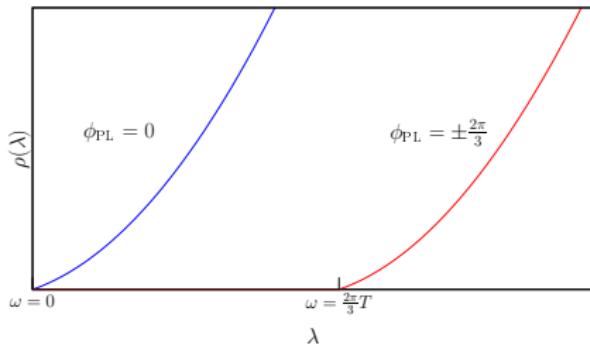
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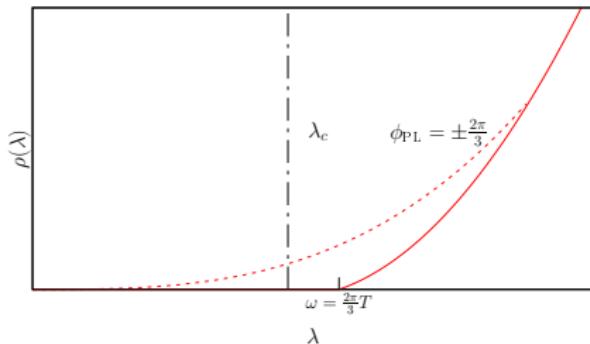


"Sea/islands" picture at  $\hat{\mu}_I = \pi$  and  $T > T_{\text{RW}}$

- fermions choose one of the complex centre sectors  $P(\vec{x}) \sim e^{\pm i \frac{2\pi}{3}}$  spontaneously breaking residual  $\mathbb{Z}_2$  symmetry
- (pseudo)gap in the spectrum  $\omega = \frac{2\pi}{3} T$  for either sector
- fluctuations away from  $e^{\pm i \frac{2\pi}{3}}$  reduce eigenvalue, lead to localisation

⇒ expect low modes to turn from delocalised to localised at  $T_{\text{RW}}$

# Roberge-Weiss transition and localisation



"Sea/islands" picture at  $\hat{\mu}_I = \pi$  and  $T > T_{RW}$

- fermions choose one of the complex centre sectors  $P(\vec{x}) \sim e^{\pm i \frac{2\pi}{3}}$  spontaneously breaking residual  $\mathbb{Z}_2$  symmetry
- (pseudo)gap in the spectrum  $\omega = \frac{2\pi}{3}T$  for either sector
- fluctuations away from  $e^{\pm i \frac{2\pi}{3}}$  reduce eigenvalue, lead to localisation

⇒ expect low modes to turn from delocalised to localised at  $T_{RW}$

## Numerical setup

Lattice discretisation of  $N_f = 2 + 1$  QCD

- physical quark masses
- finite temperature  $T$
- imaginary chemical potential  $\hat{\mu}_I = \pi$  (= PBC in temporal direction)

Details for the practitioners:

- 2-stout improved rooted staggered fermions
- tree-level improved Symanzik gauge action
- $N_t = 4, 6, 8$  with aspect ratio 6 (also 8 for  $N_t = 4$ )

Scan in temperature for  $T > T_{\text{RW}}$ , study localisation using statistical properties of the Dirac spectrum

# Localisation and spectral statistics

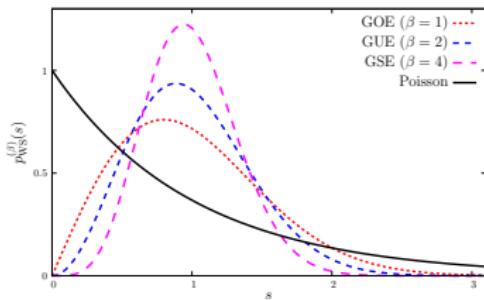
Localisation and spectral statistics are connected

- delocalised modes easily mixed by fluctuations → RMT-type statistics
- localised modes fluctuate independently → Poisson statistics

Universal analytic results available for *unfolded spectrum*

$$x_n = \frac{V}{T} \int^{\lambda_n} d\lambda \rho(\lambda)$$

average no. of evs. below  $\lambda$



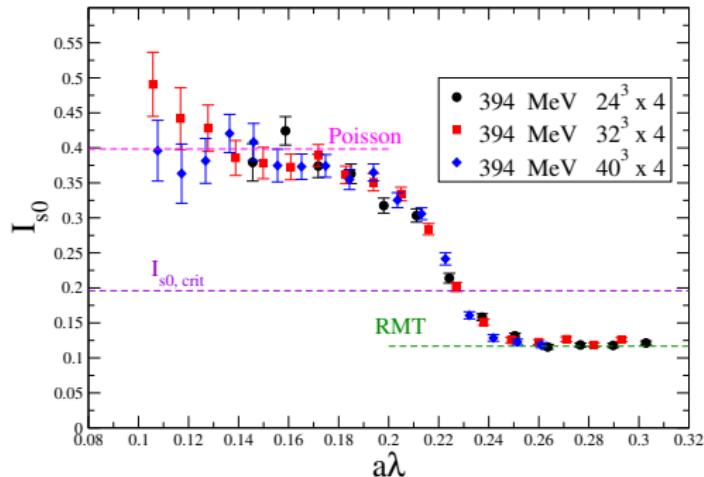
- unfolding removes non-universal, model-dependent details
- PDF of unfolded spacings known

$$s_n = x_{n+1} - x_n = \frac{\lambda_{n+1} - \lambda_n}{\langle \lambda_{n+1} - \lambda_n \rangle}$$

Measure  $p_\lambda(s)$  locally and compare with  $p_{\text{RMT}}(s)$ ,  $p_{\text{Poisson}}(s)$

$$p_\lambda(s) = \frac{\langle \sum_n \delta(\lambda - \lambda_n) \delta(s - s_n) \rangle}{\langle \sum_n \delta(\lambda - \lambda_n) \rangle}$$

# Localisation above $T_{\text{RW}}$



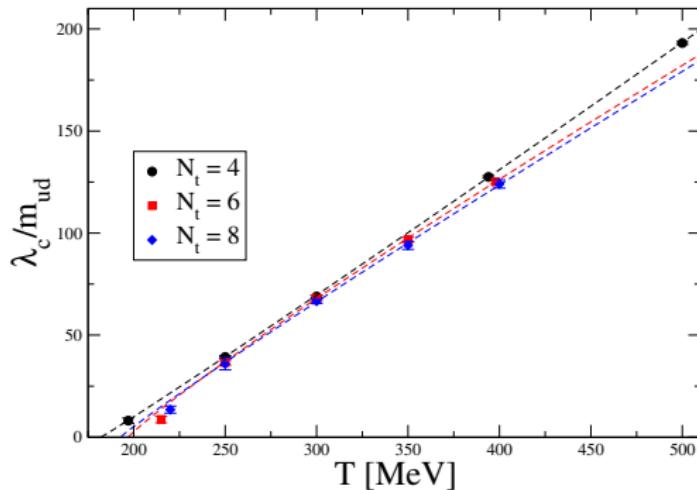
Extract features from  $p_\lambda(s)$

$$I_{s0}(\lambda) = \int_0^{s_0} ds p_\lambda(s)$$

Optimised  $s_0 \approx 0.508$ , maximises difference between RMT and Poisson

Critical value  $I_{s0}^{\text{crit}}$  known [MG et al. (2014)], use to find  $\lambda_c$  via  $I_{s0}(\lambda_c) = I_{s0}^{\text{crit}}$

# Mobility edge

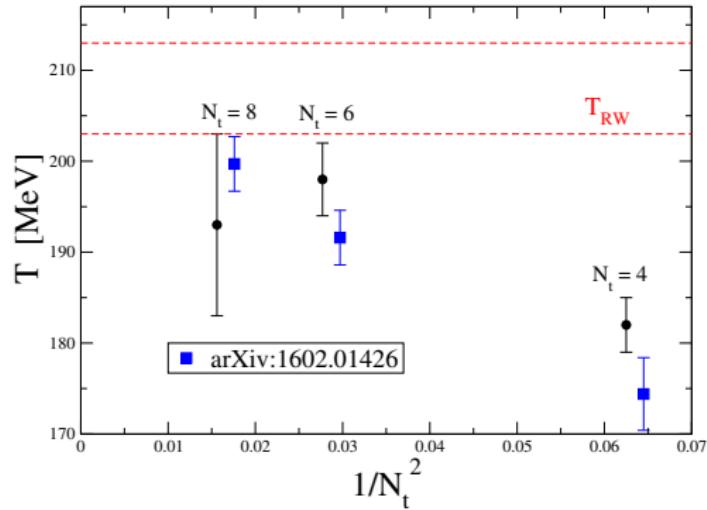


Find  $T_{\text{loc}}(N_t)$  where  $\lambda_c = 0$  for each lattice spacing  $a = (TN_t)^{-1}$  fitting to

$$\frac{\lambda_c}{m_{ud}} = A(N_t)[T - T_{\text{loc}}(N_t)]^{B(N_t)}$$

Lowest  $T$  for each  $N_t$  excluded from fit,  
large finite-size effects for  $N_t = 6, 8$

# Localisation temperature



Within errors  $T_{loc}(N_t) = T_{RW}(N_t) \implies$  strongly supports  $T_{loc} = T_{RW}$

Localised modes appear right at a genuine deconfinement transition (Roberge-Weiss transition) also in the presence of fermions

# Summary and outlook

We studied localisation in QCD at imaginary chemical potential  $\hat{\mu}_I = \pi$  above the Roberge-Weiss temperature  $T_{RW}$ , finding that:

- localised low Dirac modes are present  
     $\Rightarrow$  confirms expectations of “sea/islands” picture
- localisation appears at  $T_{RW}$   
     $\Rightarrow$  confirms strong connection with deconfinement

Open issues:

- many clues connecting localisation and deconfinement, but something still missing
  - ▶ no studies yet in models with a trivial centre
- physical meaning of localisation still unclear
  - ▶ in the chiral limit it affects (possibly kills) Goldstone excitations [MG (2020)], but no explicit realisation known so far
- is localisation how deconfinement improves chiral symmetry properties?



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