Quantum Gravity on the Lattice: a new look at an old problem

or how to waste your time on a hopeless project

Matteo Giordano

Eötvös Loránd University (ELTE)
Budapest

ELTE particle physics seminar
Budapest, 11/10/2017

Based on work in progress with S. D. Katz and D. Nógrádi

Quantising Gravity

Quantisation of gravity is one of the (if not "the") most important open problems in theoretical physics

Many different kind of approaches have been tried:

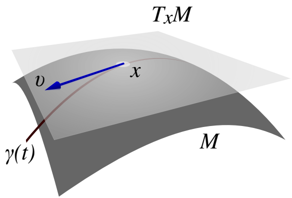
- the most straightforward (perturbative QFT quantisation) fails due to nonrenormalisability
- renormalisable extensions (including R^2 terms) have problems with unitarity
- nonperturbative quantisation requires a nonperturbative formulation to begin with
- path-integral/functional-integral methods are "traditional" in a sense (they work for the other interactions), and amenable in principle to a nonpertubative treatment

Doesn't mean we know how to do it, though

Classical General Relativity Yeah, like we know the quantum counterpart

Ingredients: manifold $\mathcal{M}$, metric $g_{\mu\nu}$, connection $\Gamma^{\mu}{}_{\nu\rho}$ entering the covariant derivative

$$D_\nu v^\mu = \partial_\nu v^\mu + \Gamma^{\mu}{}_{\rho\nu} v^\rho$$



Second order formalism: metric components are the independent variable, the torsionless metric-compatible connection is entirely determined by $g_{\mu\nu}$ (Levi-Civita connection)

Einstein-Hilbert action + cosmological constant (Minkowski signature)

$$S_{EH} = \int_{\mathcal{M}} d^4x \sqrt{-g}(m_P^2 R - \Lambda)$$

R Ricci scalar, $g = \det g_{\mu\nu}$,
$m_P^2 = 1/(16\pi G_N)$ (Planck mass)

Zero torsion: $\Gamma^{\mu}{}_{\nu\rho} = \Gamma^{\mu}{}_{\rho\nu}$
Metric compatibility: $D_\mu g_{\nu\rho} = 0$

We use signature (-,+,+,+)

First Order Formalisms

Palatini: treat metric and torsionless connection as independent

$$S_P = \int_{\mathcal{M}} d^4x \sqrt{-g} (m_P^2 g^{\mu\nu} R_{\mu\nu} - \Lambda)$$

Metric compatibility emerges from the equations of motion together with Einstein equation

Cartan: treat vierbein and metric-compatible connection as independent

$$S_{EC} = \int_{\mathcal{M}} \left[\frac{m_P^2}{2} R^{ab} \wedge e^c \wedge e^d - \frac{\Lambda}{4!} e^a \wedge e^b \wedge e^c \wedge e^d \right] \epsilon_{abcd}$$

Absence of torsion emerges from the equations of motion together with Einstein equation

Einstein-Cartan Formulation

Vierbein: $e^a{}_\mu$ connects a coordinate basis in the tangent space ($\{\mathbf{u}_\mu\} = \{\partial_\mu\}$) to an orthonormal basis ($\{\mathbf{e}_a\}$), $\mathbf{u}_\mu = e^a{}_\mu \mathbf{e}_a$

$g(\mathbf{e}_a, \mathbf{e}_b) = \eta_{ab} \qquad g_{\mu\nu} = g(\mathbf{u}_\mu, \mathbf{u}_\nu) = g(\mathbf{e}_a, \mathbf{e}_b) e^a{}_\mu e^b{}_\nu = \eta_{ab} e^a{}_\mu e^b{}_\nu$

Use the language of forms for brevity

$DV = D(V^\mu \mathbf{u}_\mu) = (dV^\mu + \Gamma^\mu{}_\nu V^\nu)\mathbf{u}_\mu = D(V^a \mathbf{e}_a) = (dV^a + \omega^a{}_b V^b)\mathbf{e}_a$

- $e^a = e^a{}_\mu dx^\mu$ vierbein one-forms
- ω^{ab} spin connection one-form (indices raised/lowered with η)
- $R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$ curvature two-form (Riemann tensor)

$$S_{EC} = \int_{\mathcal{M}} \left[\frac{m_P^2}{2} R^{ab} \wedge e^c \wedge e^d - \frac{\Lambda}{4!} e^a \wedge e^b \wedge e^c \wedge e^d \right] \epsilon_{abcd}$$

! WARNING !

Cartan action equivalent to Einstein-Hilbert for $\det(e^a{}_\mu) > 0$, a factor $\mathrm{sgn}(\det(e^a{}_\mu))$ should be included

A First Glimpse of a Gauge Structure

What do torsion and metric compatibility read in terms of vierbein and spin connection?

Torsion: $T^a = De^a$

Metric compatibility: $Dg_{\nu\rho} = dg_{\mu\nu} + \Gamma^{\alpha}{}_{\mu} g_{\alpha\nu} - \Gamma^{\alpha}{}_{\nu} g_{\mu\alpha} = 0 \Rightarrow$

$$\omega^{c}{}_{a}\eta_{cb} - \omega^{c}{}_{b}\eta_{ac} = 0$$

$\omega^{a}{}_{b}$ is a $\mathfrak{so}(3,1)$-valued one-form, e^a is a $\mathbb{R}^{3,1}$-valued one-form

Change of orthonormal frame in the tangent space = element of SO(3,1):

- $\omega^{a}{}_{b}$ transforms like a gauge field (inhomogeneously)
- e^a transforms like a vector
- $R^{a}{}_{b}$ transforms like a rank-2 tensor

Functional Integral Approach

Starting point: define a partition function $\Rightarrow$ requires an action and a functional measure

- For the action the natural choice is the Einstein-Hilbert action, or variations thereof (cosmological term, quadratic term,...)
- The measure is a more complicated issue: what one would want to do in principle is to integrate over geometries, and to do so in such a way as to preserve diffeomorphism invariance

Formally we aim at defining at least one of these partition functions:

$$Z_{EH} = \int [Dg] e^{iS_{EH}}$$

$$Z_P = \int [Dg][D\Gamma] e^{iS_P}$$

$$Z_{EC} = \int [De][D\omega] e^{iS_{EC}}$$

Problems of a Numerical Approach - Part 1 of 3027

If we knew how to formulate QG nonperturbatively, how would we compute the functional integral in practice?

Numerical calculations would do, but they come with problems:

- problematic in Minkowskian signature, requires to go over to Euclidean signature
- discretisation of the path integral breaks badly diffeomorphism invariance, make sure that it is recovered in the continuum limit
- make sure that a continuum limit even exists: requires a second-order phase transition somewhere in parameter space

divergent correlation length $\rightarrow$ system loses knowledge of the discretisation

! WARNING !

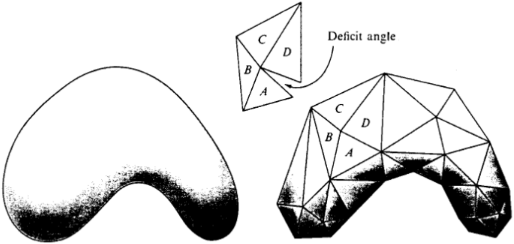
Analytic continuation to Euclidean might be a very bad idea:

- it can lead to something unrelated to general relativity, some argue
- problems in defining the Wick rotation for a general metric
- Euclidean action is unbounded

Existing Numerical Approaches

Regge calculus: sum over all piece-wise flat (Riemannian) manifolds weighted by a discretised version of Einstein action, possibly with cosmological constant and/or higher derivative terms

Dynamical variables: edge lengths of simplicial complexes



classical:

[Regge (1961)]

quantum:

[review: Immirzi (1996)]

numerical:

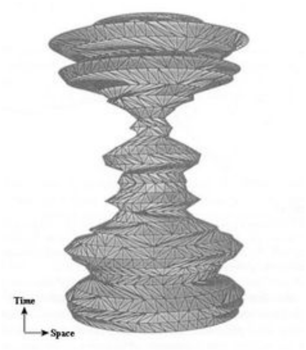
[Berg (1985)]

[Hamber and Williams (1985)]

Existing Numerical Approaches

Dynamical triangulation: a type of Regge calculus where the dynamical variables are the connectivity of the complexes

[Römer and Zöhringer (1986)][Agishtein and Migdal (1992)][Ambjørn and Jurkiewicz (1992)]



- summing over all Euclidean geometries leads to meaningless results
- Causal DT [Ambjørn and Jurkiewicz (1992)]: the set of geometries is restricted; Wick rotation under control; promising, 2nd order transitions have been found
- finding what are "all the geometries" over which "the sum over all geometries" is is part of the problem of QG

[review: Ambjørn *et al.* (2012)]

Existing Numerical Approaches

Lattice Gauge Theory: exploit formulation of relativity as a gauge theory based on Einstein-Cartan formulation, preserving the local SO(3,1) gauge invariance exactly

Formulations:

[Smolin (1979)] [Mannion and Taylor (1981)] [Kondo (1984)]

[Menotti and Pelissetto, (1986,1987)] [Caselle, D'Adda and Magnea (1987)]

Numerical:

[Caracciolo and Pelissetto (1987,1988)]

[Catterall, Ferrante and Nicholson (2012)]

Gravity as (almost) a gauge theory

GR can be set up as a kind of gauge theory of the groups $G = \mathrm{ISO}(3,1)$ (Poincaré) or $G = \mathrm{SO}(4,1)$ (de Sitter), isometries of Minkowski and de Sitter spaces

de Sitter gravity is somehow simpler: algebra-valued connection one-forms $\mathcal{A} \in \mathfrak{g} = \mathfrak{so}(4,1) \cong \mathfrak{so}(3,1) \oplus \mathbb{R}^{3,1}$ (as vector spaces)

$$\mathcal{A} = \begin{pmatrix} \omega & \frac{1}{\ell} e \\ \frac{1}{\ell} e^T & 0 \end{pmatrix} \qquad \omega \in \mathfrak{so}(3,1),\ e \in \mathbb{R}^{3,1}$$

$\mathcal{A} \to \mathcal{A}^{AB} = -\mathcal{A}^{BA}$, $A, B = 1, \dots 5$

$$\mathcal{A}^{ab} = \omega^{ab} \qquad \mathcal{A}^{a5} = \frac{1}{\ell} e^a \qquad a = 1, \dots 4$$

ℓ: length scale (in de Sitter: radius of the space)

Gravity as (almost) a gauge theory II

Associated curvature two-form splits similarly: $\mathcal{R} = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$

$$\mathcal{R}^{ab} = d\omega^{ab} + \omega^{a}{}_{c} \wedge \omega^{cb} - \frac{1}{\ell^2} e^a \wedge e^b = R^{ab} - \frac{1}{\ell^2} e^a \wedge e^b$$
$$\mathcal{R}^{a5} = \frac{1}{\ell}(de^a + \omega^{a}{}_{c} \wedge e^c) = \frac{1}{\ell}(D_\omega e)^a = \frac{1}{\ell} T^a$$

If it were a true gauge theory the action would be given by

$$S_{\text{gauge}} = \int \mathrm{Tr}\, \mathcal{R} \wedge *\mathcal{R}$$

but this is not the right action

A metric is implicitly understood in the Hodge-$*$ operation, and this has to be extracted from the connection $\mathcal{A}$ $\rightarrow$ very complicated functional of ω and e, nothing to do with Einstein-Hilbert

Gravity as (almost) a gauge theory III

SO(4,1) has the same algebra as Spin(4,1) (its double covering)

Above: connection in the fundamental representation of SO(4,1)

Now: connection in the fundamental representation of Spin(4,1)

$$\sigma_{AB} = \frac{1}{2}[\gamma_A, \gamma_B] \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \qquad \mathcal{A} = \frac{1}{2}\mathcal{A}^{AB}\sigma_{AB}$$

$$\mathcal{R} = d\mathcal{A} + \mathcal{A}\wedge\mathcal{A} = \frac{1}{2}(d\mathcal{A}^{AB} + \mathcal{A}^A{}_C \wedge \mathcal{A}^{CB})\sigma_{AB}$$
$$= \frac{1}{2}(R^{ab} - \frac{1}{\ell^2}e^a\wedge e^b)\sigma_{ab} + \frac{1}{\ell}T^a\gamma_a\gamma_5$$

Internal space duality: $\star = -i\gamma_5$ $\qquad -i\gamma_5\sigma_{ab} \propto \epsilon_{abcd}\sigma^{cd}$

$$-\alpha\mathrm{Tr}\,\mathcal{R}\wedge\star\mathcal{R} = -\alpha\epsilon_{abcd}(R^{ab} - \frac{1}{\ell^2}e^a\wedge e^b)\wedge(R^{cd} - \frac{1}{\ell^2}e^c\wedge e^d)$$
$$= \alpha\epsilon_{abcd}\left(-R^{ab}\wedge R^{cd} + \underbrace{2\frac{1}{\ell^2}R^{ab}\wedge e^c\wedge e^d - \frac{1}{\ell^4}e^a\wedge e^b\wedge e^c\wedge e^d}_{\text{EC with cosmological constant}}\right)$$

MacDowell-Mansouri Action

Setting $\alpha = \frac{m_P^2 \ell^2}{4}$, $\ell^2 = \frac{6m_P^2}{\Lambda}$

$$S_{MM} = -\frac{3m_P^4}{2\Lambda}\int \mathrm{Tr}\,\mathcal{R}\wedge\star\mathcal{R} = S_{EC} - \underbrace{\frac{3m_P^4}{2\Lambda}\int R^{ab}\wedge R^{cd}\epsilon_{abcd}}_{\text{Gauss-Bonnet topological term}}$$

Spin(4,1) symmetry broken down to Spin(3,1) explicitly by the Lagrangian

Moving over to Euclidean space:

- SO(n,1), Spin(n,1) $\to$ SO(n+1), Spin(n+1)
- γ-matrices $\{\gamma_\mu\gamma_\nu\} = 2\eta_{\mu\nu}$ $\to$ Euclidean γ-matrices $\{\gamma_\mu\gamma_\nu\} = 2\delta_{\mu\nu}$
- $\eta_{ab} \to \delta_{ab}$, $g_{\mu\nu} = e^a{}_\mu e^a{}_\nu$

$$S_{EC} \to S_{EC}^{(E)} = -\int \left[\frac{m_P^2}{2}R^{ab}\wedge e^c\wedge e^d - \frac{\Lambda}{4!}e^a\wedge e^b\wedge e^c\wedge e^d\right]\epsilon_{abcd}$$

$$S_{MM} \to S_{MM}^{(E)} = \frac{3m_P^4}{2\Lambda}\int \mathrm{Tr}\,\mathcal{R}\wedge\star\mathcal{R}$$

Lattice Formulation

Euclidean MacDowell-Mansouri action ready for lattice discretisation:

[Smolin (1979)]

- can be done both with SO(5) and Spin(5) groups, broken by the Lagrangian down to SO(4) and Spin(4)
- Poincaré gravity can be put on the lattice as well, but non-compact groups are less amenable to numerical treatment

Formulation for Spin(5) [Catterall, Ferrante and Nicholson (2012)]

Hypercubic 4D lattice, assign $U_\mu(n) = e^{iaA_\mu} \in \mathrm{Spin}(5) \cong \mathrm{Sp}(2)$ to link (n, μ)

$$U_\mu(n) = \exp\left\{ ia \left[\omega^{ab}{}_\mu(n) \frac{\sigma^{ab}}{4} + e^a{}_\mu(n) \frac{\sigma^{a5}}{2} \right] \right\} = \exp\left\{ ia \left[\omega_\mu(n) + e_\mu(n) \right] \right\}$$

$$\sigma^{AB} = \frac{1}{2i}[\gamma^A, \gamma^B] \qquad (\ell \text{ reabsorbed in } e^a{}_\mu)$$

Under a Spin(4) gauge transformation:

$$\omega_\mu \sim \text{gauge field } (\rightarrow \text{spin connection}), \; e_\mu \sim \text{vector } (\rightarrow \text{vierbein})$$

Lattice Formulation

Extract vierbeins: Iwasawa decomposition $U_\mu(n) = e^{ia\tilde{e}_\mu(n)} e^{ia\omega_\mu(n)}$

$$\tilde{e}_\mu(n) = \tilde{e}^a{}_\mu(n)\frac{\sigma^{a5}}{2}\,, \qquad \tilde{e}^a_\mu(n) = e^a{}_\mu(n) + \tfrac{1}{2}\omega^{ab}{}_\mu(n) e^b{}_\mu(n) + \ldots$$

$$\tilde{E}_\mu(n) = U_\mu(n)\gamma^5 U^\dagger_\mu(n)\gamma^5 = e^{i2a\tilde{e}_\mu(n)}$$

Define plaquette to extract curvature $(\Delta_\mu f(n) \equiv f(n+\mu) - f(n))$

$$U_{\mu\nu}(n) = U_\mu(n) U_\nu(n+\hat{\mu}) U^\dagger_\mu(n+\hat{\nu}) U^\dagger_\nu(n) = e^{ia^2 F_{\mu\nu} + \ldots}$$

$$F_{\mu\nu} = \Delta_\mu A_\nu - \Delta_\nu A_\mu + i[A_\mu, A_\nu] = R_{\mu\nu} + i[e_\mu, e_\nu] + D_{[\mu} e_{\nu]}\,,$$

$$R_{\mu\nu} = \Delta_\mu \omega_\nu - \Delta_\nu \omega_\mu + i[\omega_\mu, \omega_\nu]\,,$$

$$D_{[\mu} e_{\nu]} = D_\mu e_\nu - D_\nu e_\mu \qquad D_\mu e_\nu \equiv \Delta_\mu e_\nu + i[\omega_\mu, e_\nu]$$

Exploiting the commutation relations of σ^{AB} we find

$$\underbrace{F_{\mu\nu}}_{SO(5)} = \frac{\sigma^{ab}}{4}\left(\underbrace{R^{ab}{}_{\mu\nu}}_{SO(4)} - e^a{}_{[\mu} e^b{}_{\nu]}\right) + \frac{\sigma^{a5}}{2}\underbrace{(D_\mu e_\nu)^a}_{T^a}$$

Continuum Operators

On the lattice we can build operators that respect the local SO(4) gauge invariance exactly, but not diffeomorphisms: on our hypercubic lattice all is left is a discrete subgroup of rotations

We have then to consider in general a continuum theory where only the local invariance, rotations and (kind of) reflections are symmetries.

Allowed operators up to dimension 4 are

$$\mathrm{Tr} e_\mu e_\mu \quad (\mathrm{Tr} e_\mu e_\nu)(\mathrm{Tr} e_\mu e_\nu) \quad \mathrm{Tr}[e_\mu, e_\nu]^2 \quad \mathrm{Tr}\left(D_{[\mu} e_{\nu]}\right)^2 \quad \mathrm{Tr}\left(D_{(\mu} e_{\nu)}\right)^2$$

$$\mathrm{Tr} R_{\mu\nu} i[e_\mu, e_\nu] \qquad \mathrm{Tr} R_{\mu\nu}^2$$

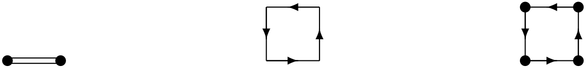
$$\epsilon_{\mu\nu\rho\sigma} \mathrm{Tr} \gamma^5 e_\mu e_\nu e_\rho e_\sigma \quad \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr} \gamma^5 R_{\mu\nu} e_\rho e_\sigma \quad \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr} \gamma^5 R_{\mu\nu} R_{\rho\sigma}$$

Reflections = spatial reflections $x^a \to -x^a$
+ gauge reflection $A_\mu \to \gamma^a A_\mu \gamma^a$

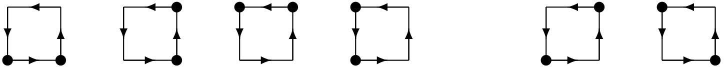
Lattice Operators I

We want Spin(4) gauge-invariant operators $\rightarrow$
trace of products of link variables along closed paths

Since γ^5 commutes with Spin(4) transformations we can place it on sites along the path; reflections require an even number of them



$\mathrm{Tr}\,\tilde{E}_\mu \qquad P_{\mu\nu} = \mathrm{Tr}\,U_{\mu\nu} \qquad \bar{P}_{\mu\nu} = \mathrm{Tr}\,\bar{U}_{\mu\nu}$



$I^{(1)}_{\mu\nu} \quad I^{(2)}_{\mu\nu} \quad I^{(3)}_{\mu\nu} \quad I^{(4)}_{\mu\nu} \qquad J^{(1)}_{\mu\nu} \quad J^{(2)}_{\mu\nu}$

$$G^{(1)}_{\mu\nu} = \mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}\gamma^5 U_{\mu\nu}\gamma^5 \qquad G^{(2)}_{\mu\nu} = \mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}\gamma^5 U^\dagger_{\mu\nu}\gamma^5$$
$$\bar{G}^{(1)}_{\mu\nu} = \mathrm{Re}\,\mathrm{Tr}\,\bar{U}_{\mu\nu}\gamma^5 \bar{U}_{\mu\nu}\gamma^5 \qquad \bar{G}^{(2)}_{\mu\nu} = \mathrm{Re}\,\mathrm{Tr}\,\bar{U}_{\mu\nu}\gamma^5 \bar{U}^\dagger_{\mu\nu}\gamma^5$$

Formal Continuum Limit

Assuming that the small-field regime $U_\mu(n) \simeq \mathbf{1}$ is reached we can formally identify the corresponding continuum operators

$$\begin{aligned}
\mathrm{Tr}\tilde{E}_\mu &= \mathrm{Tr}U_\mu\gamma^5 U_\mu^\dagger\gamma^5 = 4 - 2a^2\mathrm{Tr}e_\mu^2 + \mathcal{O}(3) \\
P_{\mu\nu} &= \mathrm{Re}\,\mathrm{Tr}U_{\mu\nu} = 4 - \tfrac{1}{2}a^4\mathrm{Tr}\left\{(R_{\mu\nu} + i[e_\mu, e_\nu])^2 + \left(D_{[\mu}e_{\nu]}\right)^2\right\} + \mathcal{O}(5) \\
G^{(1)}_{\mu\nu} &= \mathrm{Re}\,\mathrm{Tr}U_{\mu\nu}\gamma^5 U_{\mu\nu}\gamma^5 = 4 - 2a^4\mathrm{Tr}(R_{\mu\nu} + i[e_\mu, e_\nu])^2 + \mathcal{O}(5) \\
G^{(2)}_{\mu\nu} &= \mathrm{Re}\,\mathrm{Tr}U_{\mu\nu}\gamma^5 U_{\mu\nu}^\dagger\gamma^5 = 4 - 2a^4\mathrm{Tr}(D_{[\mu}e_{\nu]})^2 + \mathcal{O}(5) \\
\bar{G}^{(1)}_{\mu\nu} &= \mathrm{Re}\,\mathrm{Tr}\bar{U}_{\mu\nu}\gamma^5 \bar{U}_{\mu\nu}\gamma^5 = 4 - 2a^4\mathrm{Tr}(R_{\mu\nu} - i[e_\mu, e_\nu])^2 + \mathcal{O}(5) \\
\bar{G}^{(2)}_{\mu\nu} &= \mathrm{Re}\,\mathrm{Tr}\bar{U}_{\mu\nu}\gamma^5 \bar{U}_{\mu\nu}^\dagger\gamma^5 = 4 - 2a^4\mathrm{Tr}(D_{(\mu}e_{\nu)})^2 + \mathcal{O}(5)
\end{aligned}$$

and also $\frac{1}{16}\mathrm{Tr}(\tilde{E}_\mu + \tilde{E}_\mu^\dagger - 2)(\tilde{E}_\nu + \tilde{E}_\nu^\dagger - 2) = a^4\mathrm{Tr}e_\mu^2 e_\nu^2 + \mathcal{O}(3)$

Ten operators, seven independent, allow to extract all the continuum ones not involving the antisymmetric tensor

Lattice Operators II

$$\mathrm{Tr}\gamma^5 \tilde{E}_\mu \tilde{E}_\nu \tilde{E}_\rho \tilde{E}_\sigma = 96a^4 \det e + \mathcal{O}(5)$$

$$\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\gamma^5 U_{\mu\nu} U_{\rho\sigma} = a^4 \bigg(\frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\epsilon^{abcd5} R^{ab}{}_{\mu\nu} R^{cd}{}_{\rho\sigma} - \epsilon_{\mu\nu\rho\sigma}\epsilon^{abcd5} R^{ab}{}_{\mu\nu} e^c{}_\rho e^d{}_\sigma + 4! \det e \bigg) + \mathcal{O}(5)$$

$$\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\gamma^5 \bar{U}_{\mu\nu} \bar{U}_{\rho\sigma} = a^4 \bigg(\frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\epsilon^{abcd5} R^{ab}{}_{\mu\nu} R^{cd}{}_{\rho\sigma} + \epsilon_{\mu\nu\rho\sigma}\epsilon^{abcd5} R^{ab}{}_{\mu\nu} e^c{}_\rho e^d{}_\sigma + 4! \det e \bigg) + \mathcal{O}(5)$$

The second operator corresponds to the MacDowell-Mansouri action

For reflection symmetry replace plaquette with clover if required
There are also parity-breaking operators

Comments

- We have written down operators that in the formal continuum limit give back the desired continuum operators - but we do not know where the continuum limit is in parameter space (and if there is one at all)
- The breaking of diffeomorphism invariance led to a lot of allowed operators and a big parameter space
- Parity is not an invariance, only the combined spatial+internal parity transformation is: the quantity $\mathrm{sgn}(\det e^a_\mu)$ should multiply the Lagrangian to have them as separate symmetries
- In particular that would be required to have reflection positivity - but some form of that is still present in certain cases

Attempts at a Lattice Quantisation of Gravity

We can now choose an action and a measure, explore parameter space, and hope for the best (i.e. a second-order phase transition)

There is a natural choice for the measure, namely the Haar measure of Spin(5) - but that is a guess, and powers of $|\det e|$ might be needed

$$Z = \int [DU] e^{-S_{\mathrm{lat}}[U]}$$

We made various attempts using different actions, with similar results nonetheless

Terms different from the MacDowell-Mansouri one cannot be excluded on the lattice, and might be actually required if they are relevant at the critical point

This includes terms that do not respect diffeomorphism invariance (e.g. $\mathrm{Tr} e_\mu^2$): they can be interpreted as being part of the measure

Lattice Action

$$S_{\text{lat}} = \sum_{i=1}^{8} g_i \sum_{\text{lattice}} \mathcal{O}_i$$

$$\mathcal{O}_1 = \mathrm{Tr}\, U_{\mu\nu}(n) \qquad \mathcal{O}_2 = \mathrm{Tr}\gamma^5 U_{\mu\nu}(n) U_{\rho\sigma}(n)\epsilon_{\mu\nu\rho\sigma}$$

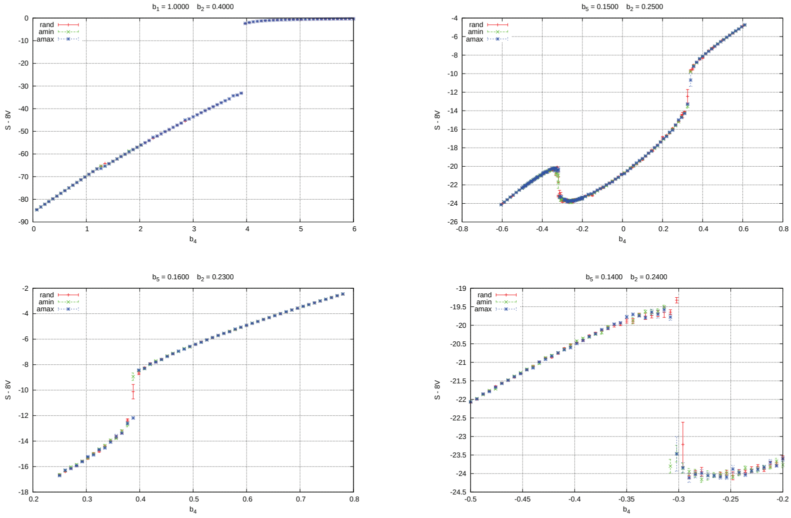
$$\mathcal{O}_3 = \mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(n)\gamma^5 U_{\mu\nu}(n)\gamma^5 \qquad \mathcal{O}_4 = \mathrm{Tr}\, U_{\mu}(n)\gamma^5 U_{\mu}(n)^\dagger \gamma^5$$

$$\mathcal{O}_5 = \mathrm{Tr}\gamma^5 U_{\mu\nu}(n) \bar{U}_{\rho\sigma}(n)\epsilon_{\mu\nu\rho\sigma} \qquad \mathcal{O}_6 = \mathrm{Tr}\gamma^5 \bar{U}_{\mu\nu}(n) U_{\rho\sigma}(n)\epsilon_{\mu\nu\rho\sigma}$$

$$\mathcal{O}_7 = \mathrm{Tr}\gamma^5 \bar{U}_{\mu\nu}(n) \bar{U}_{\rho\sigma}(n)\epsilon_{\mu\nu\rho\sigma} \qquad \mathcal{O}_8 = \mathrm{Tr}\, \bar{U}_{\mu\nu}(n)$$

coupling combination	effect in the continuum
$b_1 = g_1 = -g_8$	quadratic Riemann term removed
$b_2 = g_2 = -g_7$	Gauss-Bonnet and cosmological term removed
$b_4 = g_4$	mass term
$b_5 = g_5 = -g_6$	Gauss-Bonnet term removed: nothing left

Selected Results



Only first-order phase transitions (volume 8^4)

No Cigar

Other attempts:

- only (and independent) EC, cosmological term and Gauss-Bonnet
- action vanishing for vanishing vierbein
- suppressing torsion

In all cases we could only find signs of first-order phase transitions

Other routes not explored yet:

- instead of γ^5 to break Spin(5) in the MM term, use a scalar field Φ^A which develops a vev (spontaneous rather than explicit breaking)
- use different lattice topologies
- work in one dimension higher and study the boundary (M. Caselle at Torino University)

Conclusions and Outlook

Attempts at quantising gravity in the QFT framework have failed so far – and we did not fare any better

- Gauge formulation of gravity is amenable to numerical treatment on the lattice, but a number of problems arise
- No continuum limit found so far
- Keep trying

Open issues:

- all of them

References

- T. Regge, *Nuovo Cim. A* **19** (1961) 558
- G. Immirzi, *Nucl. Phys. Proc. Suppl.* **57** (1997) 65
- B. A. Berg, *Phys. Rev. Lett.* **55** (1985) 904
- H. W. Hamber and R. M. Williams, *Phys. Lett. B* **157** (1985) 368
- H. Römer and M. Zöhringer, *Class. Quantum Grav.* **3** (1986) 897
- M. E. Agishtein and A. A. Migdal, *Mod. Phys. Lett. A* **7** (1992) 1039
- J. Ambjørn, and J. Jurkiewicz, *Phys. Lett. B* **278** (1992) 42
- J. Ambjørn, J. Jurkiewicz and R. Loll, *Nucl. Phys. B* **610** (2001) 347
- J. Ambjørn, A. Görlich, J. Jurkiewicz and R. Loll, *Phys. Rept.* **519** (2012) 127
- L. Smolin, *Nucl. Phys. B* **148** (1979) 333
- C. Mannion and J. G. Taylor, *Phys. Lett. B* **100** (1981) 261
- K. Kondo, *Prog. Theor. Phys.* **72** (1984) 841
- P. Menotti and A. Pelissetto, *Ann. Phys. (N.Y.)* **170** (1986) 287; *Phys. Rev. D* **135** (1987) 1194
- M. Caselle, A. D'Adda and L. Magnea, *Phys. Lett. B* **192** (1987) 406, 411
- S. W. MacDowell and F. Mansouri, *Phys. Rev. Lett.* **38** (1977) 739, 1376
- S. Caracciolo and A. Pelissetto, *Phys. Lett. B* **193** (1987) 237; *Nucl. Phys. B* **299** (1988) 693; *Phys. Lett. B* **207** (1988) 468
- S. Catterall, D. Ferrante and A. Nicholson, *Eur. Phys. J. Plus* **127** (2012) 101
- F. Zappa, *Waka/Jawaka* (1972); Weather Report, *Black market* (1976); Tortoise, *Millions now living will never die* (1996)