

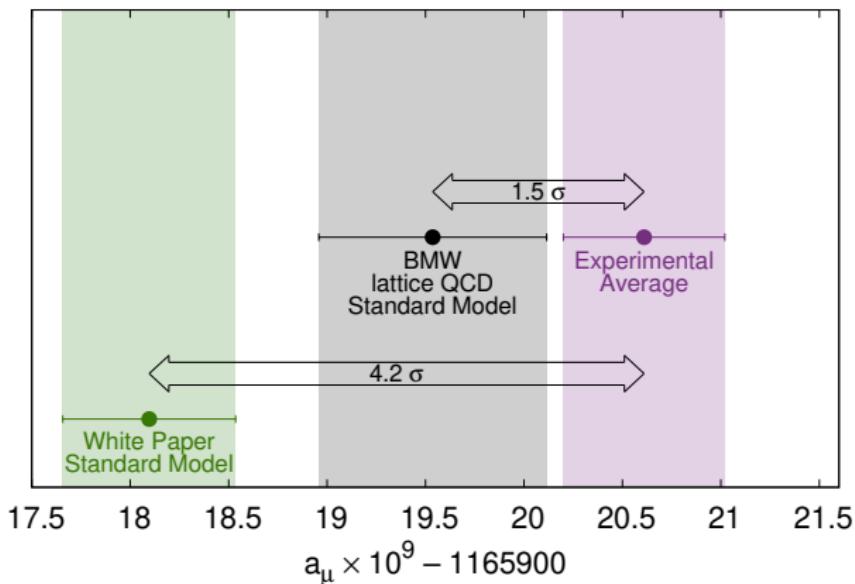
$(g - 2)_\mu$ from lattice QCD and experiments: 4.2 sigma, indeed?

Zoltan Fodor

Budapest–Marseille–Wuppertal collaboration
(BMW)

11 May, 2021

Tensions in $(g - 2)_\mu$

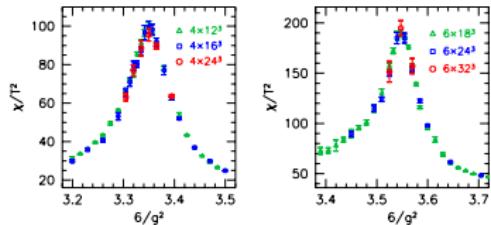


[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

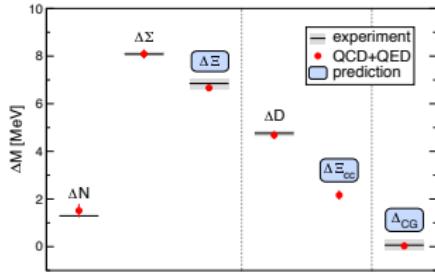
[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

Lattice QCD: examples

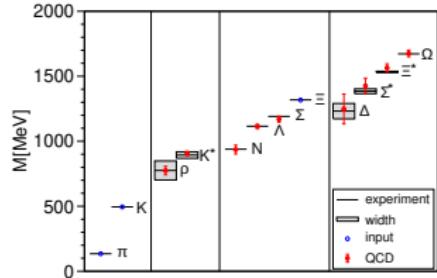
- Wuppertal–Budapest-collaboration,
The order of the quantum chromodynamics transition predicted by the standard model of particle physics,
Nature 443 (2006) 675-678



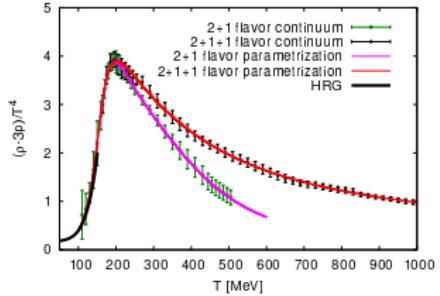
- Budapest–Marseille–Wuppertal-collaboration,
Ab initio calculation of the neutron-proton mass difference, Science 347 (2015) 1452-1455



- Budapest–Marseille–Wuppertal-collaboration,
Ab-initio Determination of Light Hadron Masses,
Science 322 (2008) 1224-1227

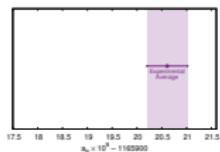


- Wuppertal–Budapest-collaboration,
Lattice QCD for Cosmology, Nature 539 (2016) 7627, 69-71

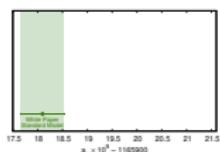


Outline

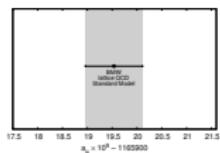
1. $(g - 2)_\mu$



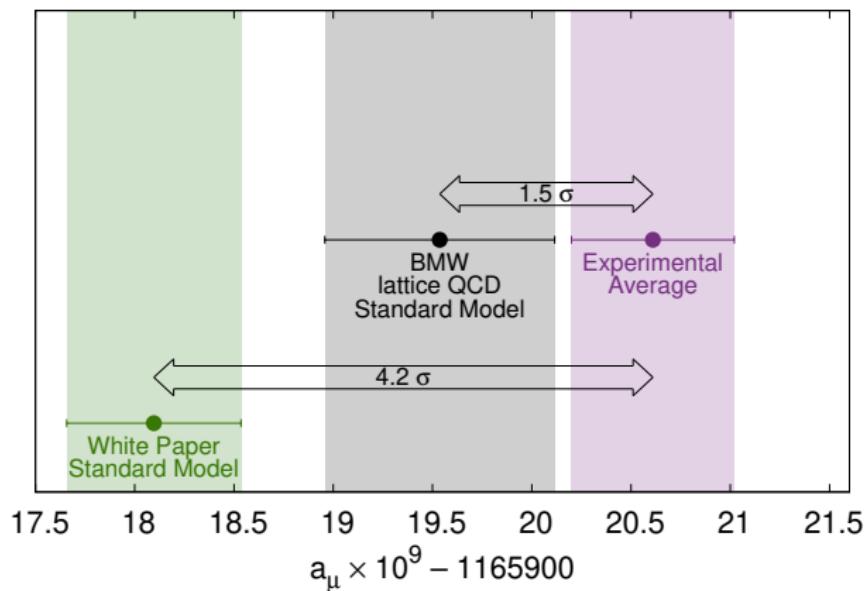
3.



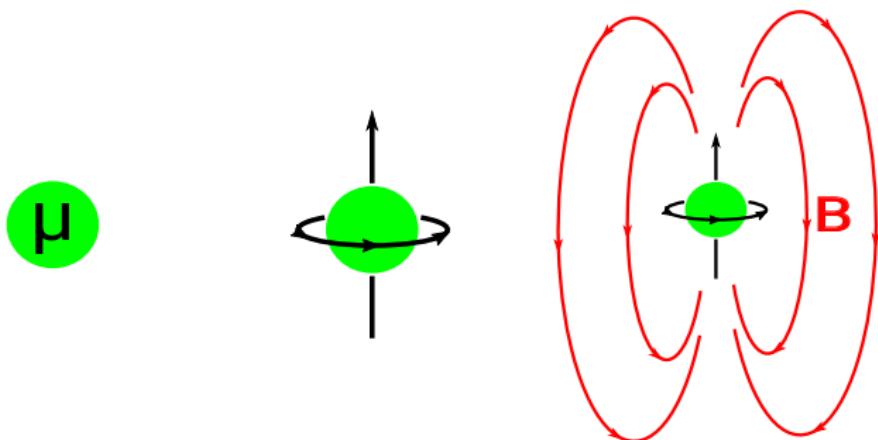
4.



5. Summary



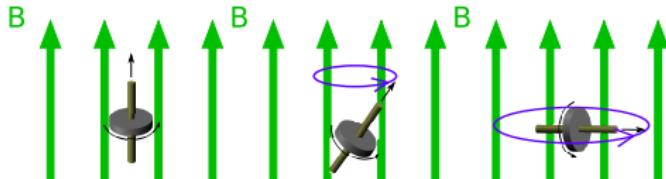
The muon



- Subatomic particle, first discovered in 1936
- Created cosmic rays entering the atmosphere
- Elementary particle, belongs to the family of charged leptons: e^- , μ^- , τ^-
 - same electric charge
 - same spin ($s = \frac{1}{2}$)
 - ≈ 207 times heavier than electron
 - finite lifetime: $\tau = 2.2 \mu s$
- Acts like a tiny magnet. How strong? $\mu = ?$

Anomalous magnetic moment

- Charged top, with homogeneous charge and mass distribution, Q, M



- Circular current \rightarrow magnetic moment μ

$$\mu = \frac{Q}{2M} \mathbf{L}$$

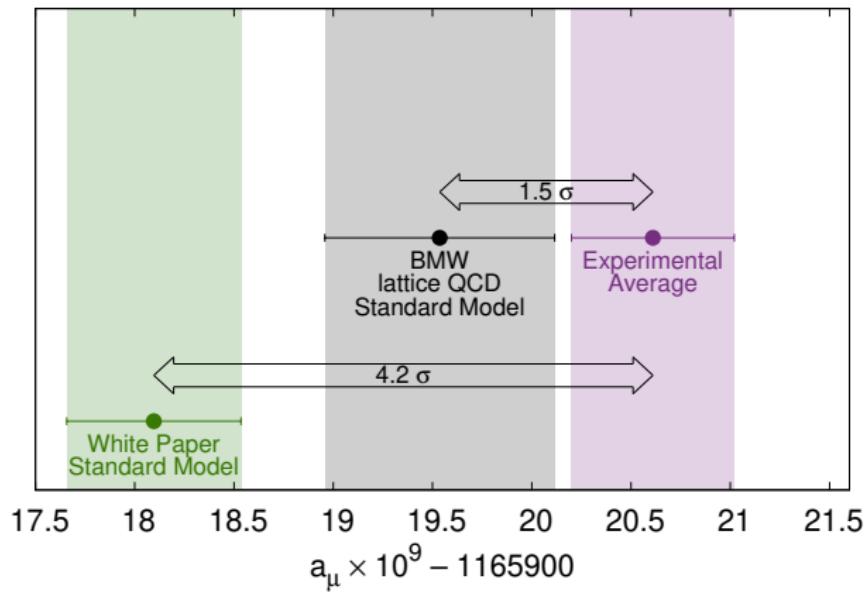
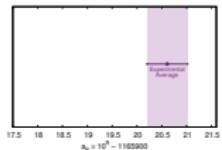
- μ is proportional to angular momentum \mathbf{L}
- Muon has spin $\frac{1}{2}$ \rightarrow described by the Dirac equation

$$\mu_\mu = 2 \frac{e}{2m_\mu} \mathbf{S}_\mu = g \frac{e}{2m_\mu} \mathbf{S}_\mu$$

- Quantum mechanics predicts: $g = 2$
- In reality: $g_\mu \approx 2.00233\dots$
- Deviation: Anomalous Magnetic Moment $a = \frac{g - 2}{2}$

Outline

2.



Experimental result

- Newly announced result at Fermilab

$$a_\mu(\text{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46 \text{ ppm})$$

- Equivalent to: bathroom scale sensitive to weight of a single eyelash.



- Fully agrees with the BNL E821 measurement

$$a_\mu(\text{BNL}) = 11\,659\,209.1(6.3) \cdot 10^{-10} \quad (0.54 \text{ ppm})$$

$$a_\mu(\text{combined}) = 11\,659\,206.1(4.1) \cdot 10^{-10} \quad (0.35 \text{ ppm})$$

- Target uncertainty: (1.6)

History of $(g - 2)_\mu$ experiments

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL	1997	μ^+	11 659 251(150)	13
BNL	1998	μ^+	11 659 191(59)	5
BNL	1999	μ^+	11 659 202(15)	1.3
BNL	2000	μ^+	11 659 204(9)	0.73
BNL	2001	μ^-	11 659 214(9)	0.72
Average			11 659 208.0(6.3)	0.54

- Brookhaven had the \$30 million magnet
- Fermilab had the most intense beam of muons in the world
- Bring the 14 m diameter magnet ring to Fermilab

The route

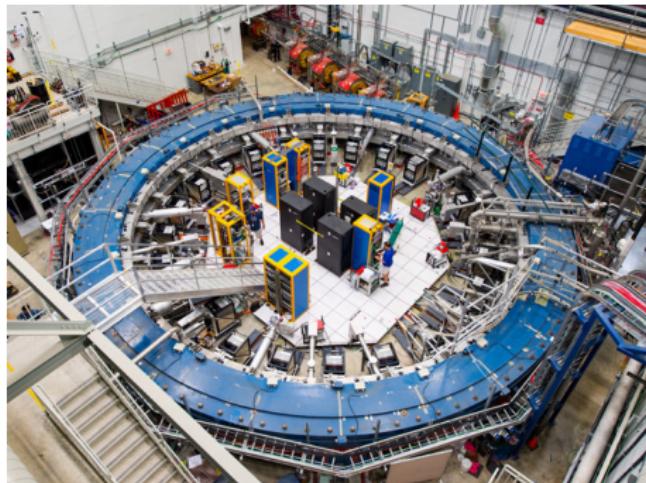


Transportation



Experiment at FNAL

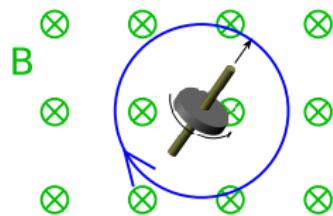
- Storage ring at Fermilab



- Ring diameter: 14 m
- Homogeneous magnetic field: $B = 1.45 \text{ T}$
- Momentum of the muon: $p_\mu = 3.094 \text{ GeV}$

Measurement principle

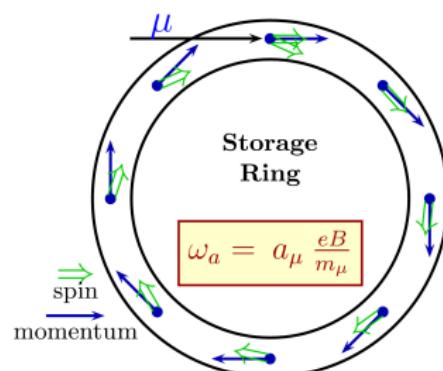
- Cyclotron motion frequency: $\omega_c = \frac{eB}{m_\mu \gamma}$ with $\gamma = \frac{1}{\sqrt{1 - v^2}}$
- Spin precession frequency: $\omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}$



- Measure: $\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m_\mu}$

- Gives $a_\mu = \frac{g_\mu - 2}{2}$ directly

- During each circle spin axis changes 12'

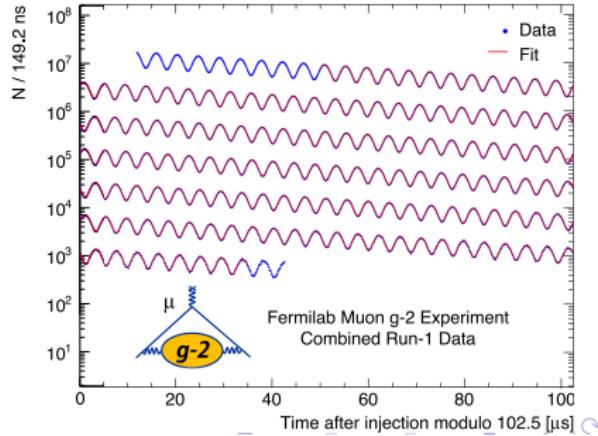


Measurement principle

- Pions produced at fixed target $p + p \rightarrow p + n + \pi^+$
- Pion decays through weak interaction \rightarrow highly polarized muons

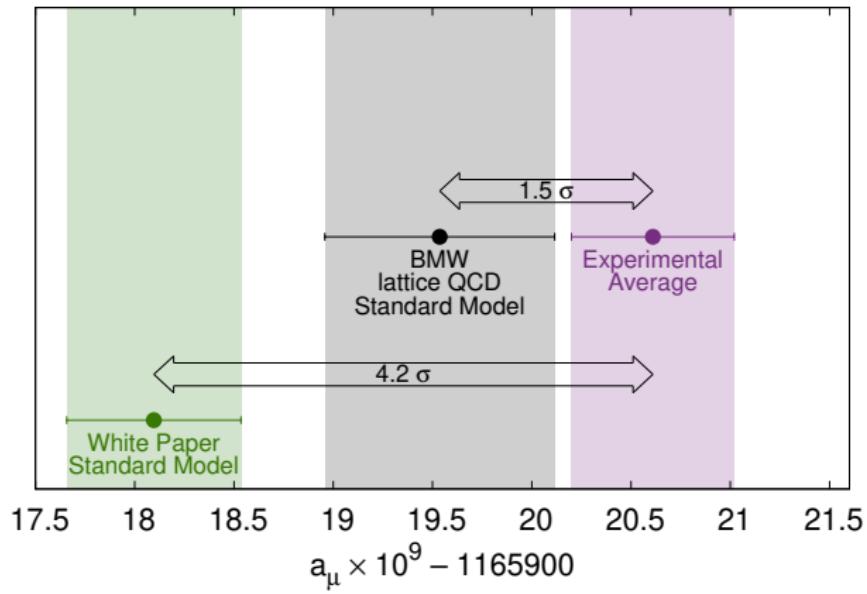
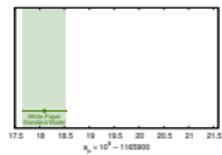
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

- Muons decay after several circles: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- Strong correlation between μ^+ spin and e^+ momentum
- Detect emitted e^+

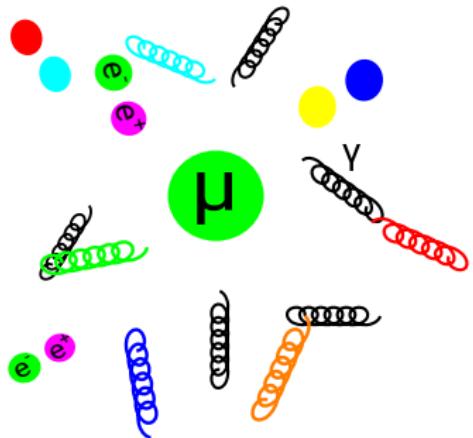


Outline

3.



Theory: Standard Model



Sum over all known physics:

- ➊ quantum electrodynamics (QED): photons, leptons
- ➋ electroweak (EW): W, Z bosons, neutrinos, Higgs
- ➌ strong (QCD): quarks and gluons

- ➎ [2006.04822] White Paper of Muon g-2 Theory Initiative

	$a_\mu \times 10^{-10}$
QED	11658471.9(0.0)
electroweak	15.4(0.1)
strong	693.7(4.3)
total	11659181.0(4.3)

Theory: QED

- $\alpha = \frac{e^2}{4\pi} \ll 1 \rightarrow$ rapidly converging series, was a key to success of QED

$$\left(\frac{g-2}{2}\right) = \left(\frac{\alpha}{\pi}\right) a^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 a^{(3)} + \dots$$

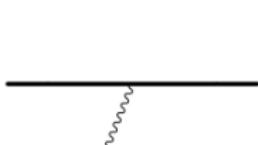
- all contributions with photons and leptons (e, μ, τ)

n-loop	$a_\mu^{\text{QED}} \times 10^{-10}$
1	11614097.330(0.008)
2	41321.762(0.010)
3	3014.190(0.000)
4	38.081(0.030)
5	0.448(0.140)
total	11658471.811(0.160)

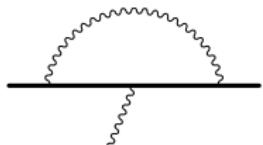
- inputs are m_μ/m_e , m_μ/m_τ and α

Theory: QED

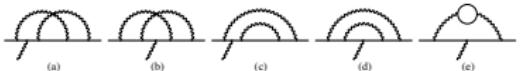
$$\alpha^0 \rightarrow g = 2$$



$$\alpha^1 \rightarrow \alpha/(2\pi)$$



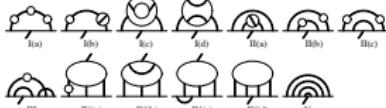
$$\alpha^2$$



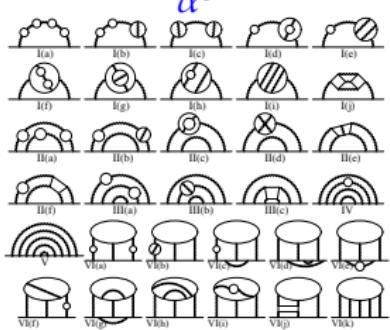
$$\alpha^3$$



$$\alpha^4$$



$$\alpha^5$$

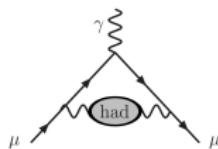


Five loops by [Kinoshita et al '15]

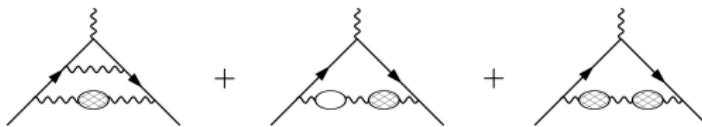
- 12672 diagrams
- automated diagram generation, numerical evaluation of integrals, only some diagrams known analytically
- not yet confirmed by other groups

Hadronic contributions

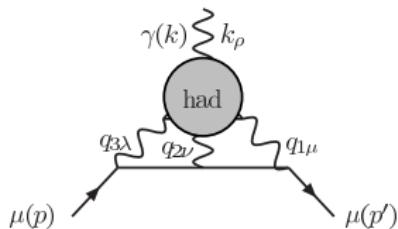
- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



- NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)



- Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



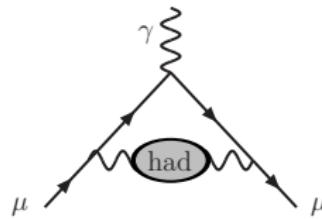
- pheno $a_\mu^{\text{HLbL}} = 9.2(1.9)$

[Colangelo, Hoferichter, Kubis, Stoffer et al '15-'20]

- lattice $a_\mu^{\text{HLbL}} = 7.9(3.1)(1.8) \text{ or } 10.7(1.5)$ [RBC/UKQCD '19 and Mainz '21]

$O(\alpha^2)$ hadronic contribution

- Hadronic vacuum polarization (HVP) of photon



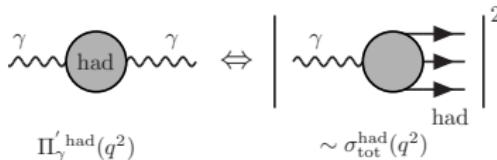
- In QED, $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137} \ll 1 \rightarrow$ rapidly converging series

$$\left(\frac{g-2}{2} \right) = \left(\frac{\alpha}{\pi} \right) a^{(1)} + \left(\frac{\alpha}{\pi} \right)^2 a^{(2)} + \left(\frac{\alpha}{\pi} \right)^3 a^{(3)} + \dots$$

- In QCD, at low energies: $\alpha_s = O(1)$
- Perturbative approach fails

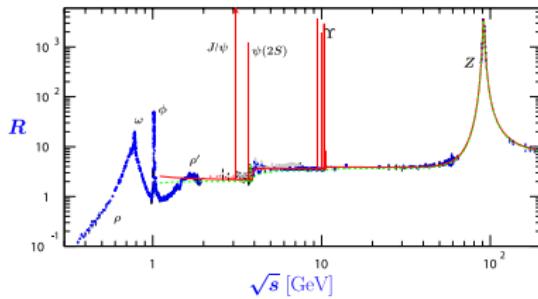
HVP from R-ratio

- Optical theorem



Use $e^+e^- \rightarrow \text{had}$ data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

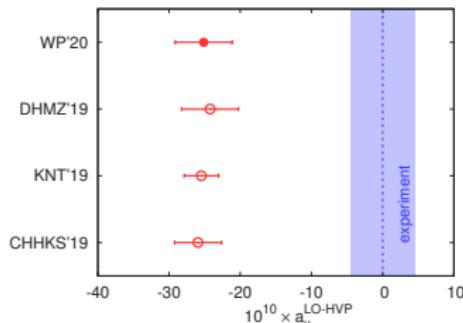
$$a_{\mu}^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_{\mu}(s) R(s)$$



LO	[Jegerlehner '18]	688.1(4.1)	0.60%
LO	[Davier et al '19]	693.9(4.0)	0.58%
LO	[Keshavarzi et al '19]	692.78(2.42)	0.35%
LO	[Hoferichter et al '19]	692.3(3.3)	0.48%
NLO	[Kurz et al '14]	-9.87(0.09)	
NNLO	[Kurz et al '14]	1.24(0.01)	

Discrepancy

- $a_\mu^{\text{exp}} - a_\mu^{\text{theory}} = 25.1(6.0)$ around 4.2σ significance



error budget:

$$(4.1)_{\text{exp}}(0.1)_{\text{QED}}(0.1)_{\text{weak}}(4.0)_{\text{HVP}}(1.8)_{\text{HLbL}}$$

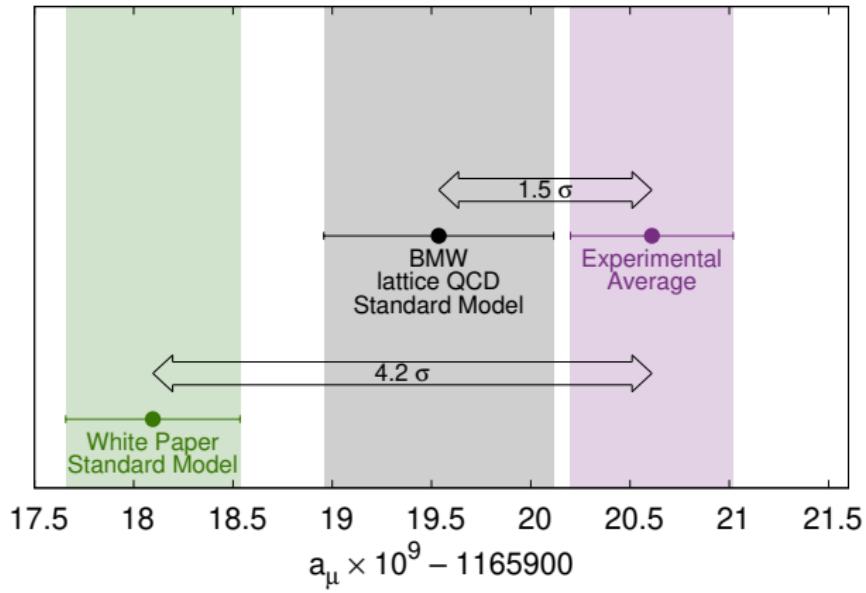
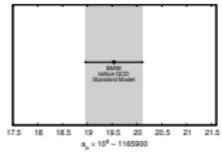
- HUGE: is about 2x electroweak contribution

- For new physics:
- FNAL(plan) + same theory errors 6σ
 - FNAL(plan) + HLbL 10% + HVP 0.2% 11σ

- For no new physics:
- 4% larger HVP, $a_\mu^{\text{LO-HVP}} = 720.0(6.8)$
 - 360% larger HLbL, $a_\mu^{\text{HLbL}} = 37.9(7.1)$

Outline

4.



Lattice QCD

- Integrate over all classical field configurations

$$\int [dU] [d\bar{\psi}] [d\psi] O e^{-S_g(U) - \bar{\psi} M(U) \psi}$$

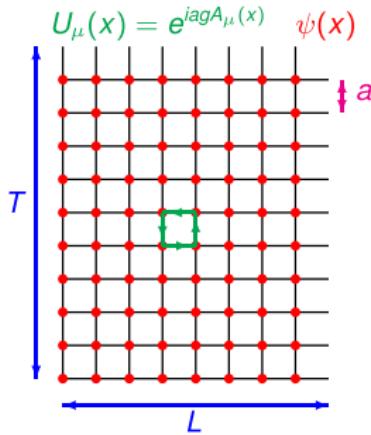
- E.g. $96^3 \times 144$ lattice $\longrightarrow \approx 4 \cdot 10^9$ dimensional integral
- Stochastic integration



- 100000 years for a laptop \longrightarrow 1 year for supercomputer

Lattice QCD

- Lattice gauge theory: systematically improvable, non-perturbative, 1st principles method
- Discretize space-time with lattice spacing: a



- quarks on sites, gluons on links
- discretize action + operators

$$\int d^4x \rightarrow a^4 \sum_x$$

$$\partial_\mu \rightarrow \text{finite differences}$$

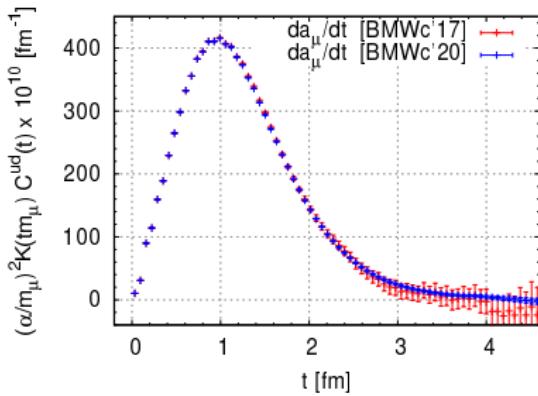
- To get physical results, need to perform:
 - 1 Infinite volume limit ($V \rightarrow \infty$) → numerically or analytically
 - 2 Continuum limit ($a \rightarrow 0$) → min. 3 different a

$a_\mu^{\text{LO-HVP}}$ from lattice QCD

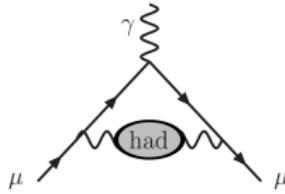
- Compute electromagnetic current-current correlator

$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt K(t) C(t)$$



$K(t)$ describes the leptonic part of diagram



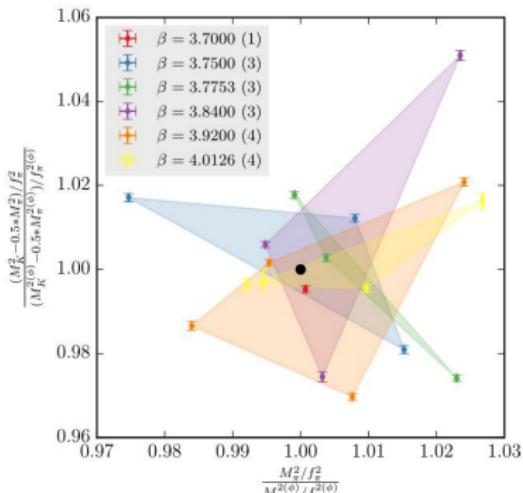
Simulation setup

- 6 lattice spacings: $0.13 \text{ fm} - 0.064 \text{ fm}$ \rightarrow controlled continuum limit
- Box size: $L \sim 6 \text{ fm}$

$L \sim 11 \text{ fm}$ at one lattice spacing \rightarrow FV effects

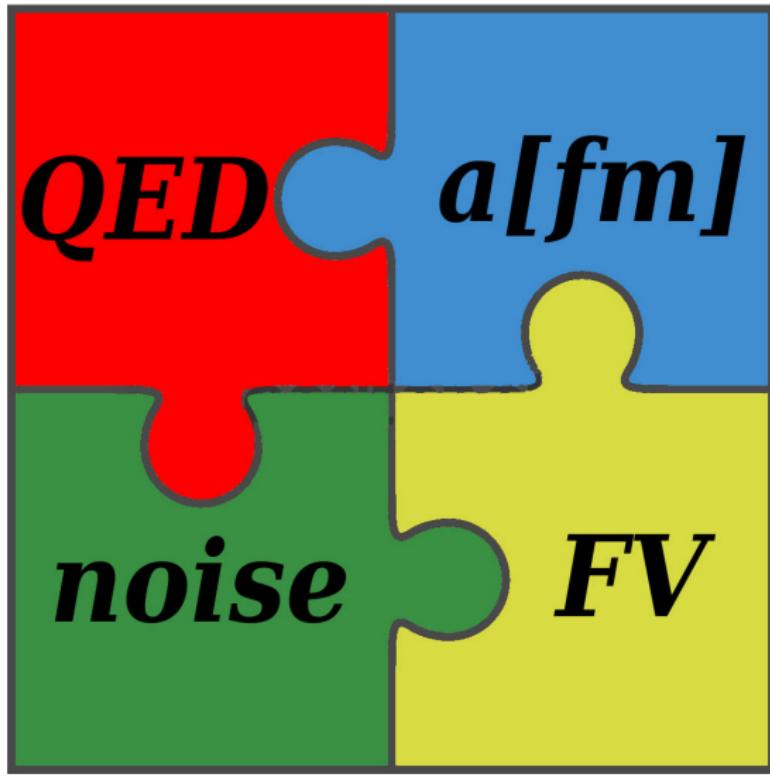
$1 \text{ fm} = 10^{-15} \text{ m} \sim \text{size of proton}$

- Quark masses bracketing their physical values



β	$a[\text{fm}]$	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980

New challenges



Scale determination

Lattice spacing a enters into a_μ determination:

- physical value of m_μ
- physical values of m_π, m_K

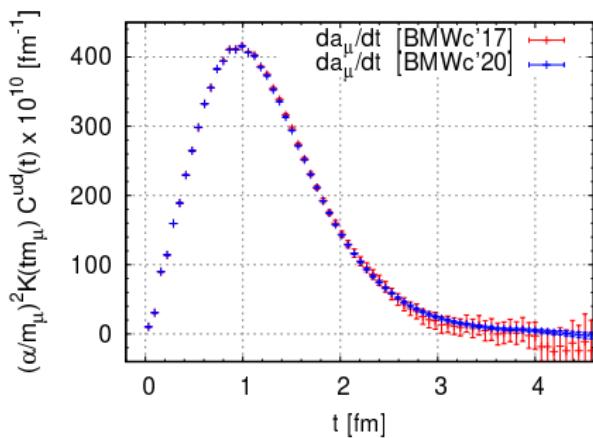
$$\rightarrow \Delta_{\text{scale}} a_\mu \sim 2 \cdot \Delta(\text{scale})$$

- ① For final results: M_Ω scale setting $\rightarrow a = (aM_\Omega)^{\text{lat}} / M_\Omega^{\text{exp}}$
 - Experimentally well known: 1672.45(29) MeV [PDG 2018]
 - Moderate m_q dependence
 - Can be precisely determined on the lattice
- ② For separation of isospin breaking effects: w_0 scale setting
 - Moderate m_q dependence
 - Can be precisely determined on the lattice
 - No experimental value
 → Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0) \rangle$ grows for large distances



- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above t_c

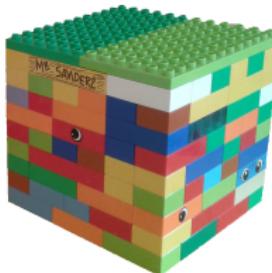
$$0 \leq C(t) \leq C(t_c) e^{-E_{2\pi}(t-t_c)}$$

→ few permil level accuracy on each ensemble

Finite-size effects

- Typical lattice runs use $L < 6 \text{ fm}$, earlier model estimates gave $\mathcal{O}(2)\%$ FV effect.

$$L_{\text{ref}} = 6.272 \text{ fm}$$



$$L_{\text{big}} = 10.752 \text{ fm}$$

- $a_\mu(\text{big}) - a_\mu(\text{ref})$

- perform numerical simulations in $L_{\text{big}} = 10.752 \text{ fm}$
- perform analytical computations to check models

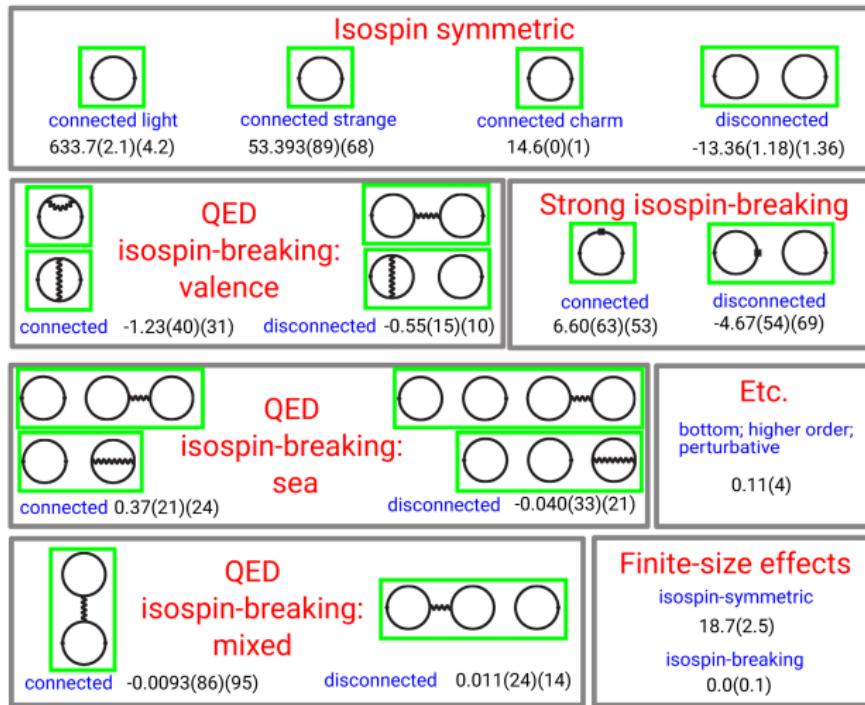
lattice	NLO XPT	NNLO XPT	MLLGS	HP	RHO
$18.1(2.0)_{\text{stat}}(1.4)_{\text{cont}}$	11.6	15.7	17.8	16.7	15.2

- $a_\mu(\infty) - a_\mu(\text{big})$

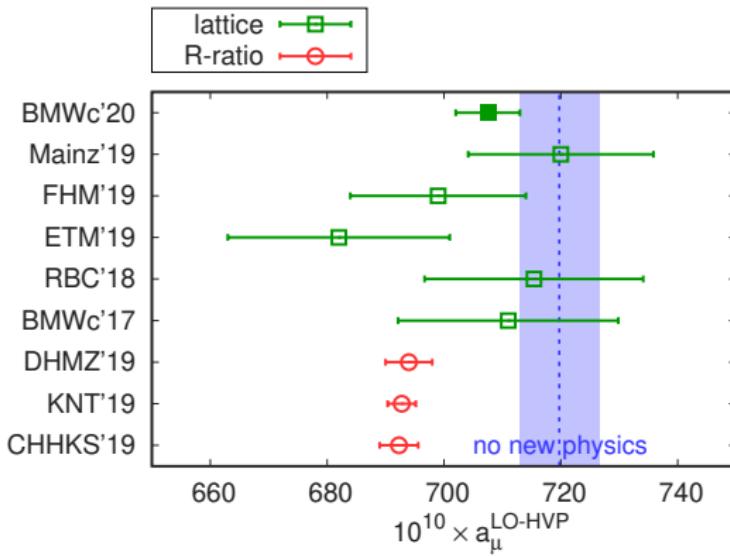
- use models for remnant finite-size effect of “big” $\sim 0.1\%$

Isospin breaking effects

- Include leading order IB effects: $O(e^2)$, $O(\delta m)$



Final result

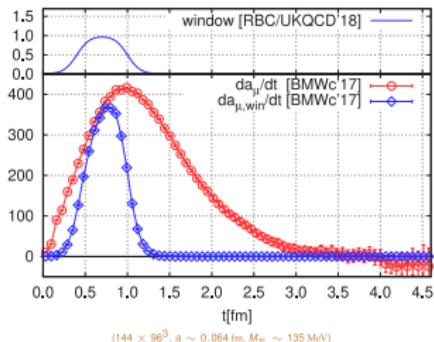


- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with new FNAL experiment
- 2.0σ larger than [DHMZ'19], 2.5σ than [KNT'19]

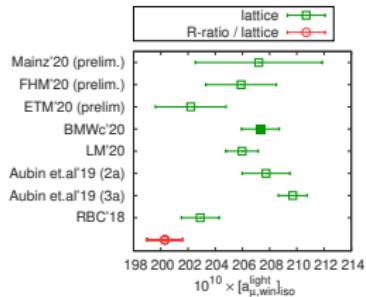
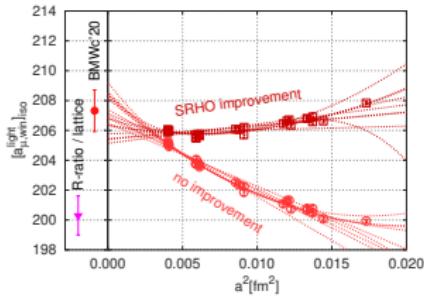
Window observable

- Restrict correlator to window between $t_1 = 0.4 \text{ fm}$ and $t_2 = 1.0 \text{ fm}$

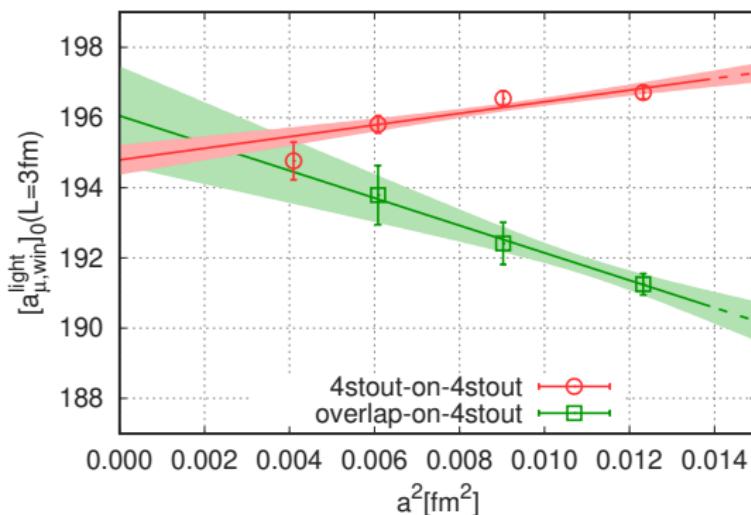
[RBC/UKQCD'18]



- Less challenging than full a_μ
 - signal/noise
 - finite size effects
 - lattice artefacts (short & long)

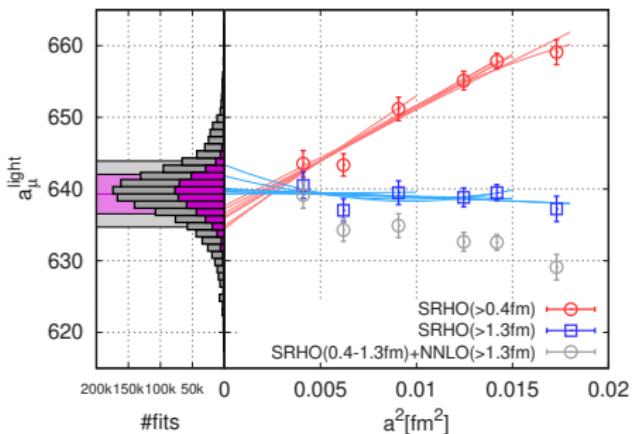
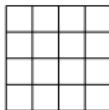


Crosscheck – overlap

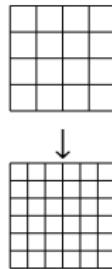
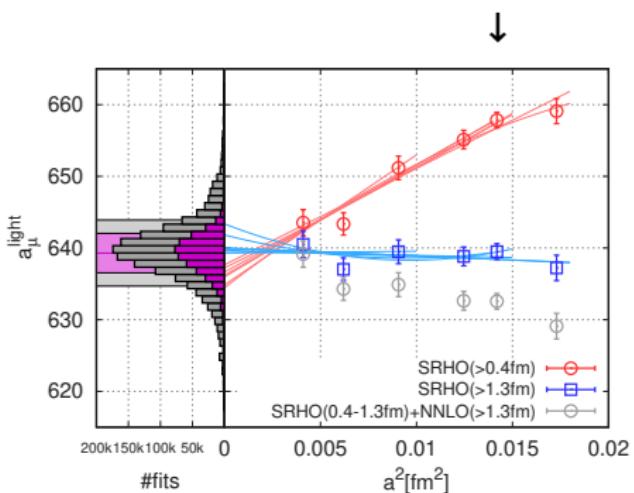


- compute $a_{\mu, \text{win}}$ with overlap valence
- local current instead of conserved \rightarrow had to compute Z_V
- cont. limit in $L = 3 \text{ fm}$ box consistent with staggered valence

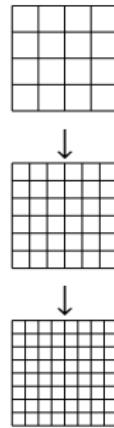
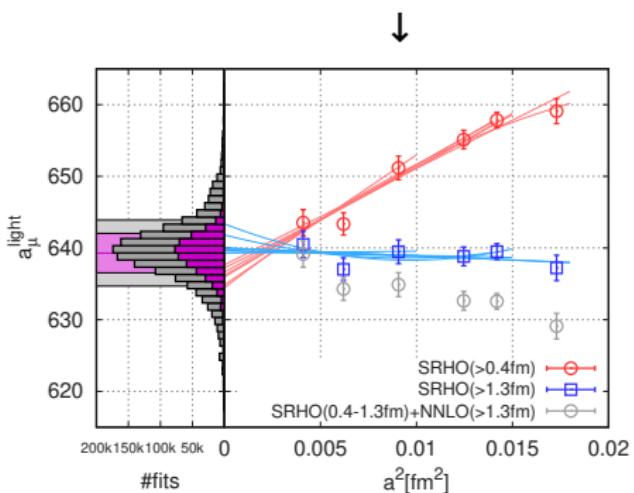
Continuum limit



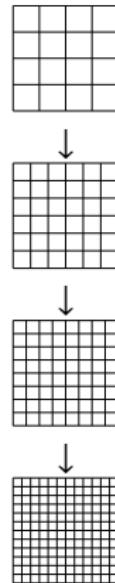
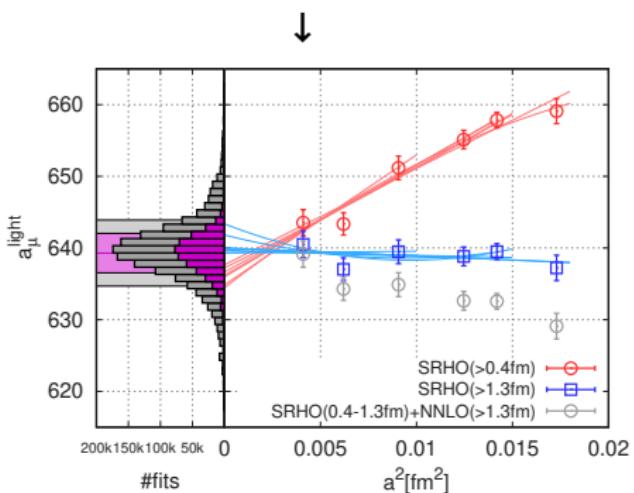
Continuum limit



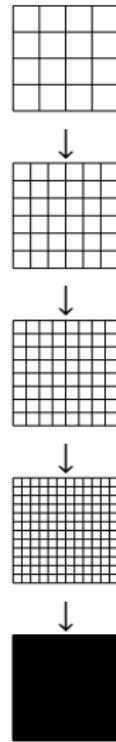
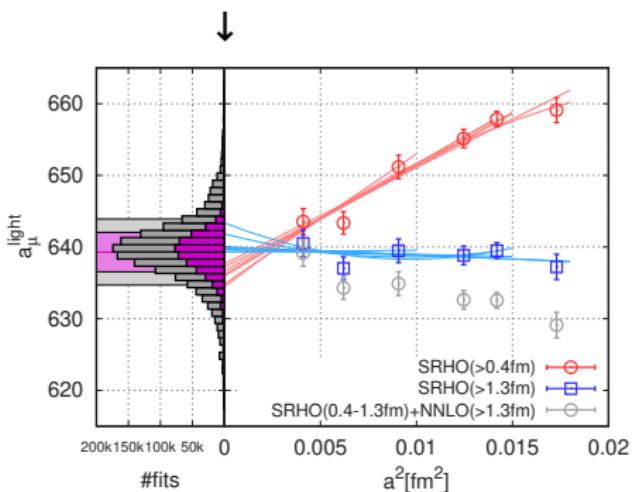
Continuum limit



Continuum limit



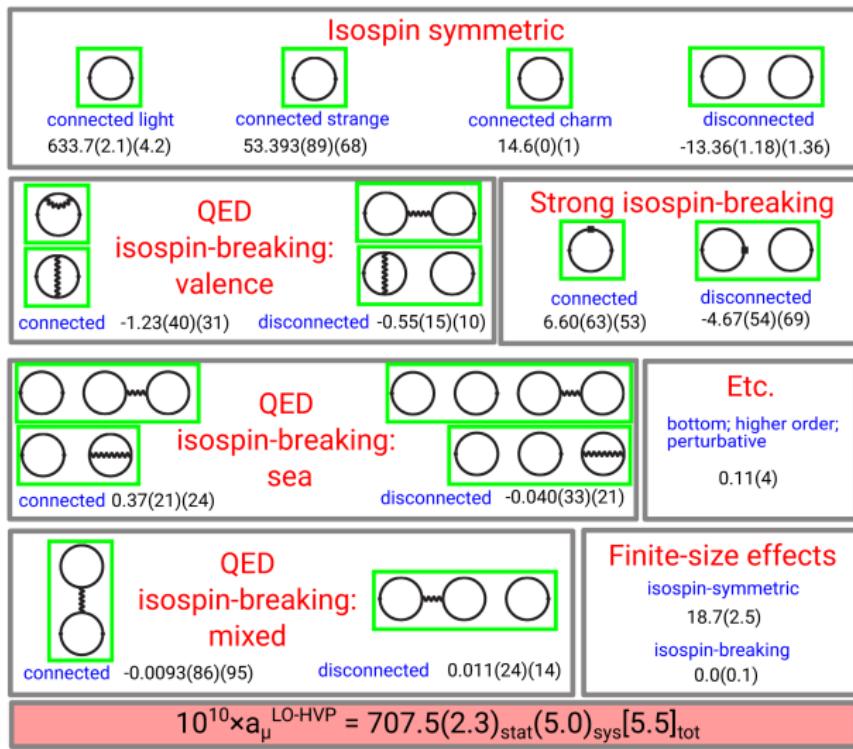
Continuum limit



Outline

5. Summary

Final result



Tensions

