Axial anomaly and hadronic properties in a nuclear medium

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GF & A. Hosaka, Phys. Rev. D **95**, 116011 (2017)
GF & A. Hosaka, Phys. Rev. D **98**, 036009 (2018)

Outline

Motivation

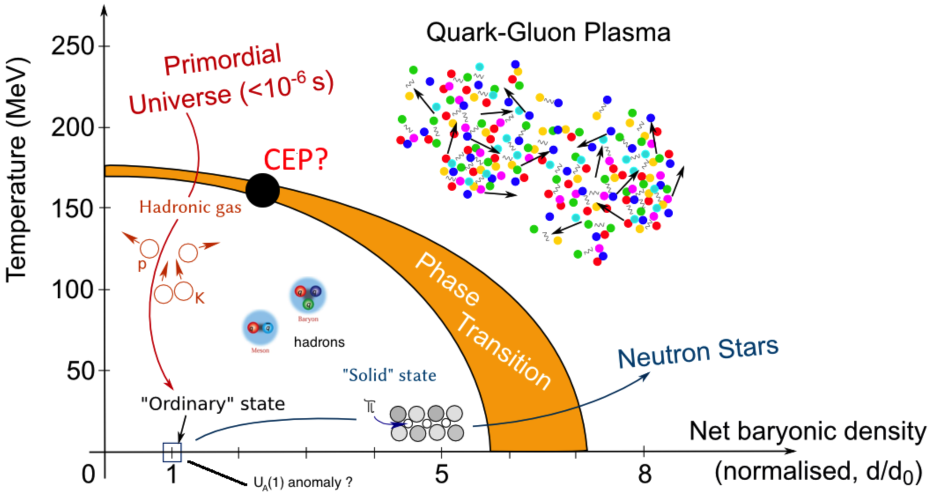
Functional Renormalization Group

Chiral effective nucleon-meson theory at finite μ_B

Numerical results

Summary

Motivation



Temperature (MeV)
250
200
150
100
50
0
Quark-Gluon Plasma
Primordial
Universe (<10^{-6} s)
CEP?
Hadronic gas
p
K
Baryon
Meson
hadrons
Phase Transition
Neutron Stars
"Solid" state
"Ordinary" state
Net baryonic density
(normalised, d/d$_0$)
1
5
8
U$_A$(1) anomaly ?

Motivation

AXIAL ANOMALY OF QCD:

- $U_A(1)$ anomaly: anomalous breaking of the $U_A(1)$ subgroup of $U_L(N_f) \times U_R(N_f)$ chiral symmetry
  $\longrightarrow$ vacuum-to-vacuum topological fluctuations (instantons)

$$\partial_\mu j_A^{\mu a} = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,[T^a F_{\mu\nu} F_{\rho\sigma}]$$

- $U_A(1)$ breaking interactions depend on instanton density
  $\longrightarrow$ suppressed at high T[1]
  $\longrightarrow$ calculations are trustworthy only at high temperature
  $\longrightarrow$ is the anomaly present at the phase transition?
- Very little is known at finite baryochemical potential (μ_B)[2]
  $\longrightarrow$ effective models have not been explored in this direction

[1] R. D. Pisarski, and L. G. Yaffe, Phys. Lett. B**97**, 110 (1980).
[2] T. Schaefer, Phys. Rev. D**57**, 3950 (1998).

η' - NUCLEON BOUND STATE:

- Effective models at finite T and/or density:
  $\longrightarrow$ effective models (NJL[3], linear sigma models[4]) predict a $\sim$150 MeV drop in $m_{\eta'}$ at finite μ_B
- Effective description of the mass drop:
  $\longrightarrow$ attractive potential in medium $\Rightarrow$ $\eta' N$ bound state
  $\longrightarrow$ Analogous to $\Lambda(1405) \sim \bar{K}N$ bound state

[3] P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
[4] S. Sakai & D. Jido, Phys. Rev. C**88**, 064906 (2013).

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- Problem with mean field calculations: they treat model parameters as environment independent constants
  $\longrightarrow$ „$A \cdot v$" type of terms decrease (A-constant, v-decreases)
  $\longrightarrow$ evolution of the „A" anomaly at finite T and μ_B?
- What is the role of fluctuations?

[3] P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
[4] S. Sakai & D. Jido, Phys. Rev. C**88**, 064906 (2013).

- Fluctuation effects in a quantum system is encoded in the effective action
- Partition function and effective action in field theory:
  [$\mathcal{S}$: classical action, ϕ: dynamical variable, $\bar{\phi}$: mean field, J: source field]

$$Z[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)}, \quad \Gamma[\bar{\phi}] = -\log Z[J] - \int J\bar{\phi}$$

- Γ contains the truncated *n-point functions*
  $\longrightarrow$ amplitudes, part. lifetimes, thermodynamics (EoS, etc.)
- How to calculate the effective action? $\Rightarrow$ perturbation theory!
  $\longrightarrow$ find a small parameter in $\mathcal{S}$ and Taylor expand
  $\longrightarrow$ fails in QCD & eff. models are not weakly coupled either
- Non-perturbative methods are necessary:
  Functional Renormalization Group (FRG)[5]

[5] C. Wetterich, Phys. Lett. B**301**, 90 (1993)

Functional Renormalization Group

- Wilsonian Renormalization Group:
  $\longrightarrow$ solves the problem of divergent pert. theory at T_C
  $\longrightarrow$ average over fluctuations but avoid IR singularities

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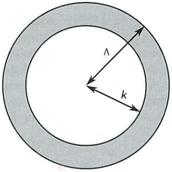
- Momentum-shell integration: we introduce an intermediate scale k and let $\phi(\vec{p}) = \phi^<(\vec{p})\Theta(k - |\vec{p}|) + \phi^>(\vec{p})\Theta(|\vec{p}| - k)$

$$Z = \int \mathcal{D}\phi e^{-S[\phi]} = \int \mathcal{D}\phi^< \int \mathcal{D}\phi^> e^{-S_< - S_> - S_{mix}[\phi^<, \phi^>]}$$

- $\phi^>$ integral is performed using pert. theory



$$Z = \int \mathcal{D}\phi^< e^{-S[\phi^<]}$$

$$S \to S - \log <e^{-[S_>] - S_{mix}[\phi^<, \phi^>]}>_>$$

- Result: $\Lambda \to k$, $m_\Lambda^2 \to m_k^2$, $g_\Lambda \to g_k$ + new vertices

Functional Renormalization Group

- Rescaling dimensionful quantities with k
  $\longrightarrow$ flow equations for individual coupling constants:

$$k\partial_k \bar{m}_k^2 = \beta_{m^2}(\bar{m}_k^2, \bar{g}_k^{(i)}, \ldots) \qquad k\partial_k \bar{g}_k^{(i)} = \beta_{g^{(i)}}(\bar{m}_k^2, \bar{g}_k^{(i)}, \ldots)$$

- Identification of relevant, irrelevant and marginal couplings
  $\rightarrow$ irrelevant op.'s die out, relevant (marginal) ones remain

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- Identification of relevant, irrelevant and marginal couplings
  $\rightarrow$ irrelevant op.'s die out, relevant (marginal) ones remain
- Fixed points: statistically self-similar behavior
  $\rightarrow$ no relevant length scale! (2nd order phase transitions)
- The WRG has had great success in
  $\longrightarrow$ describing universality in 2nd order transitions
  $\longrightarrow$ predicting critical exponents
  $\longrightarrow$ understanding the concept of effective theories

Functional Renormalization Group

- FRG generalizes the idea of the WRG: fluctuations are taken into account at the level of the quantum effective action

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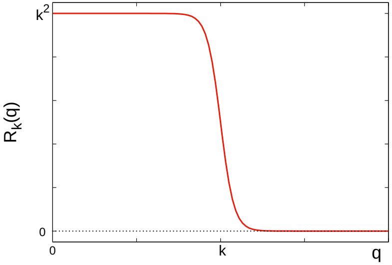
- Introduction of a flow parameter k and inclusion of fluctuations for which $q \gtrsim k$

$$Z_k[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)} \times e^{-\frac{1}{2}\int \phi R_k \phi}$$

$\longrightarrow$ regulator: mom. dep. mass term suppressing low modes

$\longrightarrow$ take the $k \to 0$ limit



- WRG is a *special choice* of FRG (sharp regulator)

Functional Renormalization Group

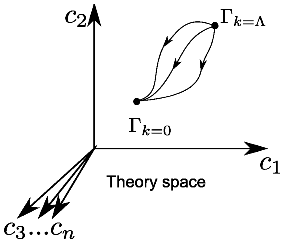
- Scale-dependent effective action:

$$\Gamma_k[\bar\phi] \quad = \quad -\log Z_k[J] - \int J\bar\phi - \frac{1}{2}\int \bar\phi R_k \bar\phi$$

$\longrightarrow k \approx \Lambda$: no fluctuations $\quad \Rightarrow \Gamma_{k=\Lambda}[\bar\phi] = \mathcal{S}[\bar\phi]$

$\longrightarrow k = 0$: all fluctuations $\quad \Rightarrow \Gamma_{k=0}[\bar\phi] = \Gamma[\bar\phi]$

- The scale-dependent effective action interpolates between classical- and quantum effective actions



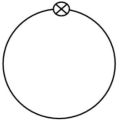
- The trajectory depends on R_k but the endpoint does not

- Choice of $R_k \leftrightarrow$ choice of scheme

Functional Renormalization Group

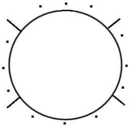
- Flow of the effective action is described by the Wetterich equation:

$$\partial_k \Gamma_k = \frac{1}{2}\int \mathrm{Tr}\left[\partial_k R_k (\Gamma_k^{(2)} + R_k)^{-1}\right] = \frac{1}{2}$$



- Slightly different form: [$\tilde{\partial}_k$ acts only on R_k]

$$\partial_k \Gamma_k = \frac{1}{2}\tilde{\partial}_k \int \mathrm{Tr}\,\log[\Gamma_k^{(2)} + R_k] = \frac{1}{2}\tilde{\partial}_k \sum$$



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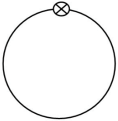

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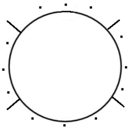

- One-loop structure:

  $\longrightarrow$ RG change in the n-point vertices are described by one-loop diagrams [propagators are dressed!]

  $\longrightarrow$ functional integro-differential equation [exact!]

  $\longrightarrow$ approximations are necessary

- Local potential approximation (LPA):

$$\Gamma_k[\bar{\phi}] = \int_x \left(\frac{Z_k}{2} \partial_i \bar{\phi} \partial_i \bar{\phi} + V_k(\bar{\phi}) \right)$$

$\longrightarrow$ leading order of the derivative expansion

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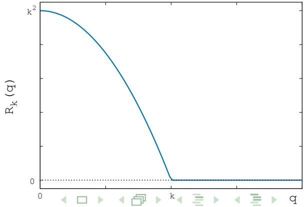
- Regulator optimization: maximize the convergence radius in an amplitude expansion of V_k
  $\longrightarrow$ $R_k^W = \lim_{\Lambda\to\infty} \Lambda^2 \Theta(k^2 - q^2)$ is not optimal!
- Optimal regulator for LPA:
  $R_k(q) = Z_k(k^2 - q^2)\Theta(k^2 - q^2)$



3 FLAVOR CHIRAL NUCLEON-MESON MODEL:

- Effective model of chiral symmetry breaking: order par. M
  [excitations of M: π, K, η, η' and a_0, κ, f_0, σ]

$$\begin{aligned}
\mathcal{L}_M &= \mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] - \mathrm{Tr}\,[H(M^\dagger + M)] \\
&+ V_{ch}(M) + A \cdot (\det M^\dagger + \det M) \\
\mathcal{L}_{\omega+N} &= \frac{1}{4}(\partial_i \omega_j - \partial_j \omega_i)^2 + \frac{1}{2} m_\omega \omega_i^2 + \bar{N}(\partial\!\!\!/ - \mu_B \gamma_0) N, \\
\mathcal{L}_{\mathrm{Yuk}} &= \bar{N}(g_Y \tilde{M}_5 - i g_\omega \omega\!\!\!/) N
\end{aligned}$$

$\longrightarrow$ nucleon mass: entirely from Yukawa coupling

- Goal: calculation of the effective action Γ

$\longrightarrow$ Particular interest: finite μ_B

$\longrightarrow$ How does the anomaly behave toward the nuclear liquid-gas transition?

Chiral effective nucleon-meson theory at finite μ_B

- Fluctuations are included in the quantum effective action Γ_k:

$$\Gamma_k = \int_x \Big(\mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] - \mathrm{Tr}\,[H(M^\dagger + M)] + \bar{N}(\partial\!\!\!/ - \mu_B \gamma_0) N + \frac{1}{4}(\partial_i \omega_j - \partial_j \omega_i)^2 + \frac{1}{2} m_\omega^2 \omega_i^2 + \bar{N}(g_Y \tilde{M}_5 - i g_\omega \omega\!\!\!/) N + \underline{V_k} \Big)$$

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- We fluctuation effects in the mesonic potentials:

$$V_k = \underline{V_{ch,k}(M)} + \underline{A_k}(M) \cdot (\det M^\dagger + \det M)$$

- The chiral potential splits into two parts:

$$V_{ch,k}(M) = V_k^{3fl}(M) + V_k^{2fl}(\tilde{M})$$

- Projecting the flow equation onto chiral invariants lead to flows of $V_k^{3fl}(M)$, $V_k^{2fl}(\tilde{M})$ and $A_k(M)$

$$\partial_k \Gamma_k = \frac{1}{2} \int_{q,p}^{(T)} \mathrm{Tr}\left[\partial_k R_k(q,p)(\Gamma_k^{(2)} + R_k)^{-1}(p,q)\right]$$

Chiral effective nucleon-meson theory at finite μ_B

- Baryon Silver Blaze property:
  $\longrightarrow$ no change in the effective action for $T = 0$ if
  $\mu_B < m_N - B \equiv \mu_{B,c}$

[6] M. Drews and W. Weise, Prog. Part. Nucl. Phys. 93, 69 (2017).

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- At $\mu_B = \mu_{B,c}$:[6]
  $\longrightarrow$ 1st order phase transition from nuclear gas to liquid
  $\longrightarrow$ nuclear density jumps from zero to $n_0 \approx 0.17\,\mathrm{fm}^{-3}$
  $\longrightarrow$ non-strange chiral condensate jumps from f_π to $v_{ns,nucl}$
  (Landau mass $M_L \approx 0.8 m_N \Rightarrow v_{ns,nucl} \approx 69.5\,\mathrm{MeV}$)

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- The first order transition is related to the condensation of the timelike component of the ω vector particle

- ω couples to v_{ns} that couples to v_s
  $\longrightarrow$ jump in all order parameters

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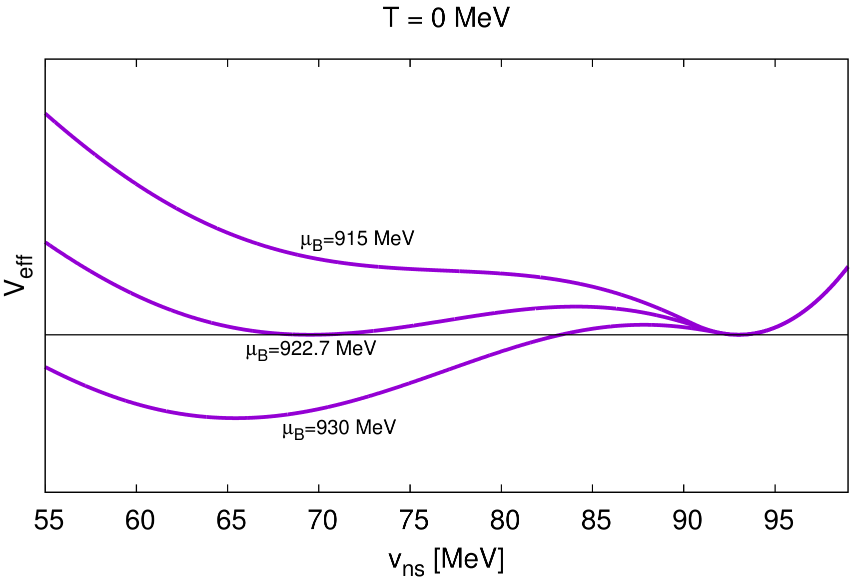
PARAMETRIZATION:

- The model consists of the following parameters:
  $\rightarrow V^{3fl}(M)$: m^2, g_1, g_2
  $\rightarrow V^{2fl}(\tilde{M})$: b_i $(i = 1..4)$ [non-renormalizable interactions!]
  $\rightarrow$ explicit breaking, anomaly: h_0, h_8, A
  $\rightarrow \omega + N$: $g_\omega^2/m_\omega^2, g_Y$
- 12 parameters in total. Input:
  $\rightarrow$ masses in the vacuum: m_π, m_K, m_η, $m_{\eta'}$, m_{a_0}, m_N
  $\rightarrow$ normal nuclear density: n_0
  $\rightarrow$ critical chemical potential: $\mu_{B,c}$
  $\rightarrow$ nucleon mass drop in the medium: Δm_N
  $\rightarrow$ 2 PCAC relations (decay constants f_π, f_K)
  $\rightarrow$ temperature of the critical endpoint T_{CEP}
  [Compression modulus: prediction! $K = \frac{9n_0}{\partial n_0/\partial \mu_B} \approx 287\,\mathrm{MeV}$]

Numerical results



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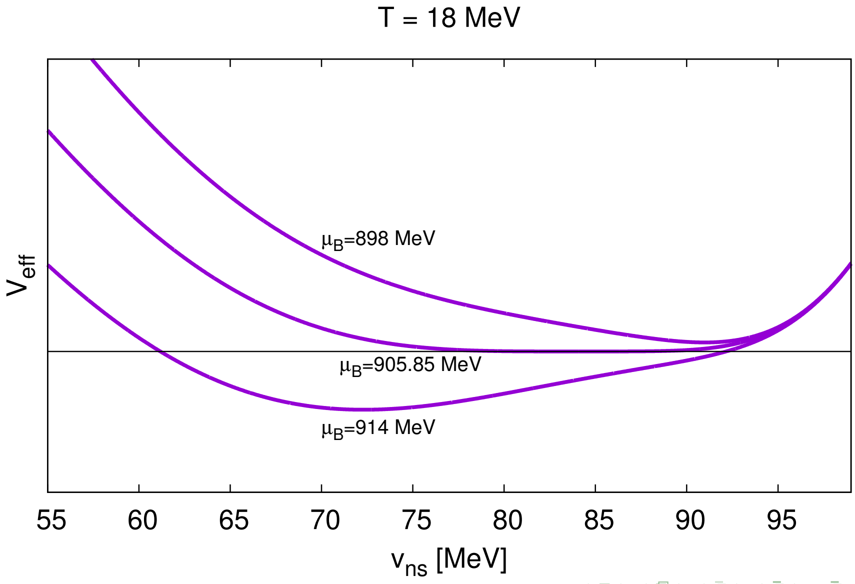

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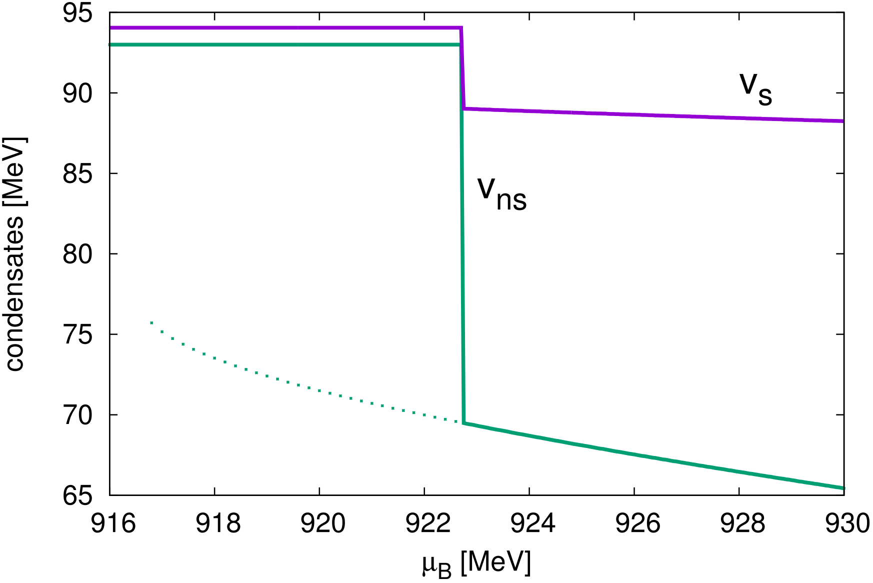

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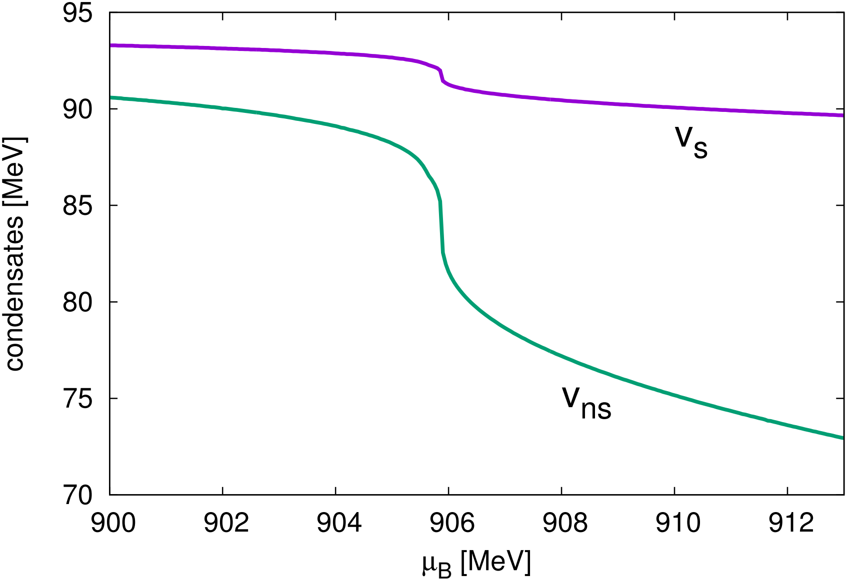

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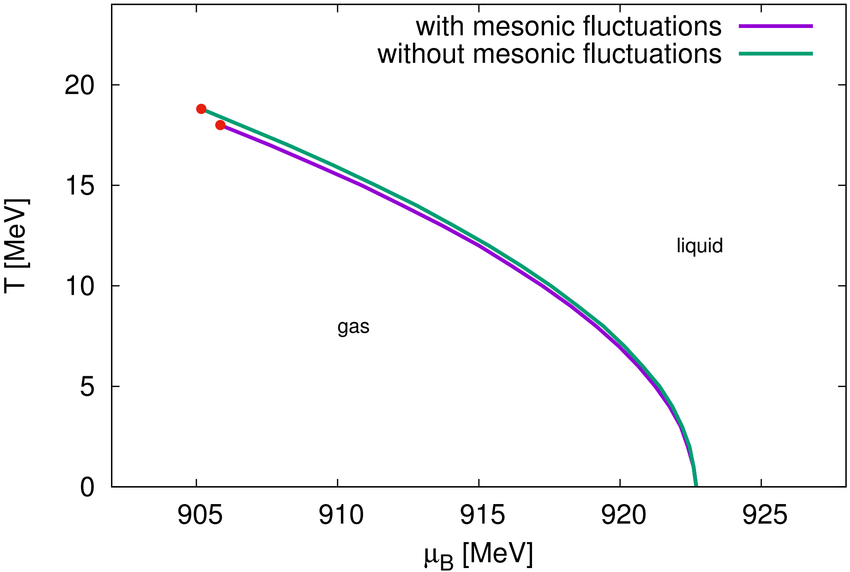

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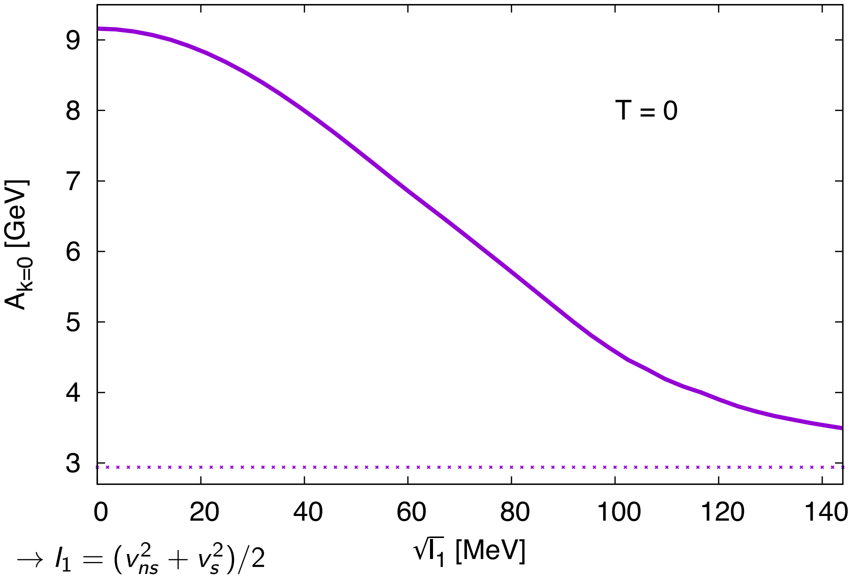
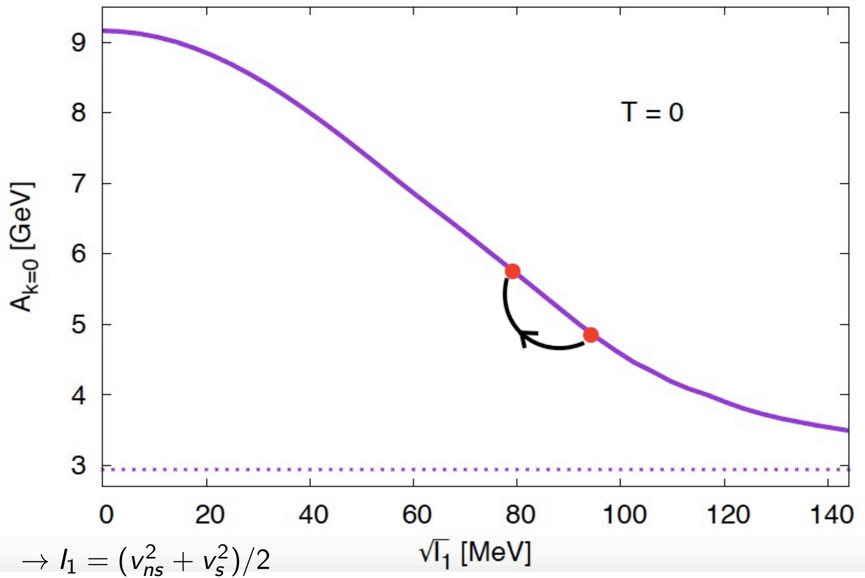


$\rightarrow I_1 = (v_{ns}^2 + v_s^2)/2$



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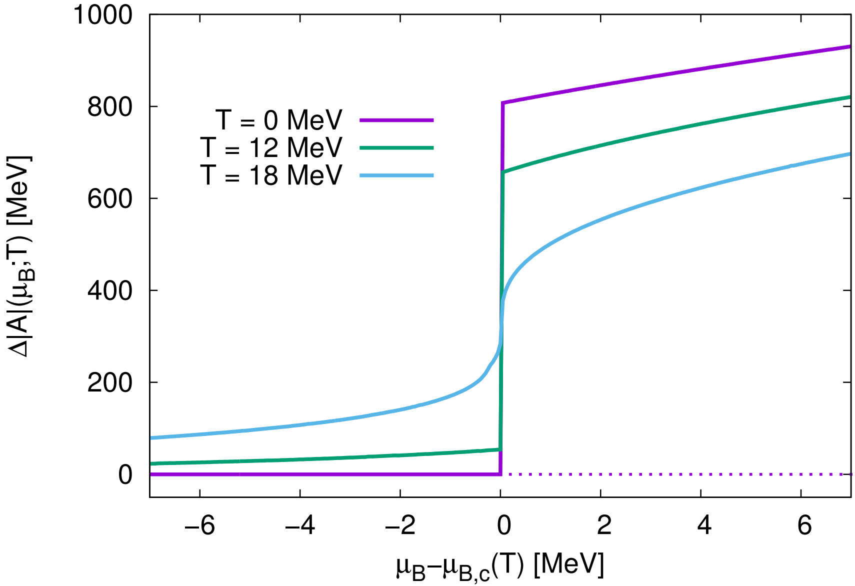

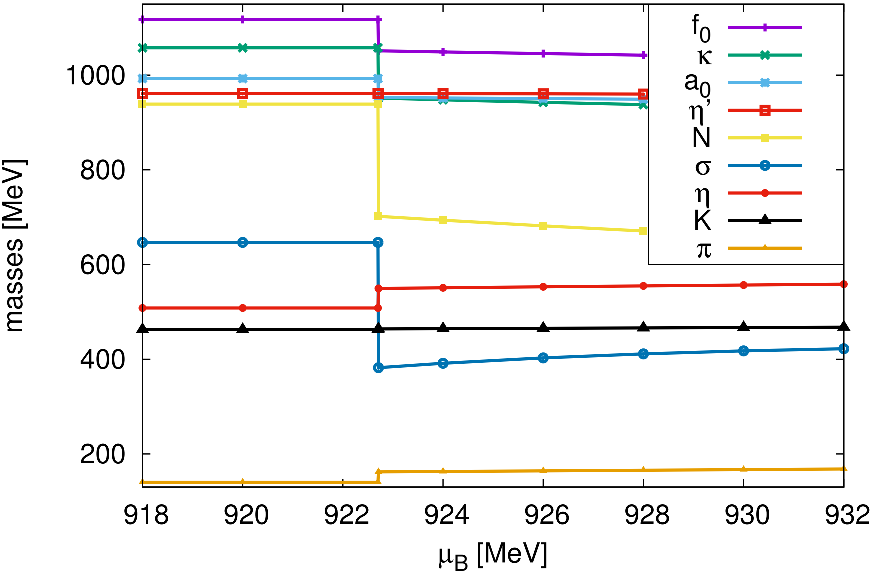

masses [MeV]
μ_B [MeV]
1000
800
600
400
200
918
920
922
924
926
928
930
932
f_0
κ
a_0
η'
N
σ
η
K
π

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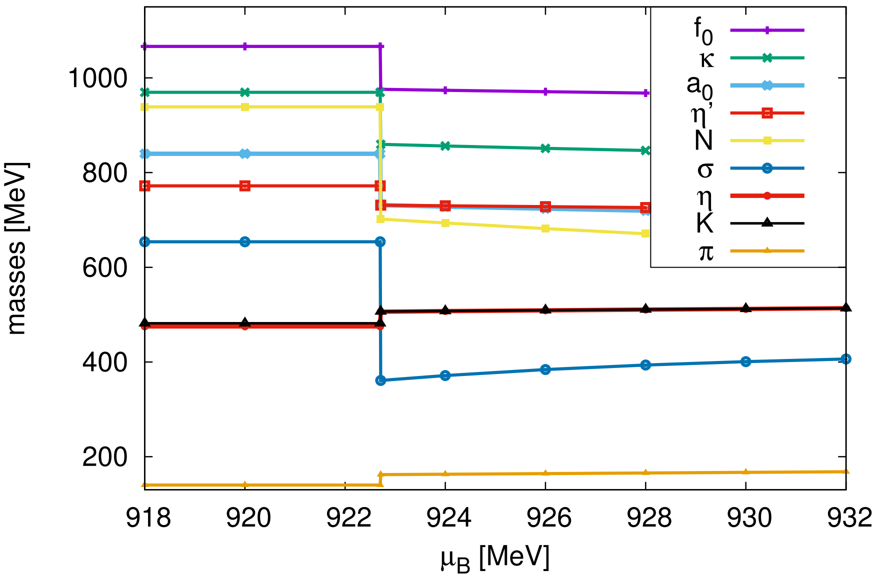

Numerical results

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- Current effect: mesonic quantum fluctuations, not instanton contributions
  $\longrightarrow$ backreaction of the anomaly on itself
  $\longrightarrow$ mean field theory is questionable
- Even the bare anomaly coefficient A can depend explicitly on T and μ_B!
  $\longrightarrow$ competition between instantons and mesonic loop effects
  $\longrightarrow$ extension: assume a form of $A = A(T, \mu_B)$

Summary

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- Findings:

  $\longrightarrow$ mesonic fluctuations make the anomaly coefficient condensate dependent

  $\longrightarrow$ (partial) restoration of chiral symmetry seem to increase the anomaly ($\Delta |A| \gtrsim 15\%$ relative difference)

  $\longrightarrow$ nuclear transition: $\sim 20\%$ drop in (n.s.) chiral cond.

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- Important:

  $\longrightarrow$ no instanton effects have been included!

  $\longrightarrow$ environment dependence of the bare anomaly coefficient could be relevant!