Hadron Resonances – Quark Matter

viewed by nonlinear Regge trajectories

I. Szanyi[1,3,4], T.S. Biró[1], L. Jenkovszky[2] and V. Libov[2]

[1] Wigner Research Centre for Physics, Budapest, Hungary

[2]Bogoljubov Institute for Theoretical Physics, Kiiv, Ukraine

[3]Eötvös University Budapest,

[4]Carlo-Roberto University Gyöngyös

10 October 2023, ELTE Particle Physics Seminar Talk by TSB

My recent and previous works

that this talk utilizes

- I. Szanyi, T. Biró, L. Jenkovszky, V. Libov: *Nonlinear Regge trajectories and saturation of the Hagedorn spectrum* Phys.Rev.C. 107 (2023) 024904
- T. S. Biró, Z. Schram, L. Jenkovszky: *Entropy production during hadronization of a quark-gluon plasma* EPJ A 54 (2018) 17
- T. S. Biró, A. Jakovác: *QCD above T_c: Hadrons, partons and the continuum* Phys.Rev.D 90 (2014) 094029
- T. S. Biró, A. Peshier: *Limiting temperature from a parton gas with power-law tailed distribution* Phys.Lett.B 632 (2006) 247

Outline

1. Regge Trajectories Models
2. Counting Resonance States
3. Inside the proton
4. Relating to Quark Matter Thermodynamics

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Linear Regge trajectory

Strings rotating ends with lightspeed

Mass and angular momentum for homogeneous strings

$$M = \int_{-L/2}^{+L/2} \frac{\kappa dx}{\sqrt{1 - v^2(x)}}, \qquad J = \int_{-L/2}^{+L/2} \frac{x v(x)\, \kappa dx}{\sqrt{1 - v^2(x)}}.$$

Variational minimum, $M - \omega J$ min., at total stretch: $v(x) = \omega x$, $v(\pm L/2) = \pm 1$.

$$M = \frac{\kappa}{\omega} \int_{-1}^{+1} \frac{dz}{\sqrt{1 - z^2}}, \qquad J = \frac{\kappa}{\omega^2} \int_{-1}^{+1} \frac{z^2\, dz}{\sqrt{1 - z^2}},$$

Results in

$$M = \frac{\pi\kappa}{\omega}, \qquad J = \frac{\pi\kappa}{2\omega^2}, \qquad J = \frac{1}{2\pi\kappa} M^2 = \alpha M^2.$$

Energy-Angular Momentum limit

two massless quarks orbit, interacting by a linear potential

Meson mass $\approx$ total energy = kinetic + potential:

$$M = 2|p| + 2\kappa|r| \geq 2\sqrt{2\kappa|r| \cdot 2|p|}$$

It follows for the angular momentum

$$J = 2|p| \cdot |r| \leq \frac{M^2}{8\kappa} \approx 0.785\,\frac{M^2}{2\pi\kappa}.$$

Nonlinearity in Regge trajectories

$\alpha(s) = J/M^2$ is not constant

Collison of equal mass particles in CMS:
$p_1 = (\sqrt{s}/2, \vec{q}/2) \quad p_2 = (\sqrt{s}/2, -\vec{q}/2)$

With impact distance parameter b, orthogonal to $\vec{q}$:

$$J = 2\frac{b}{2}\frac{q}{2} = \frac{1}{2}bq \sim b\sqrt{s-4m^2}$$

Several of such threshold contributions deliver another Regge law:

$$\alpha(s) = \lambda - \sum_i c_i\sqrt{s_i - s} + \ldots$$

Many threshold $\alpha(s)$ model

Saturation of Re α and $N(m)$

$$\alpha(s) \;=\; \lambda - \sum_{s_i > s} c_i \sqrt{s_i - s} + \ldots - i \sum_{s_i < s} c_i \sqrt{s - s_i}.$$

The higher s the more terms in the Im part of the sum.

Mass density

$$\rho(m) \;=\; \mathrm{e}^{m/T_H} \frac{d}{dm} \mathcal{R}e\, \alpha(m^2) \;=\; \mathrm{e}^{m/T_H} \sum_{s_i > m^2} \frac{c_i m}{\sqrt{s_i - m^2}}.$$

Max s_i taken $\rightarrow$ max m^2 fixed $\rightarrow$ $\rho(m) = 0$ above $\rightarrow$ $N(m)$ saturates.

Re α also staurates, after the highest s_i is surpassed.

High energy tail properties

What a spectrum are we describing?

- Below the saturation of Re α, the exp factor dominates, $N(m)$ exponentially grows.
- $\mathcal{I}m\,\alpha(m^2)$ grows like m beyond the highest s_i.
- That means increasing widths of resonances; ending with a continuum.
- An overall positive $\mathcal{R}e\,\alpha > 0$ version (Jenkovszky)

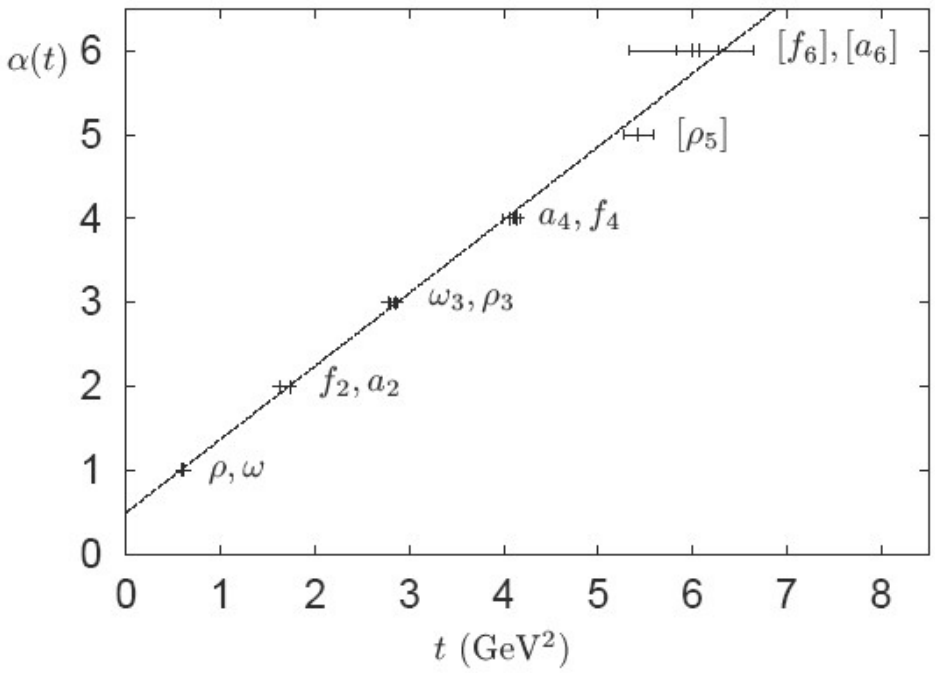
$$\alpha(s) = \frac{1}{1 + \frac{1}{\lambda}\sum_i c_i\sqrt{s_i - s}}$$

Experimental Mass Spectrum

for linear Regge trajectory model

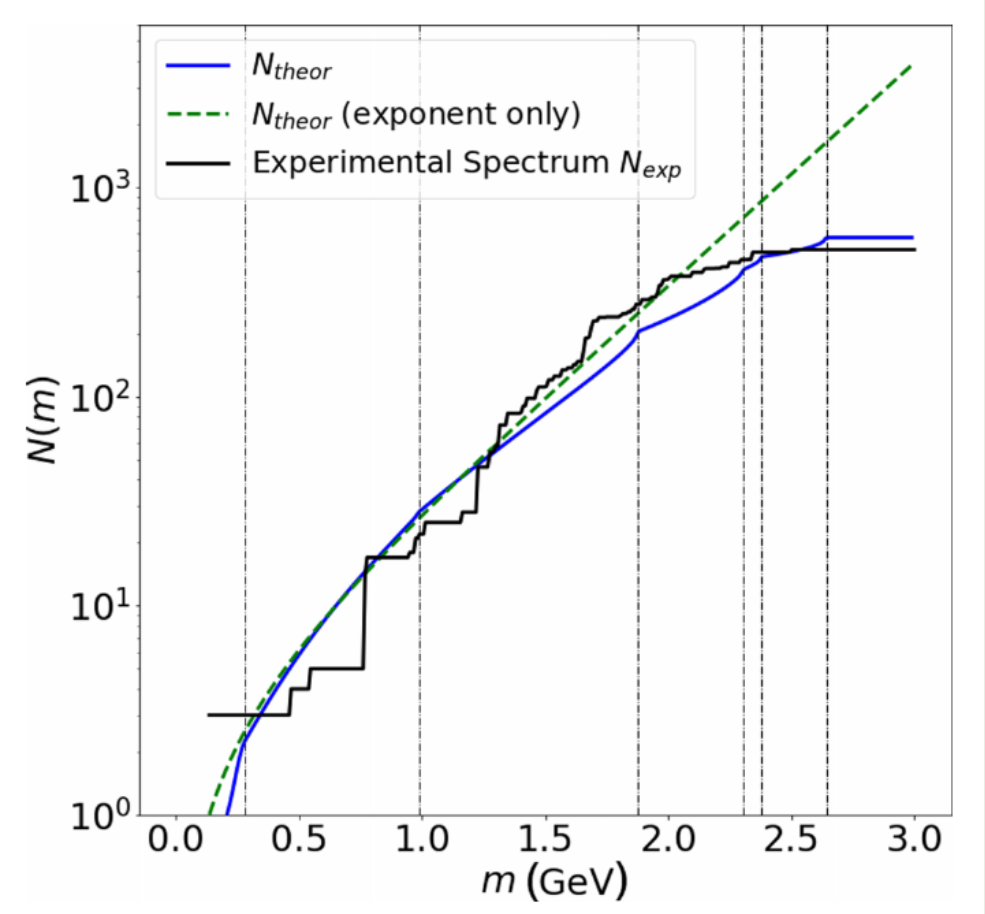
Linear particle trajectories

Plot of spins of families of particles against their squared masses:



Number of resonances below m

In the many threshold modified model



Unitarity: ”optical” theorem

amplitude, cross section, form factor

S matrix and T matrix: $S = 1 + iT, \qquad |out>= S|in>$

Unitarity: $SS^\dagger \;=\; (1+iT)\,(1-iT^\dagger) \;=\; 1 + TT^\dagger + i(T - T^\dagger) \;=\; 1$

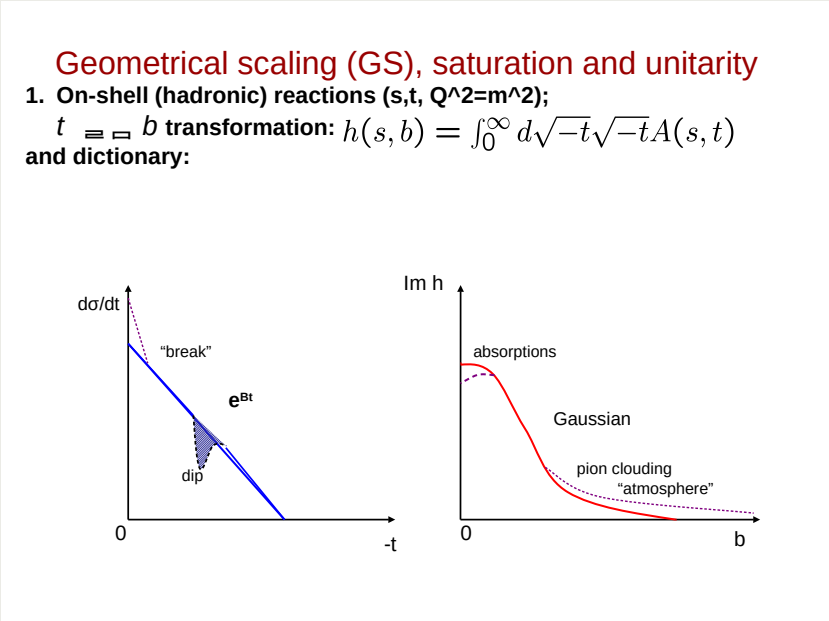
Consequence: $TT^\dagger = i(T^\dagger - T) = 2\,\mathcal{I}m\,T.$

In terms of cross sections:

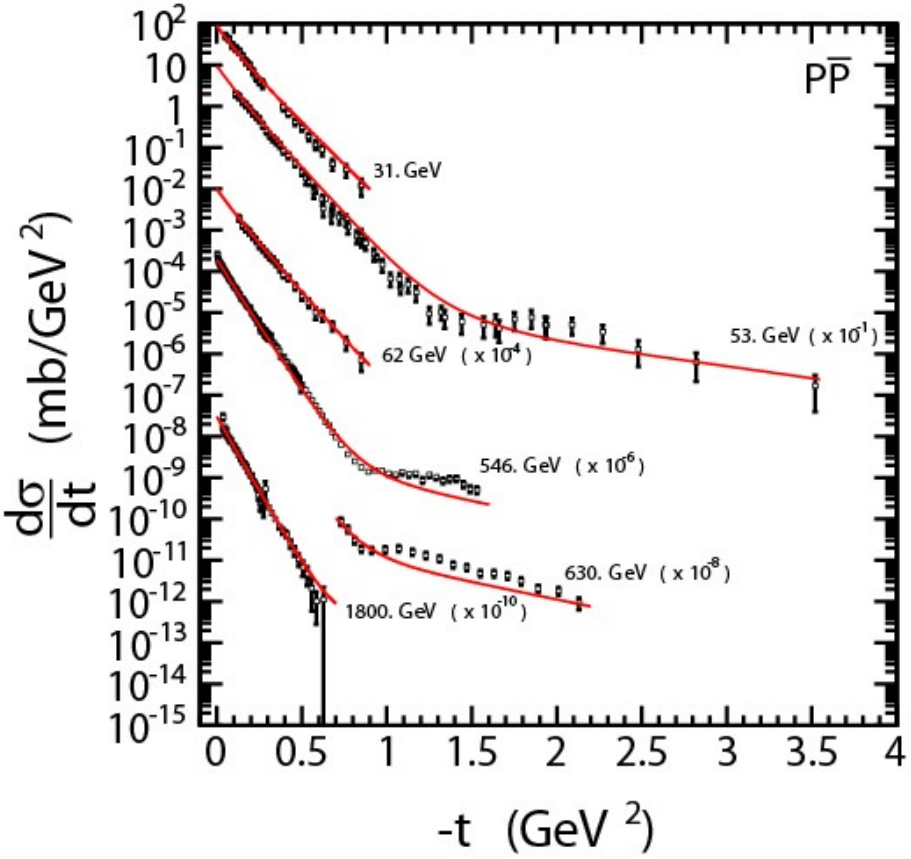
$$|out>= \mathrm{e}^{ikz} + \frac{\mathrm{e}^{ikr}}{r}\, f(\Omega)$$

$$\frac{d\sigma}{d\Omega} = |f(\Omega)|^2$$

$$\sigma_{\mathrm{tot}} = \frac{4\pi}{k} f(0)$$



Reggeon (Pomeron, Odderon)



Threshold Behavior

formula

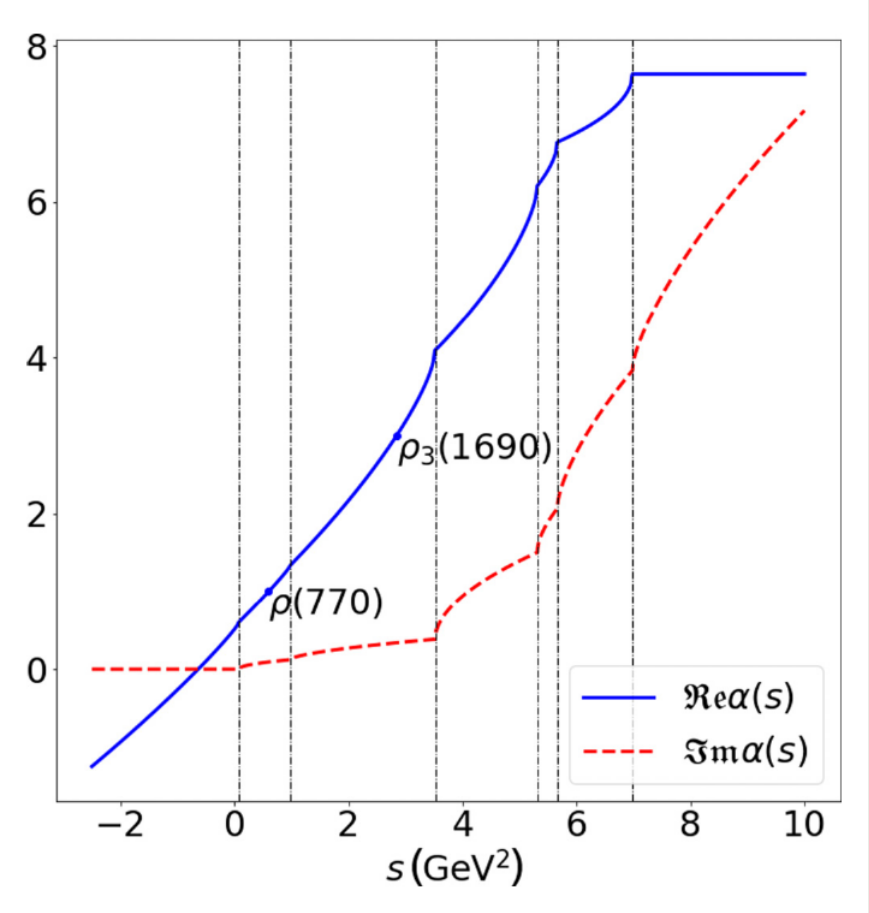
Specifically, Ref. [33] defines trajectories as

$$\alpha(s) = \lambda - \sum_i \gamma_i \sqrt{s_i - s}, \tag{10}$$

where all two-particle stable thresholds are included in the sum. For the ρ-meson trajectory, two meson-meson and four baryon-antibaryon channels are taken into account, as listed

Threshold Behavior

$\alpha(s)$



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3 Inside the proton

4 Relating to Quark Matter Thermodynamics

Hagedorn Mass Spectrum Forefactor

Deriving $\rho(m) \sim m^3$

It was suggested in Refs. [20–22] that for fixed isospin and hypercharge a cubic density of states, $\rho(m) \sim m^3$, fits the data. Moreover, as argued in Ref. [20], the cubic spectrum can be related to collinear Regge trajectories. Indeed, following the arguments of Burakovsky [23], on a linear trajectory with negative intercept, $\alpha(t) = \alpha' t - 1$, some integer values of $\alpha(t) = J$ correspond to states with negative spin, $J = \alpha(t_J)$, with squared masses $m^2(J) = t_J$. Since a spin-J state has multiplicity $2J + 1$, the total number of states with spin $0 \leqslant J \leqslant j$ at $t = m(j)^2$ is given by

$$N(j) = \sum_{J=0}^{j} (2J + 1) = (j + 1)^2 = \alpha'^2 m^4(j). \qquad (3)$$

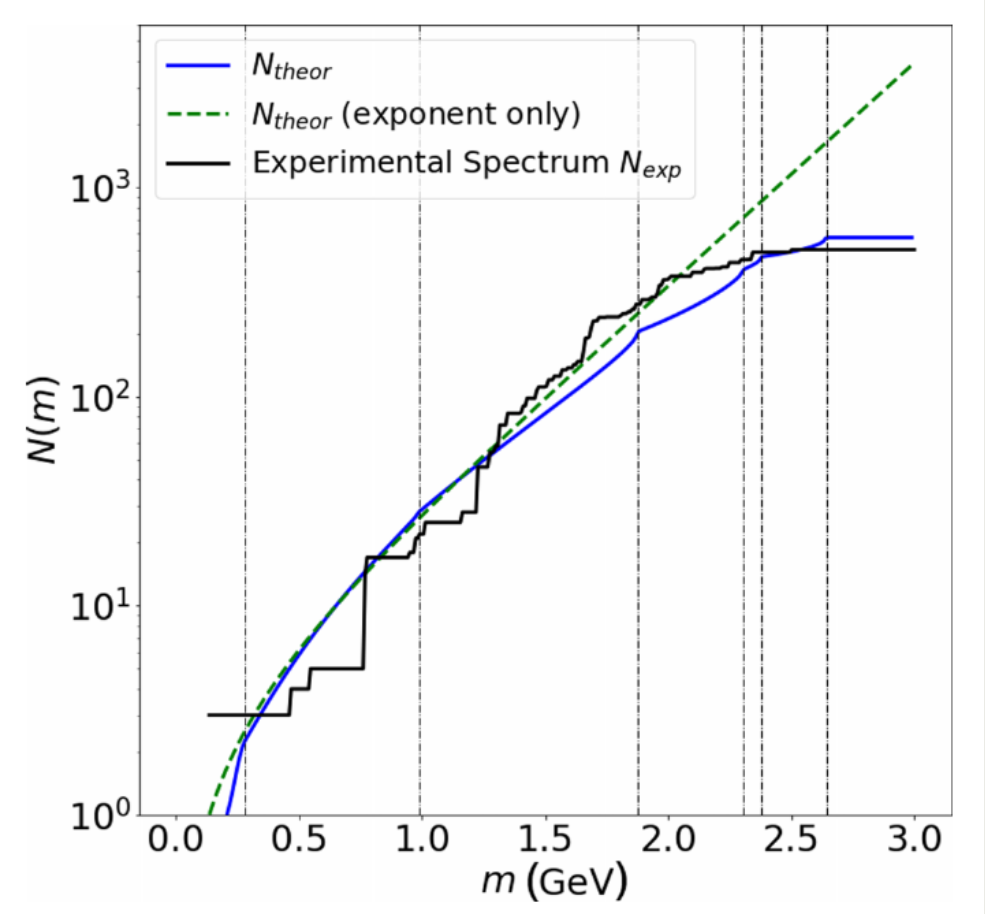
Hence, the density of states per unit mass interval is obtained as the derivative of this cumulative quantity,

$$\rho(m) = \frac{dN(m)}{dm} = 4\alpha'^2 m^3, \qquad (4)$$

and it grows as the cubic power of the mass. Consequently, for a finite number of collinear trajectories, N, the corresponding mass spectrum is given as

Number of resonances with mass below m

Saturation



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Inside the proton

mass, spin and pressure

$$\langle p' | T_{\mu\nu}(0) | p \rangle \;=\; \overline{u}(p')\,[M + J + D]\,u(p),$$

with $P = (p + p')/2$, $\Delta = p' - p$ and $t = \Delta^2, s = 4P^2$.

$$\text{mass} \quad \text{term} \qquad M = A(t)\frac{P_\mu P_\nu}{M_N}$$

$$\text{spin} \quad \text{term} \qquad J = J(t)\frac{P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}}{2M_N}\, i\Delta^\rho$$

$$\text{pressure} \quad \text{term} \qquad D = d(t)\frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N}$$

Generalized Parton Distribution

scatterer distribution in space

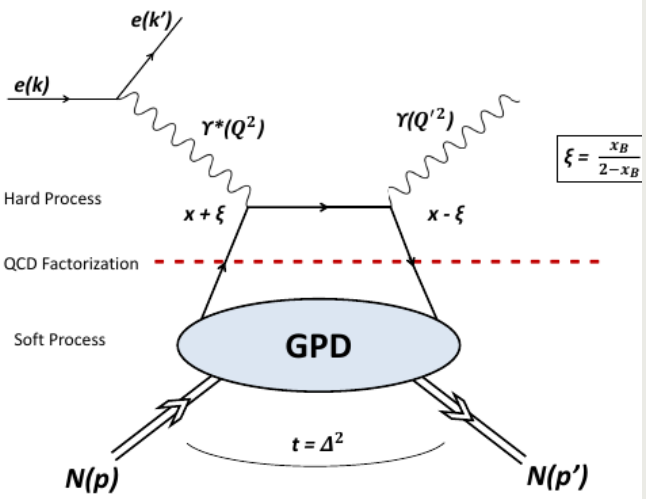


Figure 1: Diagram of deeply virtual Compton scattering (DVCS) with QCD factorizatic separating nonperturbative "soft" and perturbative "hard" dynamics.

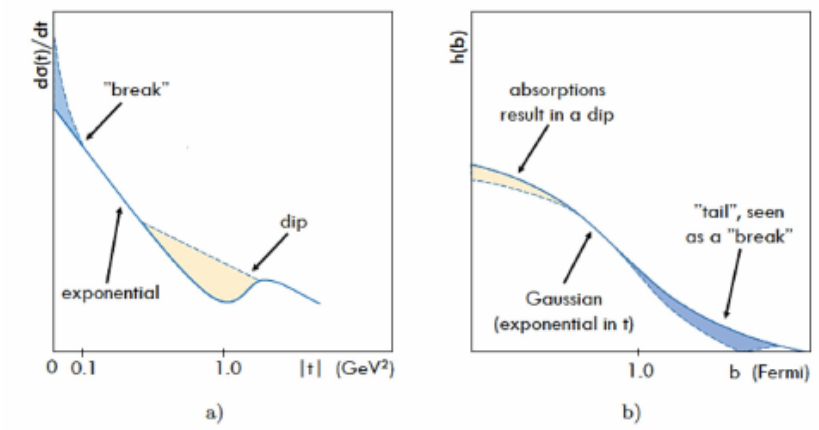
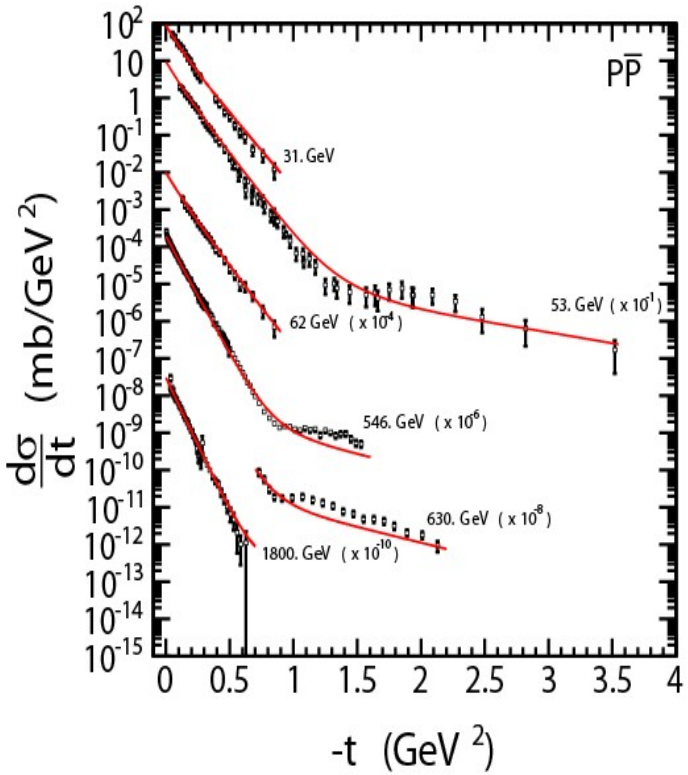
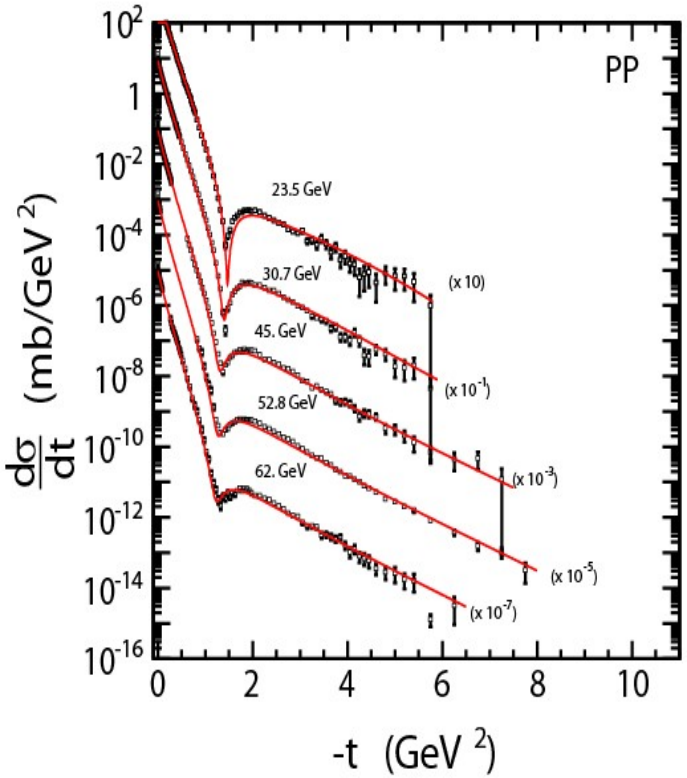


Figure 3: Typical pattern of high-energy proton-proton elastic differential cross-section (a) and of the corresponding proton profile, its F-B image (b). The small-$t \approx -0.1$ GeV^2 "break" is a consequence [19] of the threshold singularity in the trajectory/amplitude, as shown in the diagram, Fig. 2, producing the proton atmosphere (right).

Pressure inside the proton

experimental cross sections





Pressure inside the proton

R.Fiore, L.Jenkovszky, M.Oleksiienko; arxiv 2112.00605

On matter and pressure distribution in nucleons

$D(t) = \frac{18}{25} \int \frac{j_0(r\sqrt{t})}{2t} p(r) d^3 r,$ Fit to data: $D(t) \approx -\frac{36}{25}\left(1 - \frac{t}{25}\right)^3$

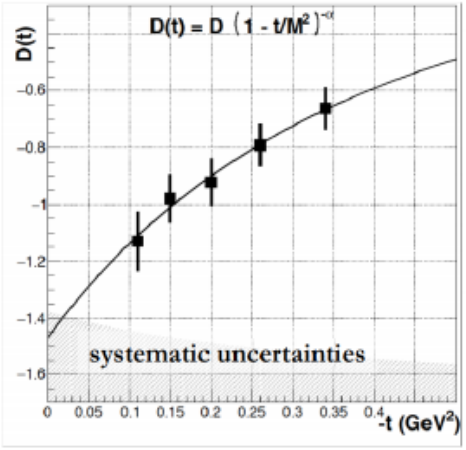


Figure 5: $D(t)$ from CLAS 6 GeV data [4]

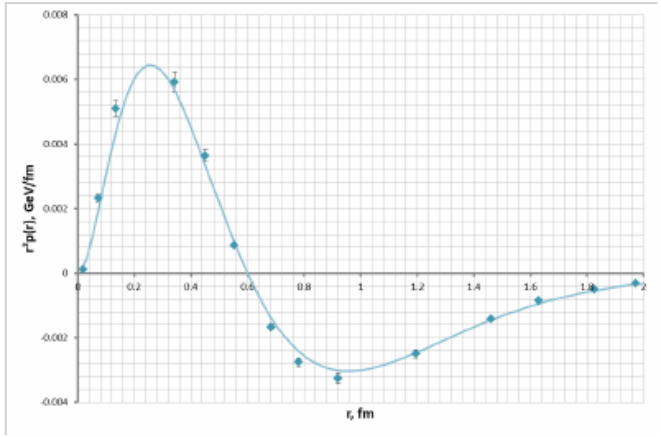


Figure 6: The curve $r^2 p(r)$, where $p(r)$ is the pressure distribution inside the proton according to the formula (26). It is compatible with the experimental data [4].

Fit to pressure: $p(r) = A(B - r)\mathrm{e}^{-r/a}$ with $A = 1 GeV/fm^4, B = 0.6 fm, a = 0.2 fm.$

Lattice QCD

measures pressure inside the proton

Pressure Distribution and Shear Forces inside the Proton
by P.E.Shanahan and W.Detmold.

Phys.Rev.Lett. 122 (2019) 072003

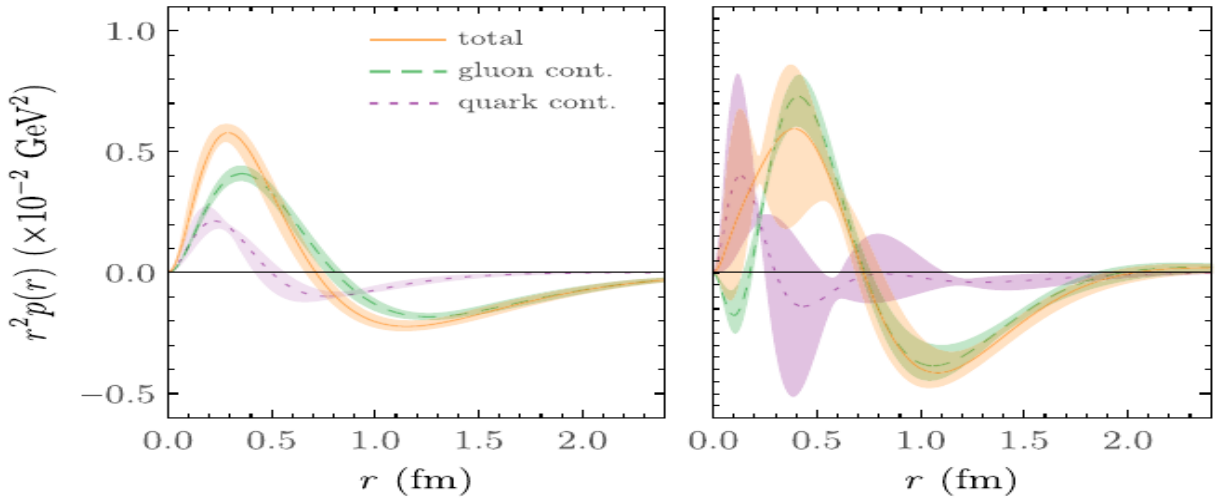


FIG. 2. (Left) Pressure distribution of the proton computed using tripole parametrizations of the LQCD quark D-term GFF and the LQCD gluon D-term GFF. The contributions from the quark and gluon terms are represented by the purple dotted and green dashed bands, respectively, while the total is denoted by the orange solid band. (Right) The same quantities, determined based on modified z-expansion parametrizations of the D-term form

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Mass Spectrum Formula

at finite temperature too

$$p = \sum_i g_i p(m_i) = \int_{M_1}^{M_2} dm\, \rho(m)\, p(m),$$

$$p(m) = \frac{T^2 m^2}{2\pi^2} K_2\left(\frac{m}{T}\right),$$

Improved (generalized) Bag Model

pressure and entropy density for $\rho(m) \sim m^k$

$$\epsilon = \int_0^\infty \rho(m) T^4 \sigma(m/T) dm = \lambda_k T^{k+5}, \qquad (13)$$

and obtain the corresponding pressure and sound velocity square as follows:[2]

$$p = \frac{\lambda_k}{k+4} T^{k+5}, \quad c_s^2 = 1/(k+4). \qquad (14)$$

Mass Spectrum vs Pressure

Meijer–Bessel transformation

Fájl Szerkesztés Nézet Előzmények Könyvjelzők Eszközök Súgó

Meijer transform - Encyclopedia of Mathematics – Mozilla Firefox

https://encyclopediaofmath.org/wiki/Meijer_transform

The Meijer K- transform (or the Meijer–Bessel transform) is the integral transform

$$F(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty K_\nu(xt)\sqrt{x}t f(t)dt.$$

If the function f is locally integrable on $(0, \infty)$, has bounded variation in a neighbourhood of the point $t = t_0 > 0$, and if the integral

$$\int_0^\infty e^{-\beta t}|f(t)|dt, \quad \beta > \alpha \geq 0,$$

converges, then the following inversion formula is valid:

$$\frac{f(t_0+0)+f(t_0-0)}{2} =$$

$$= \lim_{\lambda\to\infty} \frac{1}{i\sqrt{2\pi}} \int_{\beta-i\lambda}^{\beta+i\lambda} I_\nu(t_0x)(t_0x)^{1/2}F(x)dx.$$

For $\nu = \pm 1/2$ the Meijer K- transform turns into the Laplace transform.

The Meijer transform and Meijer K- transform were introduced by C.S. Meijer in [1] and, respectively, .

References

11:14

Lattice QCD EoS

Pressure → Mass Gap

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **32** (2006) S205–S212 doi:10.1088/0954-3899/32/12/S26

Equation of state for distributed mass quark matter

T S Biró, P Lévai, P Ván and J Zimányi

KFKI Research Institute for Particle and Nuclear Physics, H-1525 Budapest PO Box 49, Hungary

E-mail: tsbiro@sunserv.kfki.hu

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Online at stacks.iop.org/JPhysG/32/S205

Lattice QCD EoS

Pressure → Mass Gap

Here $\rho(m) = \frac{1}{T_c} f(\frac{m}{T_c})$ and $p(T) = \kappa T^4 \sigma(\frac{T_c}{T})$.

Meijer transform ($g = T_c/T$) and inverse transform ($t = m/T_c$):

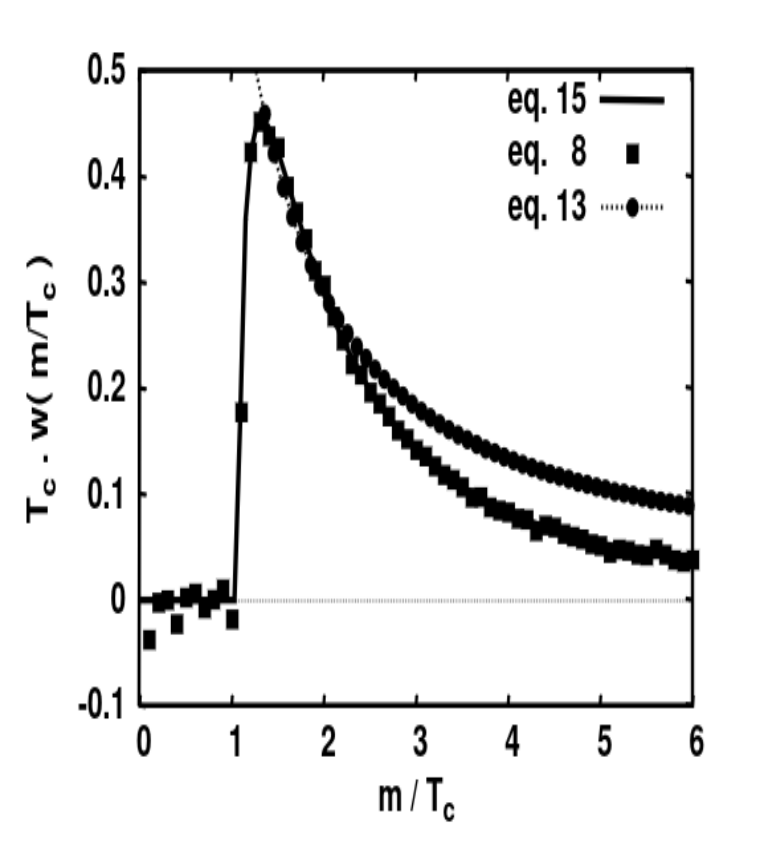
$$\sigma(g) = \int \rho(m) \frac{p(m,T)}{p_{SB}(T)} dm = \int_0^\infty f(t) \frac{g^2 t^2}{2} K_2(gt)\, dt.$$

$$f(t) = \frac{2}{i\pi} \int \sigma(z) \frac{I_2(zt)}{zt}\, dz.$$

Here $\sigma = p/P_{SB}$

Lattice QCD EoS

Pressure → Mass Gap



1. eq.8 squares: numerical back Meijer trf. from LQCD data
2. eq.15 cont. line: analytic formula
3. eq.13 dashed line w dots: pQCD to 4th order + LL

Summary

- Experimental Regge trajectories in fact are not linear ever.
- Bag constant is not a constant, $\rho(m)$ has an m^3 factor.
- Pressure inside the proton changes sign $\rightarrow$ negative at some temperatures.
- Lattice QCD qm data hint for a mass gap in terms of ideal gas eos.

Appendix

Mandelstam Variables

For $2 \to 2$ collisions; last column: equal masses case.

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = 2m^2 + 2E_1 E_2 (1 - \cos\Theta_{12}) \geq 4m^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 = 2m^2 - 2E_1 E_3 (1 - \cos\Theta_{13}) \leq 0$$

$$u = (p_1 - p_4)^2 = (p_3 - p_2)^2 = 2m^2 - 2E_1 E - 4(1 - \cos\Theta_{14}) \leq 0$$

Adding all six terms:

$$2(s+t+u) = 3(m_1^2 + m_2^2 + m_3^2 + m_4^2) + 2p_1p_2 - 2p_1p_3 - 2p_1p_4 + 2p_3p_4 - 2p_2p_4 - 2p_2p_3$$

$$2(s+t+u) = 3(m_1^2+m_2^2+m_3^2+m_4^2)+p_1(p_2-p_3-p_4)+p_2(p_1-p_4-p_3)+p_3(p_4-p_1-p_2)+p_4(p_3-p_2-p_1)$$

$$2(s+t+u) = 2(m_1^2 + m_2^2 + m_3^2 + m_4^2).$$

Property: $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 = 4m^2$;